Inverse and Reciprocity Methods for Machinery Noise Source Characterization and Sound Path Quantification Part 1: Sources

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In this article and in a forthcoming companion article some recently explored experimental approaches to the characterization of the noise source strength of machinery and to the ranking of transmission paths are reviewed. They form an addition to the more conventional approaches of the analysis of noise control problems in terms of source-transmission path-receiver schemes. In this first article source strength descriptors are defined both for airborne and for structure-borne sound. Their common basis is that the physical sources are modelled in terms of fictitious elementary sources, such as acoustical monopoles or mechanical point forces. In three of the four methods discussed, the strength of these equivalent substitution sources is determined indirectly. Advantages as well as limitations are considered. One practical advantage concerns the feature that the proposed descriptors are much less affected in strongly different installation environments than the more conventional source strength descriptors. Another practical advantage is that the use of elementary substitution sources as source models facilitates the very convenient application of experimental reciprocity techniques for transmission path ranking. Validation experiments related to applications in ships, road vehicles and office machines are briefly discussed.

1. INTRODUCTION

A conventional approach for the analysis of noise control problems uses the source-transmission path-receiver scheme. In many situations this type of analysis is complicated by the fact that there is a multitude of simultaneous sources. Moreover, even for a single noise source like a machine, there can be a multitude of partial sources and "parallel" transmission paths. Therefore, the development of cost-effective noise reduction strategies often requires detailed knowledge of the contributions of the partial sources and transmission paths to the radiated sound at distant receiver positions. Generally speaking, this type of analysis requires a system modelling in terms of inputs I, which genuinely characterize the sources themselves and of output-input ratios O/I. These latter are transfer functions TF, which characterize the transmission paths. Such an analytical approach facilitates the effective specification of required noise source quietening and of improved sound or vibration isolation in one or more paths.

The treatments in this article will be limited to systems with supposedly linear behaviour, i.e. to systems for which the output response may be modelled as a linear superposition of all contributions of the partial sources and transmission paths. Then in loose mathematical terms the following model equations apply:

- *single source, single transmission path:*

$$O = TF \times I \tag{1}$$

- *single source, multiple transmission paths, e.g.:*

$$O = \sum_{j=1}^{m} \left(TF_j \times I \right) \tag{2}$$

- *multiple sources, multiple transmission paths, e. g.:*

$$O = \sum_{i=1}^{n} \left\{ \sum_{j=1}^{m} \left(TF_{ij} \times I_i \right) \right\}. \tag{3}$$

As is conventional practice for linear system analysis, frequency domain formulations will be used in this article. However, the above equations may then still represent a variety of models. They cover, for example, discrete frequency formulations, which take into account all relevant phase relationships. But they also cover relative bandwidth formulations (e.g. 1/3-octave bands), which use mean squared bandfiltered inputs and outputs and frequency band averages of the squared magnitude of frequency response functions $TF(f)$.

This first article is devoted to a discussion of some rather unconventional source descriptors I, for situations where more conventional approaches become rather impractical or are not compatible with practical methods for transfer function determination.

In a second article some applications of these source strength descriptors to the problem of path ranking will be discussed[1]. In that article the experimental elegance will be underlined of reciprocity measurements for the determination of transfer functions TF, which are compatible with the newly defined source descriptors I .

2. SOURCE CHARACTERIZATION WITH ACOUSTICAL MONOPOLES

The most widely used methods for determining airborne sound source strengths, measure either sound pressure levels at prescribed distances from a machine or radiated sound power. These quantities may be relevant for rather general purposes, such as product specification or meeting legislative requirements. But they are often unsuitable for the type of diagnostic analysis expressed by equations (1)-(3). One reason may be a large variation of the source strength quantities under modified installation environments. Another reason may be the lack of suitable transfer functions which are compatible with such source strength descriptors. The successful use of loudspeakers as substitution sources for diagnostic analysis of airborne sound transmission is limited to cases in which the spatial reproduction of the original sound field is simple (e. g. for relatively small machines in reverberant spaces). To avoid these practical shortcomings, unconventional methods have been developed. They use source models with fictitious monopole sources, which are distributed over the radiating surfaces. These can be either correlated or uncorrelated.

2.1. Method of correlated equivalent monopoles

In figure 1 the principle is shown of a method proposed by Mason and Fahy[2]. It is based upon Cremer's description of a synthesis method using directional Green's functions[3]. It uses the velocity distribution over the sound radiating surface as a source strength descriptor. The machine's surface is divided into incremental areas ΔS_i with normal velocity v_i. Each sub-area is seen as an acoustical point source with volume velocity $Q_i = v_i dS_i$. The source velocity v_i can be measured on each sub-area, for example, with the machine installed on a test bed. Transfer functions from each point source to the far field receiver position depend on their acoustical surroundings. They need to be determined experimentally with the source in situ. These measurements become practical even for sources in small spaces, like within an enclosure, when the principle of reciprocity is used. In the reciprocal experiment (see Fig. 1b) an omnidirectional source with volume velocity Q'_r is placed at the original receiver point and the sound pressures p'_i are measured on each subarea of the passive structure. The tacit assumption behind this application of reciprocity is that the small fictitious piston sources on the source surface may be replaced by fictitious monopoles directly against this surface. With a proper choice of the mesh size this is acoustically correct in most practical situations. The partial contribution to the far field radiated sound due to the airborne sound radiation from the engine or from certain parts of it, may be obtained by using the following variant of Eq. (2):

$$p_r = \sum_i (v_i \times \Delta S_i)\left[\frac{p'_i}{Q'_r}\right] . \quad (4)$$

If the transmission path is changed, e.g. by altering an enclosure, the source part of Eq. (4) remains unaffected, but the transfer function part has to be determined for the new situation.

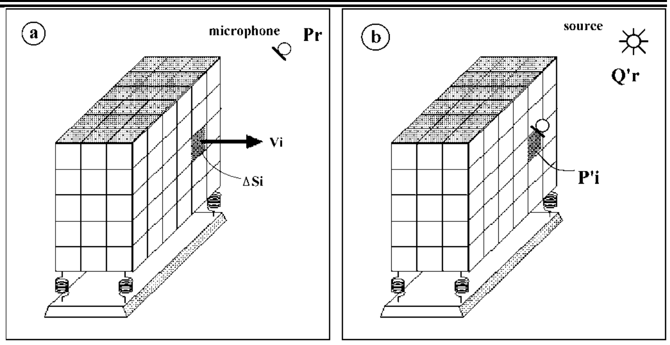

Figure 1. Method of correlated monopoles. a) piston source model and "direct" measurement of transfer functions. b) reciprocal measurement of transfer functions.

Concerning the practicability of this method, one may expect that at low frequencies and for simple vibration patterns the method may work well. For example, for the analysis of interior noise in cars caused by the rather low frequency rigid body vibrations of the car engine, the method is expected to be far superior to the use of a single loudspeaker source as substitution source[4]. But, on the other hand, for high frequencies and for complex structural shapes and vibration fields, the large amount of data needed, makes this method impractical. Also non-steady sources, like vehicle engines under run-up conditions, cannot be handled because the phase relations between the various v_i cannot be defined.

2.2. Method of uncorrelated equivalent monopoles

A method which models the acoustical source using uncorrelated monopoles on the surface of the vibrating structure has been proposed by Verheij [5-7]. This method has the advantage that in principle fewer measurements are needed than for the correlated monopole method and that phase can be neglected. The advantage of reciprocal measurements of transfer functions is retained. In contrast with the correlated monopole method, the accuracy for the uncorrelated monopole method is expected to improve at higher frequency and for complex vibrating structures. For compact radiating structures (i.e. with dimensions small compared to the wavelength of sound) often the sound radiation is rather directional. In such cases, of course, neglecting phase would be erroneous.

2.2.1. Source strength definition of uncorrelated monopoles

The first step is to divide the radiating surface into m sub-areas, which are considered as partial sources, see Fig. 2a. On truck engines, for example, such parts can be valve and distribution covers and oil sumps. The assumption is made that on each sub-area the sound radiation of the structure may be replaced by that of $n(j)$ uncorrelated monopole sources each with the same strength. The acoustical source strength of each sub-area is defined as the total squared volume velocity of the $n(j)$ monopole sources, i.e.

$$Q_{eq}^2(j) = n(j) \times Q_{eq,i}^2(j) \quad (5)$$

The question of whether or not this equivalent source strength is independent of the acoustical surroundings will be addressed later. First it will be described how the transfer functions between the partial sources and a receiver position are defined and can be measured.

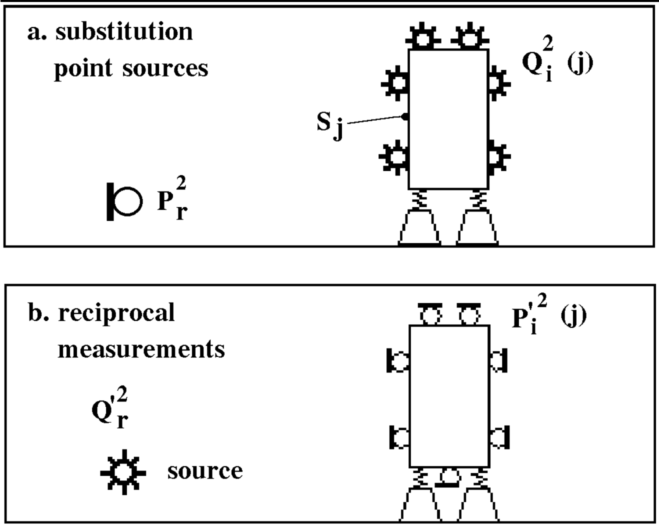

Figure 2. Method of uncorrelated monopoles. a) subdivision in partial source areas S_j. b) reciprocal measurement of transfer functions.

2.2.2. Transfer functions

The transfer function between a sub-area and a receiver position is defined as the average transfer function over the $n(j)$ monopole positions. Because direct measurement of the transfer functions is often too difficult, the reciprocity principle is applied in the same way as mentioned for the correlated monopole method, as follows (see Fig. 2b):

$$T_{j,r} = \frac{1}{n(j)} \sum_{i=1}^{n(j)} \left[\frac{p_i'^2}{Q_r'^2(j)} \right] \tag{6}$$

The radiated sound due to the m radiating sub-areas is found from

$$p_r^2 = \sum_{j=1}^{m} p_r^2(j) = \sum_{j=1}^{m} T_{j,r} \times Q_{eq}^2(j) \tag{7}$$

As in equations (1)-(3), Eq. (7) shows clearly the distinction between source strengths and transfer system properties. The acoustical strengths of the uncorrelated monopoles can be determined in different manners. Two of them are briefly discussed now.

2.2.3. Determination of the source strength of uncorrelated monopoles

One method of determining the source strength, as defined in Eq. (5), is from sound intensity measurements[5]. This is valid in situations in which the source radiates about the same power as in the free field (e.g. on an engine test rig). For each sub-area the radiated sound power can be determined from sound intensity measurements on a measurement plane close to it. Now the equivalent volume velocity is found from equating the measured sound power with the estimated power radiated by the uncorrelated monopoles. Assuming that for most of the fictitious point sources, the radiation resistance equals that for a monopole on an acoustically hard baffle and radiating into a half-space, it may be easily proven that

$$Q_{eq}^2(j) = n(j) \times Q_{eq,i}^2(j) \approx P(j) \frac{2\pi c}{\rho\omega^2} \tag{8}$$

where c denotes the speed of sound in air, ρ the density and ω the radian frequency.

Recently an alternative method for determining the equivalent volume velocities was investigated[7]. The reason was the impracticality of the sound intensity method in the case of rapidly changing sources. An example of such conditions is a vehicle engine on a test rig under fast run-up conditions, which are representative for pass-by noise tests. For that case measurements of sound intensity at a large number of discrete points would necessitate a large number of repeatable runups, which was considered impractical. In the local enclosure method the partial source $S(j)$ under consideration, radiates sound into a temporarily attached local enclosure, the interior space of which is effectively shielded from the other parts of the engine. The method consists of three steps. First, when the engine is running the mean square sound pressure $p_{1,k}^2$ inside the enclosure is measured at q different positions, see Fig. 3a. Next it is assumed that the real source can be replaced by $n(j)$ uncorrelated monopoles, see Fig. 3b. The transfer functions between monopoles and microphones may be written as

$$H_{i,k}^2(j) = \frac{p_{2,k}^2}{Q_i^2(j)} \tag{9}$$

where for the sound pressure the index number 2 is used to distinguish from measurements with the engine in operation (Fig. 3a). Again it turns out to be practical to measure these transfer functions reciprocally (see Fig. 3c). From each of the q microphone responses in Fig. 3a a raw estimation of the partial volume velocity may be obtained. However, a more smoothed estimation follows from averaging over all q available microphone responses in Fig. 3a as follows:

$$\hat{Q}_{eq}^2(j) = \frac{n(j)}{q} \sum_{k=1}^{q} \left\{ p_{1,k}^2 \left(\sum_{i=1}^{n(j)} H_{i,k}^2(j) \right)^{-1} \right\} \tag{10}$$

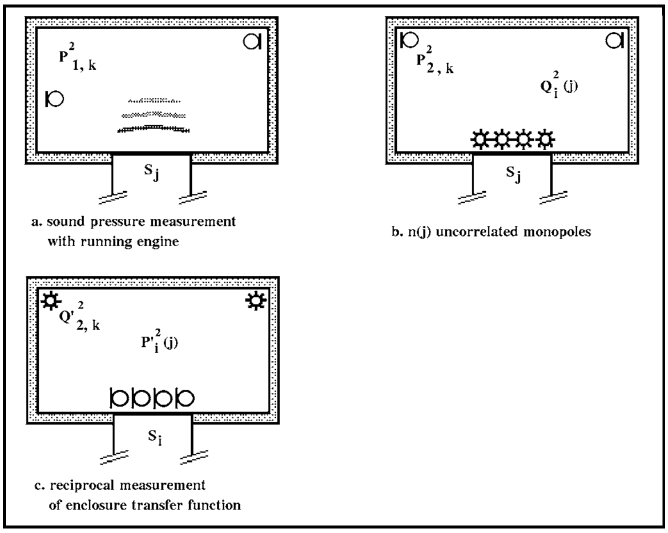

Figure 3. Principle of determining the equivalent volume velocity using transfer functions within a locally attached enclosure.

Of course, estimations of the source strength according to both methods, i. e. according to Eq. (8) and Eq. (10) should be consistent.

2.2.4. Validation studies

"Mathematical" and physical experiments have been performed to investigate the validity and feasibility of the proposed source modelling method.

A laboratory experiment is described on the airborne sound transmission from an engine simulator into a water tank[5]. The purpose behind this validation experiment, was the application of the proposed source descriptor method in ships. There the objective is to determine the contribution of airborne sound transmission from machinery to the underwater sound. In the laboratory experiment, the underwater sound pressure levels, which were solely determined by airborne sound transfer, were compared with the predictions on basis of Eq. (8).

The same ideas have been applied on an internal combustion engine in an automotive test room[8]. Again the purpose of the experiments was to compare measured sound pressure levels with predictions from Eq. (7).

In both publications the equivalent source strength was determined with the intensity method and in both cases the agreement between measured levels and the predicted results according to Eq. (7), was quite good. This holds[8] for $400\ \text{Hz}<f<2000\ \text{Hz}$, even in narrow bands, whereas it holds[5] for one-third octave results for $200\ \text{Hz}<f<4000\ \text{Hz}$. From these experiments with a combustion engine and with an engine simulator, both installed in relatively large spaces, the uncorrelated monopole method appears to provide a valid source strength descriptor.

With the application for truck engines in mind, the question has been studied, whether or not this would remain the case when a source is installed in a completely different acoustical environment, e.g. inside a rather tightly fitting enclosure. Some investigations on this aspect have been reported[6]. These concern both "mathematical experiments" and laboratory experiments. The latter were done with an engine simulator, without and with enclosures.

In the mathematical experiments a series of computations have been performed on the sound radiation of baffled plates into a half-space and into a shallow cavity. Generally speaking, these computational results were quite promising in supporting the hypothesis that the uncorrelated monopole source strength is invariant for drastic changes in radiation resistance of the plates.

The laboratory experiments with an engine simulator without and with enclosures will be discussed in the article on transmission path quantification[1]. The results from an analysis on basis of Eq. (7) imply that the proposed source strength descriptor remains unaltered when the acoustical surroundings of the engine simulator changes drastically.

Source strength data have been compared, which were obtained with both the intensity method and the local enclosure method[7]. This was done with an engine simulator with which the fast run-up of a truck engine during a pass-by test was simulated. These tests were part of a research project to apply the type of analysis according to Eq. (7) to heavy road vehicle engines. For that application it is the intention to use source strength data determined on an engine test rig. Figure 4 shows the equivalent source strength levels of the oil sump of the engine simulator, determined from three different experiments. Because of the transient nature, the source strength varies with time. The data shown correspond with time averages over 1/16 s at apparent engine speeds of about 1000 and 1625 r.p.m. The results derived from sound intensity measurements were obtained from measurements at 32 measurement points rather close to the oil sump. The results obtained from the local enclosure method (see Eq. (10)) were obtained from experiments with two different enclosures. One had a volume of 0.3 m^3, the other 1.2 m^3. In both enclosures 4 microphones were placed in corners, as in Fig. 3a. In the reciprocity experiments 22 microphones were positioned against the oil sump, as in Fig. 3c. The miniature sound source, which was used for these reciprocal experiments, has been described[9].

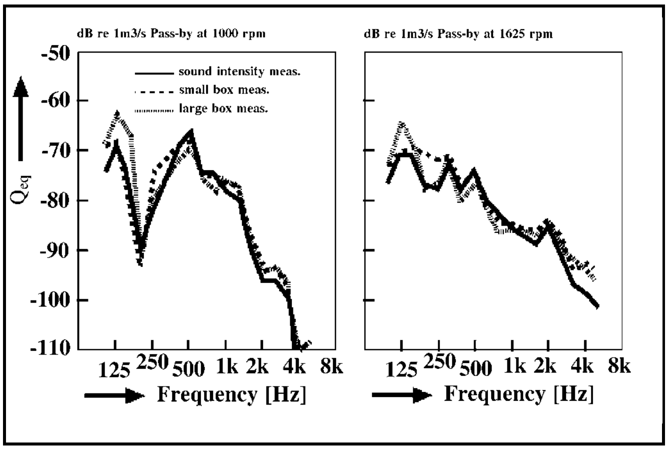

Figure 4. Equivalent volume velocity levels (1/3-octaves) of truck engine oil sump, determined from three different experiments.

It is seen in Fig. 4 that for $315\ \text{Hz}<f<3150\ \text{Hz}$ (the most important frequency range for exterior noise from trucks) there are only minor differences in the source strength estimates.

Also measured and calculated data are reported for the A-weighted sound pressure levels as a function of running speed[7]. For the calculations according Eq. (7), the three with speed varying source strengths spectra were used, from which the results in Fig. 4 form only a small part. The predicted and measured levels are equal within 1.5 dB(A) for the apparent speed variation from 1000-2250 r.p.m. Because the sound spectrum of the engine simulator was made equal to that of a genuine truck engine, this close agreement seems to be a representative result.

2.3. Discussion

The foregoing discussions of source characterization were limited to machines or machinery components. However, the potential application of the methods is, of course, much wider. For example, the boundaries of a vehicle interior may be considered as partial sources in a way analogous with machinery surfaces. Then the corresponding transfer functions are, for example, those between the various boundary parts and the ear positions of the drivers seat.

One of the attractive features of both equivalent monopole concepts is their compatibility with transfer functions which can be determined by rather simple reciprocal experiments.

In cases where the phase relationships cannot be ignored (i.e. the correlated monopole method), there is a practical need for further development of measurement methods for volume velocity of vibrating structures. Recent work on this topic has been reported[10,11].

The uncorrelated monopole method is attractive for its relative simplicity, especially for radiators with complex shape and vibration behaviour. Recent research has also shown its elegance as input quantity for numerical modelling of sound transfer problems[12].

3. SOURCE CHARACTERIZATION WITH MECHANICAL POINT FORCES

3.1. Survey of the problem

The most widely used methods for determining structure-borne sound source strengths, involve measured vibration levels. Often these are measured at positions close to where the sources are connected to the receiver structures. However, here the problems are even greater than for airborne sound sources. Typically there is a large variety of receiver structures to which a source may be connected. This causes a big scatter in source vibration levels in different surroundings. Another complication is caused by the fact that the vibrational response, even at a single point is characterized by six motional degrees of freedom. This does complicate the definition of source strength descriptors with broadly valid values. It also restricts the development of analysis procedures of the type expressed by equations (1)-(3).

An exception is the case of resiliently mounted machinery[13]. If the vibration isolators are sufficiently soft, the vibration levels on the machine side of the isolators will be virtually equal to the free vibrations. They can be measured for all relevant degrees of freedom, either retaining or ignoring phase relationships. To quantify the transmission via vibration isolators to a far field position, transfer functions *TF* are needed, which may be combined with these source inputs. These are found by multiplying transfer impedances of the isolators with transfer functions of the receiver structure, see e.g.[4,14,15]. The transfer functions of the receiver structure are usually of the type (p_r/F) and are often measured reciprocally as (v/Q_r'). In such a reciprocal measurement an omnidirectional sound source with volume velocity Q_r' is placed at the original receiver point and the free velocity v caused by this source is measured at the position of the original excitation force and in the same direction as this force.

For situations with strong coupling between source and receiver structures, the above approach is impossible. Conventionally, for linear source and receiver structures, the complex interaction between source and receiver is analysed with impedance-type methods. Source characterization methods, which belong to this category are the multidimensional mechanical equivalents of the Thévenin or Norton concepts from electrical network theory. These methods consider the mechanical forces and velocities that determine the interaction between the source and the receiver structure at their connecting interfaces.

Mondot and Petersson[16] have formulated this approach in a special manner, namely in terms of the power flow from a source into a receiver. This power flow is expressed as being determined by the product of two factors, namely a source descriptor and a coupling function. The source descriptor contains solely source properties, namely free (i.e. dynamically unloaded) source terminal velocities and corresponding source mobilities. The coupling function contains source as well as receiver mobilities. When analyzing the transmission to the far field with this method, a variant of Eq. (1) has to be used. Thus in addition to the power which is injected into the receiver structure, a transfer function *TF* for the receiver structure is needed, for example of the type $P_{rad} / P_{mech,in}$. Input power estimation requires determination of the "free velocities" of the source, either directly or indirectly. Generally speaking, also mobility measurements at all connection points and for all relevant vibration directions are needed. Because of the complexity of this task, past and current research aims at demonstrating the validity and applicability of simplified versions of this method for typical applications, see e.g.[17]. Nevertheless, especially in cases where strong coupling exists between adjacent connection points or between translational and rotational degrees of freedom at the same point, the mobility measurements will be often too demanding to become practical outside the laboratory. Moreover, the measurement of free source velocities is often completely impracticable.

A completely different approach to the problem circumvents the analysis of the complex interaction phenomena in the contact areas between source and receiver structures. This approach defines fictitious mechanical point forces as substitution sources which act upon the outside of a machinery casing or body. In this way the source structure becomes part of the transmission system. Except from this crucial difference, this equivalent source modelling approach is rather similar to that described earlier in the previous chapter on airborne sound. In a similar way as for the equivalent monopole concept, the source modelling by mechanical point forces facilitates the use of structural-acoustic reciprocity techniques for the measurement of transfer functions. Again two variants of the concept have been explored, one using uncorrelated equivalent point forces and the other using correlated point forces.

3.2. Methods of uncorrelated equivalent point forces

Source modelling with uncorrelated point forces has been extensively used in transmission path analysis of machinery noise in ships in relation to underwater sound. This concerned mainly experiments with multi-path sound transmission in which the use of a machine in operation was impracticable. Examples are scale model studies of insertion losses of alternative resilient mounting system configurations and shipboard path ranking experiments while blocking transmission in one or more paths. These applications were concerned with rather large machines such as diesel engines and main gear boxes. This implies that in the frequency range of interest a large number of resonant vibration modes of the source structure determine its vibration response.

For such machines it appears rather well possible to replace the complex internal excitation mechanisms by a rather limited set of artificial external forces, which together are able to reproduce the far field sound and (or) structural vibrations, which were originally generated by the running machines. Such equivalent forces can be found by using Eq. (1) in an 'inverse' manner, i.e.

$$F_{eq} = TF^{-1} \times O \tag{11}$$

One requirement for a proper source descriptor is that equivalent forces obtained according to Eq. (11) ought to be invariant for different receiver positions where O can be measured, for example, in air or on structures. The source descriptor should also be invariant for the way in which the machine is installed. Thus it should be invariant whether mounted rigidly or resiliently and whether in a ship or on test rig in the machine factory. The validity of such assumptions have been confirmed to some extent for a number of diesel engines by Verheij[18]. However, the values of the equivalent forces may be different at different positions of the machine. One observation reported[18] is that for similar excitation positions distributed over the length of a cylinder block of an auxiliary diesel engine the equivalent forces are closely equal. But another observation reported in the same paper shows that values of equivalent forces averaged over positions on the cylinder head will differ sometimes from the averaged value on the cylinder block. A plausible explanation is that for excitation of a reverberant machine structure, the values of the equivalent forces will be different for positions with significantly different driving point mobilities. However, it is expected that frequency band averaged equivalent forces inject roughly the same power at positions with different point mobilities, as long as they are not applied close to the contact points with the receiver structure. Therefore, an equivalent power variant has been suggested as a more spatially invariant source quantity[19,20]. It is obtained by multiplying the equivalent forces obtained from Eq. (11) with the real part of the driving point mobilities at the corresponding fictitious excitation positions.

3.3. Method of correlated equivalent point forces

In practice important categories of small size and compactly built machines behave as 'compact' sources. Their vibrations are by definition determined by a small number of rigid and (or) non-rigid body modes. For such machines the method of correlated equivalent point forces has been proposed as source descriptor by Verheij et al.[21] As in the case of uncorrelated forces the internal excitation of a machine is modelled by means of a fictitious set of external forces exerted upon the housing of the machine. But in contrast with the uncorrelated point forces concept, now the directions of the equivalent forces and their phase relationships form essential elements of the source descriptor. Again these are called 'equivalent', if together they can reproduce the original vibration fields, both on the source structure and on receiver structures. And an 'ideal' set of forces would be independent of the installation environment (i.e. boundary conditions of the source). In this section a brief outline of the method will be given, together with some preliminary results and with a discussion of some fundamental and practical aspects.

3.3.1. Determination of forces

The procedure for determining the correlated equivalent point forces is a variant of an inverse method for force identification. The special feature is that it is concerned with a fictitious set of forces and that, in principle, there may be many different sets of pseudo-forces, which fulfill the requirements of equivalence. The steps of the procedure are shown in figure 5, for a source connected to a receiver structure.

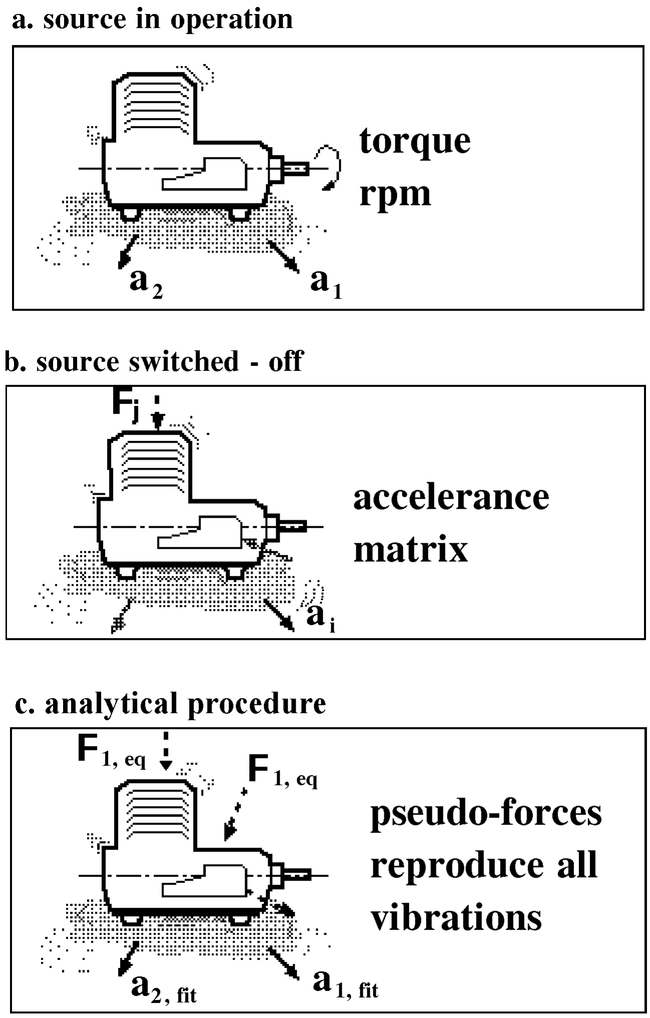

Figure 5. Procedure for determination of pseudo-forces: a) Measure accelerations **a**; b) Measure accelerance matrix **A**; c) Determine pseudo-forces $\mathbf{F}_{pseudo}$.

Step 1: Put the source in normal operation and measure a vector **a** of (translatory and/or rotatory) accelerations a_i at m positions, including their phase. These positions may be on the source structure, on the receiver structure or on both.

Step 2: Switch the source off. Select n positions on the source structure, where forces and/or torques F_j can be applied. Measure the $m \times n$ accelerance matrix **A**, which contains the complex frequency response functions a_i/F_j.

Step 3: Determine the vector $\mathbf{F}_{pseudo}$ of n equivalent pseudo-forces analytically. If, as is preferable, $m>n$ is taken, the pseudo-inverse $\mathbf{A}^+$ is used as follows[22]:

$$\mathbf{F}_{pseudo} = \mathbf{A}^+\mathbf{a} \tag{12}$$

This procedure provides a least squares solution for an overdetermined problem and may be considered as just another variant of Eq. (1). Still this procedure will lead to unstable solutions if **A** has a large condition number. Therefore, a well-known procedure based on singular value decomposition is used to avoid the construction of a pseudo-inversion of an ill-conditioned matrix[22]. This procedure is as follows. The $m \times n$ matrix to be inverted can be written as

$$A = USV^H \tag{13}$$

The $m \times n$ pseudo-inverse can be constructed as follows:

$$A^{+} = VS^{+}U^{H} \tag{14}$$

For $m>n$ the matrix **S** is an $n \times n$ diagonal matrix of non-negative real numbers. These are called the singular values of **A**. The number of non-zero singular values equals the rank of **A**, whereas the ratio of the largest and smallest non-zero singular value equals the condition number. Therefore, relatively small singular values make (pseudo-)inversion of a matrix ill-conditioned. A stable and more reliable solution can be found by setting some of the smaller singular values equal to zero before calculating the pseudo-inverse. The remaining number of singular values equals the rank of $\mathbf{A}^{+}$ and is called 'effective rank' of **A**. Following Powell[23,24], singular values will be set equal to zero if they are smaller than a threshold ε. This is the norm of an error matrix **E**, where the elements of **E** represent the random errors in the elements of **A** as follows:

$$E_{ij}(f) = 3\sqrt{\left\{\frac{1-\gamma_{ij}^{2}(f)}{2n_{av}\gamma_{ij}^{2}(f)}\right\}}\left|A_{ij}(f)\right| \tag{15}$$

This formula incorporates the coherence function and the number of samples over which an average is taken in measuring the accelerances. The factor three originates from the fact that this estimate for the error is taken as three times the supposed standard deviation.

Selection of positions and of directions of pseudo-forces. In practice the questions have to be answered, how many pseudo-forces are needed and where. If a source vibrates as a rigid body, any internal excitation mechanism can be modelled with a set of maximally three orthogonal external forces along lines through the centre of mass and of three orthogonal torques. However, because it is easier to apply just forces and because some of the preferred excitation positions are not always suitable from an experimental point of view, another more practical set of external forces can be applied. It will be 'equivalent' as long as it can be transformed into the orthogonal set by a linear transformation matrix **T.** When non-rigid body modes are involved as well, a rigorous and general approach to such questions might be possible by using modal analysis. The practicability of such an approach is currently under investigation. Here a brief discussion will be presented of a more intuitive approach, which has been followed in an pilot experiment on an electrical drive of a copier machine. Figure 6 shows the freestanding drive motor with integrated gearbox. An accelerance matrix was measured for eight forces and twelve accelerometer positions, which were well-distributed over the source surface[25,26]. An indication of the maximum number of relevant degrees of freedom of the source structure is the number of singular values which exceed the threshold based on Eq. (15). The solid line in figure 7 shows this number. One sees that below 1000 Hz, six is a typical value. This indicates that the rigid body modes are dominant in this frequency range. The dashed line in figure 7 represent the number of singular values which are equal to or larger than one-fourth of the largest singular value. It is seen that this number almost never exceeds five. Near 2400 Hz it is even lowered to one since there the response is largely dominated by a single non-rigid body mode and therefore by one (mathematical) degree of freedom. Further experiments with this drive showed that source modelling with five pseudo-forces was as effective as with eight forces[25].

Figure 6. Electromechanical drive of copier machine, mass 2.45 kg.

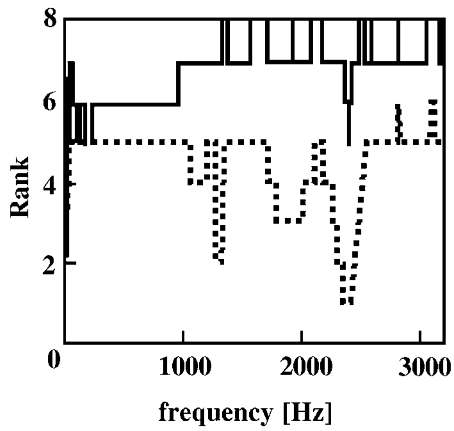

Figure 7. Effective rank of accelerance matrix (- - - with SV's zero if $SV<SV_{max}/4$).

Influence of installation environment. Another question is: how invariant is a solution of Eq. (11) for the installation environment of the source? This problem can be better understood if one considers what will happen with the rank of the accelerance matrix **A**, which is measured on the source structure. For example, what would happen if the source of figure 6 were directly attached to an infinite plate? Then at low frequencies, where the source behaves as a rigid body, the effective rank of **A** would become 3. The large in-plane impedances of the plate will suppress three degrees of freedom. An opposite effect may also occur. At about 2400 Hz, where in figure 7 a sudden drop in effective rank is observed, the drastic change in the boundary conditions would shift the natural frequencies of the source structure. Therefore, at this frequency an increase of the effective rank is to be expected. Generally speaking, it may be expected that the solution , as defined by Eq. (11), is to a certain extent dependent on the installation environment. However, the above discussion indicates what the most critical situations are. These are installations in which the relative suppression of certain rigid body modes is very different and which have a very different effect on source structure resonances. If such extreme situations are excluded, the scatter is hoped to be moderate.

Multiple source mechanisms. The internal excitation in a machine may result from different mechanisms. In the drive of Fig. 6 there is both mechanical noise from the gear transmission and noise from the electric motor. Therefore, the elements of the acceleration vector **a** will contain contributions from (partially) incoherent source mechanisms. This might be observed from coherence function between responses. If a coherence function is significantly below unity, the responses are only partially coherent. This would make the measured phases between the elements of **a** dependent on the choice of the reference channel and therefore unreliable for the purpose of Eq. (11). In such situations a principle component technique can be used to decompose the vibration field into parts, which are incoherent with each other, but fully coherent in themselves. The technique can be based on an eigenvalue and eigenvector decomposition[27] or on a singular value decompo- sition[28] of the cross-spectrum matrix of the accelerations. Figure 8 shows the singular values of a 8×8 cross-spectrum matrix measured on the drive of figure 6 in loaded condition. Only the values at multiples of 50 Hz are given, because these frequencies correspond to the noisiest components. It is seen that the highest singular value exceeds the second largest one by a factor one-hundred at many frequencies. This implies that there the level of the dominant source mechanism is some 20 dB stronger than those from other mechanisms. Notice: it is not necessarily the same mechanism which is dominant at each frequency.

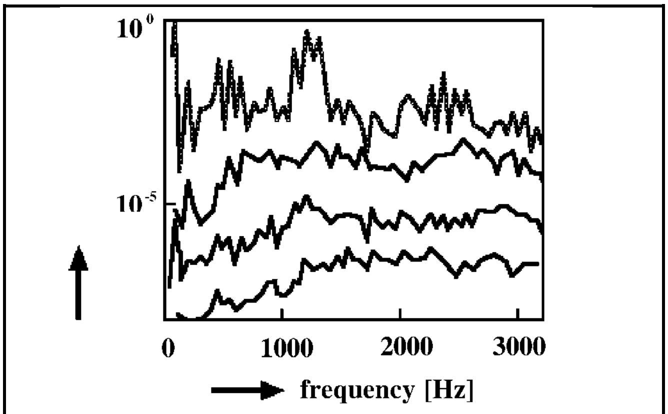

Figure 8. Singular values of the cross-spectrum matrix of the drive shown in figure 6.

3.3.2. Transfer functions

For diagnostic analysis transfer functions are needed in addition to the pseudo-forces. If the radiated sound is of interest the appropriate transfer functions are of the type p/F_j. Often the reciprocal measurement of these transfer functions is more convenient than the direct measurement. See 3.1 for the corresponding reciprocity relation.

For a single receiver position the sound pressure due to the n pseudo-forces is obtained from:

$$p = \boldsymbol{HF}_{pseudo} \tag{16}$$

where **H** denotes the row-vector of n (complex valued) frequency response functions p/F_j.

However, if a transformation is made of the column vector of pseudo-forces into, for example, a vector of three orthogonal "forces" and three orthogonal "couples", then

$$F' = \boldsymbol{TF}_{pseudo} \tag{17}$$

$$p = \mathbf{H}'\mathbf{F}' \tag{18}$$

where . Eqs. (16) and (18) can also be used to calculate the contribution of the individual pseudo-force components to the radiated sound. This is done by putting all elements in the pseudo-forces vector equal to zero, except one.

3.3.3. Validation studies

So far investigations of aspects as discussed in 3.3.1 and 3.3.2 have been performed for a small electromechanical drive (Fig. 6) and for a hydraulic pump[21,25,26]. Some results of transmission path analysis will be discussed later[1]. Here some source data of the hydraulic pump will be briefly discussed. In a first experiment pseudo-forces were determined with the pump installed in a refuse vehicle. These measurements were performed in the workshop of the vehicle manufacturer. Later a second series of measurements was performed on the same pump, but now installed in a test rig of the pump manufacturer as shown in figure 9.

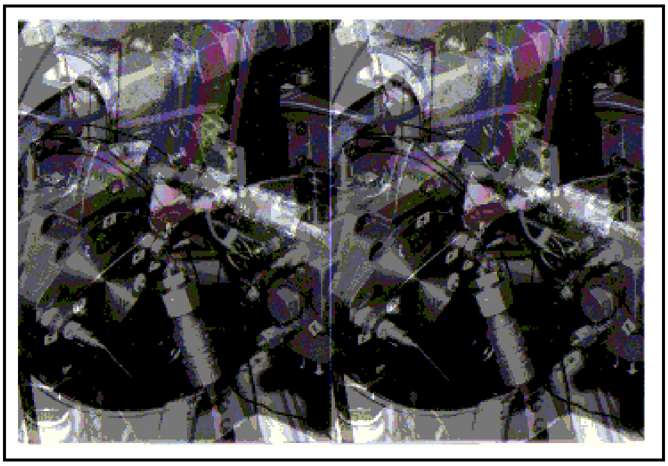

Figure 9. Hydraulic pump in test rig (variable displacement pump; 9 pistons; mass: 27 kg).

On the test rig seven force positions and ten response positions on the pump were used In the refuse vehicle the same positions were used, but eight further response positions were added on the vehicle frame onto which the pump is mounted.

Figure 10 shows measured and calculated average responses on the vehicle frame. For the calculations two sets of pseudo-forces were used. One set was determined from the measurements on the pump in the test rig and the other set from the measurements on the pump in the vehicle. Predicted and measured acceleration levels agree very well for both sets of pseudo-forces.

Figure 11 shows the two sets of pseudo-forces after transformation according to Eq. (17) into a set of six orthogonal forces and force-couples. The vibration spectrum is dominated by harmonics of the piston frequency. With nine pistons and a speed of 1385 r.p.m., these frequencies are 207.8 Hz and multiples. The data are only presented for these piston frequency harmonics. For the six lower harmonics the differences between the two sets are small for most of the components. At higher frequencies larger differences are seen. However, the rather good fits in figure 10 for both sets, show that they are still equivalent solutions in a least squares sense.

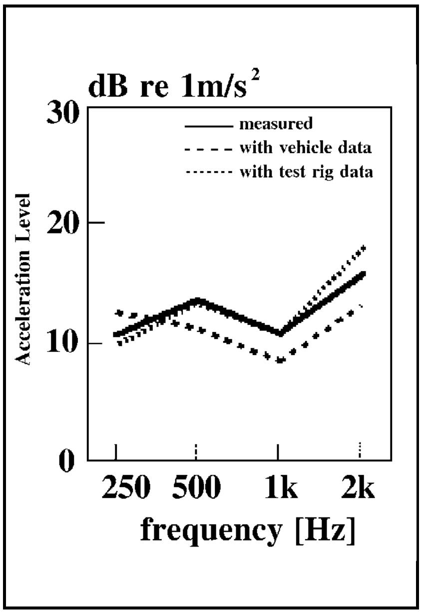

Figure 10. Measured and calculated acceleration levels on vehicle frame.

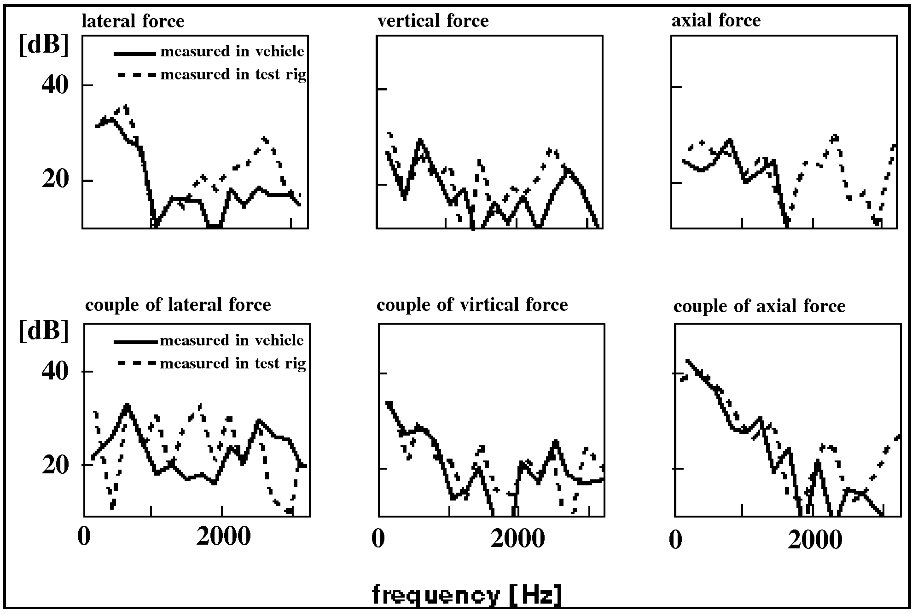

Figure 11. Orthogonal pseudo-forces and pseudo-couples on hydraulic pump: ——— measured in vehicle; - - - - measured in test rig (arbitrary reference values).

Also the experiments on the electromechanical drive[25] showed promising results with respect to this source modelling method. A similar set of six orthogonal "forces" and "couples"' as for the pump were determined for three very different installations. Measured sound pressure levels with the source installed in a copier frame were compared with predictions using the different sets of pseudo-forces[26]. A quite good agreement was found, but in a few third-octave bands a greater scatter was seen than for the vibration responses in figure 10. Several types of repeatability tests showed that, at least partially, this could be explained by the fact that the source itself was less stable and repeatable than the pump. This make the whole procedure of pseudo-force estimation statistically less certain.

3.4. Discussion

The foregoing discussion was concerned with alternative source descriptors which have certain advantages compared to impedance-coupling methods. Multi-directional point mobility measurements are not needed and measurements on the source in operation do not require impractical installation conditions. Still the multi-directional and multi-path nature of noise control problems can be properly handled. The method of correlated point forces seems especially promising for 'compact' sources. As shown by the experiments on the hydraulic pump, the equivalent forces determined on a test rig provided an accurate prediction of vibration levels in a vehicle. However, the formulation of the method according to Eq. (11) implies that a single source may be characterized by many sets of forces, which are all equivalent. For certain applications this is not a disadvantage. Moreover, for the frequency range in which the source behaves as a rigid body, it is possible to transform these equivalent sets into the same set of six orthogonal forces and couples. This makes interpretation of the source descriptor easier and less ambiguous. However, for the frequency range where non-rigid body behaviour is important, unambiguous interpretation is not possible. This aspect is a topic of further research.

Another topic which needs further attention is the statistical aspects of source behaviour. This aspect needs thorough study for all source descriptor methods, but especially for those that make use of phase relationships between responses. Information on this aspect seems totally missing from the open literature.

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