ON THE IMPORTANCE OF MEASUREMENT UNCERTAINTY IN CONFORMITY ASSESSMENT OF ACOUSTIC QUANTITIES

Massimo Garai

Department of Industrial Engineering, University of Bologna, viale Risorgimento 2, I-40136 Bologna, Italy
e-mail: massimo.garai@unibo.it

In the various fields of applied acoustics, a common problem is the assessment of the conformity of measured values of acoustic quantities to limit values. This problem becomes particularly critical when considering that measured values are always affected by a measurement uncertainty, while national regulations generally set absolute limit values, ignoring the uncertainty. From this point of view, the solution is not trivial and calls for the adoption of appropriate decision rules. Building on the background of ISO IEC GUIDE 98-4, this work presents the problem in an easy-to-understand form, review the basic decision rules and proposes a simple methodology to operate in a consistent way. Some examples taken from the everyday work of acousticians are discussed.

1. Introduction

One of the most common activities in the various fields of applied acoustics (environment, buildings, workplace etc.) is the measurement of specified acoustic quantities to verify the respect of limit values, prescribed by laws, regulations, technical standards or contractual agreements. While the limit values are usually specified without uncertainty, the measured values are always affected by a measurement uncertainty [1], [2], [3], [4]. When it is not possible to apply the analytic procedure recommended in [1], reliable data exist about the repeatability and reproducibility of the most common measurement procedures: see for example [6], [7]. Therefore it is astonishing that in most cases the assessment is still done without considering the measurement uncertainty. This may be due to the fact that the conformity assessment of the measured values to the limit ones is not so trivial. The correct solution implies the choice of appropriate decision rules, which in turn depends on the goal of the assessment. In most cases the role of these rules is underestimated; actually, too often they are implicitly assumed under the influence of habits and beliefs extraneous to the correct scientific reasoning. In spite of this unsatisfying situation, it is possible, relying on metrology and statistics, to outline a method for an optimal approach to the problem. Building on the background of ISO IEC GUIDE 98-4 [2], this work presents the problem in an easy-to-understand form, review the main decision rules and proposes a simple methodology to operate in a consistent way. This methodology has been included in UNI/TS 11326-2:2015 [5]. Some examples taken from the everyday work of acousticians are discussed.
2. Coverage interval and tolerance interval

The measurement uncertainty should always be clearly declared, e.g. as expanded uncertainty accompanied by a confidence level. The preferred confidence level generally is 95% [1], [4], with the possible exception of building acoustics; for example the Italian standard UNI 11367 recommend a confidence level of 84% [8]. The measurement uncertainty can be determined following two different approaches: the analytic approach, preferable when it is possible to calculate each uncertainty component [1], [4], or the experimental approach [1], [3], when it is not possible an analytical calculation but it is still possible a reasonable estimate, e.g. using the repeatability and reproducibility of the method. In all cases the measurements allows to estimate a probability density function (PDF). If it is symmetric and unimodal, as usually happens, it has a mean value equal to the measured value $y$, a standard deviation equal to the standard uncertainty $u(y)$ and the “coverage interval” equal to two times the expanded uncertainty $U(y)$ at 95% confidence level (Fig. 1).

Thus, the measurements gives the coverage interval $[y–U, y+U]$. This has to be compared with the “tolerance interval” given by the stated limit value(s): when the values of the measurand lay within the tolerance interval they are considered acceptable. The following cases are possible:

- Two-sided tolerance interval: it is delimited by a finite lower limit value ($T_L$) and a finite upper limit value ($T_U$);
- One-sided tolerance interval with a single lower tolerance limit: it is delimited by a finite lower limit value ($T_L$) and an upper limit value equal to $+\infty$;
- One-sided tolerance interval with a single upper tolerance limit: it is delimited by a lower limit value equal to $-\infty$ and a finite upper limit value ($T_U$).

In practice, in applied acoustics one-sided tolerance intervals are more common than two-sided ones. For example, according to the Italian decree DPCM 5-12-1997 [9]:

- The weighted sound reduction index of a party wall between two dwellings must have a minimum value of 50 dB: the tolerance interval is delimited by $T_L = 50$ dB and $T_U = +\infty$;
- The weighted impact sound pressure level on a floor between two dwellings must have a maximum value of 63 dB: the tolerance interval is delimited by $T_L = -\infty$ and $T_U = 63$ dB.
3. Decision rules

Relying on the above statements, the problem of the compliance of a measured value with one or more limit values becomes the problem of conformity assessment of a coverage interval to the tolerance interval. The coverage interval is given by the mean value of the measurement and its (expanded) measurement uncertainty, the latter being considered symmetrical around the mean value and independent from it. The tolerance interval is given by the stated limit value(s), considered exact. This conformity assessment is done adopting a “decision rule” in accordance with metrology and statistics [2]. In order to reach an unambiguous decision, it is necessary to select a decision rule, or better combination of decision rules, exhaustive, coherent and fitted to the specific case.

In order to support the choice done in Section 4, the most common decision rules are presented here. For the sake of brevity, only the case of a one-sided tolerance interval with a single upper tolerance limit is illustrated. The conformity assessment to a one-sided tolerance interval with a single lower tolerance limit can be inferred by analogy. The conformity assessment to a two-sided tolerance interval can be built joining the two one-sided cases.

3.1 Simple acceptance or rejection rules

The “simple acceptance” rule consists in considering the measurement result \( y \) as “conforming” simply if it is inside the tolerance interval. The “simple rejection” rule consists in considering the measurement result \( y \) as “non-conforming” simply if it is outside the tolerance interval. In Fig. 2, the cases 1 and 2 are conforming while cases 3 and 4 are non-conforming. These rules are the most elementary and correspond to considering the measurement result alone, without its uncertainty (as it were exact). In other worlds, uncertainty and confidence level, while existing, are not taken into account.

![Figure 2. Possible situations in the conformity assessment to an upper tolerance limit. \( T_U \).](image)

<table>
<thead>
<tr>
<th>Class of land use</th>
<th>Day time (06-22)</th>
<th>Night time (22-06)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specially protected areas</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Predominantly residential areas</td>
<td>55</td>
<td>45</td>
</tr>
<tr>
<td>Areas of mixed type</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>Areas of intense human activity</td>
<td>65</td>
<td>55</td>
</tr>
<tr>
<td>Predominantly industrial areas</td>
<td>70</td>
<td>60</td>
</tr>
<tr>
<td>Exclusively industrial areas</td>
<td>70</td>
<td>70</td>
</tr>
</tbody>
</table>
This is the most applied rule until now in acoustics, but working in this way the probability of an incorrect decision may be as large as 50% when the measured value lays very close to the tolerance limit. For example, the Italian decree DPCM 14-11-1997 [10] divides the territory in six classes and for each class assigns an upper overall noise limit, depending also on the time of the day (Table 1). For the class IV, during day time, the upper limit value is \( L_{A_{eq},T} = 65 \) dB(A). If a continuous equivalent sound pressure level of 65 dB(A) is measured, with a standard measurement uncertainty of 1 dB(A), the statement that it “complies with class IV” has a probability of 50% to be wrong. In fact (see Fig. 3):

\[
P(L_{A_{eq},T} > 65) = \int_{65}^{+\infty} P(L_{A_{eq},T}) \, dL_{A_{eq},T} = 0.50.
\]

Repeating the same measurement many times, due to the unavoidable measurement uncertainty one would find values smaller, equal or greater than 65 dB(A); there is no guarantee to respect the upper limit value of 65 dB(A) for class IV, as the DPCM 14-11-1997 requires.

The two rules of simple acceptance and simple rejection can be assumed together, resulting exhaustive, i.e. no ambiguous results are left. Whatever the risk of an incorrect decision could be, it is equally shared among the parties; for this reason these rules are also called “shared risk” rules.

### 3.2 Stringent acceptance or rejection rules

The “stringent acceptance” rule consists in considering the measurement result \( y \) as “conforming” if and only if its coverage interval is completely inside the tolerance interval (case 1 in Fig. 2). The “stringent rejection” rule consists in considering the measurement result \( y \) as “non-conforming” if and only if its coverage interval is completely outside the tolerance interval (case 4 in Fig. 2).

With the stringent acceptance rule one chooses to minimize the risk of a “false acceptance”, requiring that the conformity be assessed only if the probability density function of the measurand values has inside the tolerance interval a cumulative probability greater or equal to the chosen confidence level. With the stringent rejection rule one chooses to minimize the risk of a “false rejection”, requiring that the non-conformity be assessed only if the probability density function of the measurand values has outside the tolerance interval a cumulative probability greater or equal to the
chosen confidence level. If the two rules of stringent acceptance and rejection are assumed together, an “ambiguity interval” exists: when the best estimate of the measurand $y$ is inside the interval $[T_U - U, T_U + U]$ no decision can be taken (cases 2 and 3 in Fig. 2).

### 3.3 Relaxed acceptance or rejection rules

The “relaxed acceptance” rule consists in considering the measurement result $y$ as “conforming” if and only if its coverage interval is not completely outside the tolerance interval (cases 1, 2, 3 in Fig. 2). The “relaxed rejection” rule consists in considering the measurement result $y$ as “non-conforming” if and only if its coverage interval is not completely inside the tolerance interval (cases 2, 3, 4 in Fig. 2). With the relaxed acceptance rule one chooses to minimize the risk of a “false rejection”, requiring that the conformity be assessed unless the PDF of the measurand values has outside the tolerance interval a cumulative probability greater or equal to the chosen confidence level. With the relaxed rejection rule one chooses to minimize the risk of a “false acceptance”, requiring that the non-conformity be assessed unless the PDF of the measurand values has inside the tolerance interval a cumulative probability greater or equal to the chosen confidence level. The two rules of relaxed acceptance and rejection cannot be assumed together, otherwise a “contradiction interval” would exist: when the best estimate of the measurand $y$ is inside the interval $[T_U - U, T_U + U]$ it should be conforming (relaxed acceptance) and non-conforming (relaxed rejection) at the same time (cases 2 and 3 in Fig. 2).

### 4. Selection of the decision rule

From the above considerations is clear that the simple acceptance and rejection rules cannot always guarantee a low probability of incorrect decisions and thus should be avoided when dealing with regulatory limits. Actually, the proliferations of legal disputes about the respect of some acoustic limit values can often be traced back to the assumption, implicit or explicit, of these decision rules. Moreover, international standardization bodies are strongly committed to a better consideration of the measurement uncertainty (see for example [11]) and this is another reason to abandon the simple acceptance and rejection rules, which do not take it into account. If a set of decision rules leaving no space for ambiguity is desired, then the two stringent acceptance and rejection rules cannot be assumed together, as explained in Section 3.2. In a similar way, the two relaxed acceptance and rejection rules cannot be assumed together, because they may result in contradictory situations, as explained in Section 3.3. Therefore, only two combinations of the basic rules are left:

A) Stringent acceptance + relaxed rejection.
B) Relaxed acceptance + stringent rejection.

It is worth noting that the choice depends on the goal of the assessment, goal that should be always declared explicitly. Moreover, it should be clear that the choice of the proper decision rules is not part of the measurement procedure, instead it should be done before the measurements.

In practice, in applied acoustics two cases are possible:

- A-type case: the stringent acceptance + relaxed rejection rule is adopted, because the conformity assessment has the goal of **ascertain that the limit values are respected**: in this case one want be sure (at the given confidence level) that appropriate actions are taken to protect those who could suffer the side effects of the failure to meet the limit values.
- B-type case: the relaxed acceptance + stringent rejection rule is adopted, because the conformity assessment has the goal of **ascertain the failure to respect the limit values**: in this case one want be sure (at the given confidence level) that the limit values have been exceeded before taking an action (e.g. imposing a fine) against the responsible of such non-compliance.
4.1 A-type cases

Once the A-type decision rule has been chosen, in the conformity assessment of a measured value \( y \) with expanded uncertainty \( U \) to an upper limit value \( T_U \), assumed exact, the following cases are possible (one-sided comparison with \( T_U \)):

a. **Conformity assessed** (at the given confidence level): the sum of the measured value and its expanded uncertainty is less or equal than the upper limit value (case 1 in Fig. 2).

b. **Non-conformity assessed** (at the given confidence level): the difference between the measured value and its expanded uncertainty is greater than the upper limit value (case 4 in Fig. 2).

c. **Presumed non-conformity** (at the given confidence level): the interval defined by the measured value plus or minus its expanded uncertainty includes the upper limit value (cases 2 and 3 in Fig. 2).

In the case (c) the probability that the results be greater than the upper limit value should be declared. In this case it may be taken into consideration, when possible and practicable, reducing the measurement uncertainty so as to fit case (a) or (b).

4.2 B-type cases

Once the B-type decision rule B has been chosen, in the conformity assessment of a measured value \( y \) with expanded uncertainty \( U \) to an upper limit value \( T_U \), assumed exact, the following cases are possible (one-sided comparison with \( T_U \)):

a. **Conformity assessed** (at the given confidence level): the sum of the measured value and its expanded uncertainty is less or equal than the upper limit value (case 1 in Fig. 2).

b. **Non-conformity assessed** (at the given confidence level): the difference between the measured value and its expanded uncertainty is greater than the upper limit value (case 4 in Fig. 2).

c. **Presumed conformity** (at the given confidence level): the interval defined by the measured value plus or minus its expanded uncertainty includes the upper limit value (cases 2 and 3 in Fig. 2).

In the case (c) the probability that the results be greater than the upper limit value should be declared. In this case it may be taken into consideration, when possible and practicable, reducing the measurement uncertainty so as to fit case (a) or (b).

5. Examples and discussion

It seems obvious that an A-type rule should be preferred, because it is more precautionary for the exposed people, but it is worth noting that the exceedance of regulatory noise limits may imply some sort of sanction according to national laws and, from this point of view a B-type rule is more precautionary for the responsible of the excess of noise. So in each case it should be carefully established whether the goal of the conformity assessment is the protection of exposed people or the “protection” of the alleged responsible of excessive noise. Some examples will clarify the point.

In building acoustics an A-type rule is usually preferred, because it is more precautionary for the exposed people. For example, in Italy the DPCM 5-12-1997 [9] has the “goal of reducing the human exposure to noise” (art. 1). The decree states a lower limit value \( T_L = 50 \text{ dB} \) for the weighted apparent sound reduction index of a party wall between two dwellings. Suppose an in situ measurement result is \( R'_w = 50 \text{ dB} \). According to ISO 12999-1 [7], Table 3, the standard measurement uncertainty is \( \sigma_{\text{situ}} = 0.9 \text{ dB} \), giving a one-sided expanded uncertainty at 95% confidence level of:

\[
U = k \cdot \sigma_{\text{situ}} = 1.645 \cdot 0.9 = 1.5 \text{ dB}
\]

Thus the value to be compared with the lower limit value is:
A non-conformity is assessed: the party wall doesn’t comply with the requirement of DPCM 5-12-1997.

In the evaluation of the occupational noise exposure an A-type rule is always applied because it is more precautionary for the exposed people. In fact, the Directive 2003/10/EC [11] lays down “minimum requirements for the protection of workers from risks to their health and safety arising or likely to arise from exposure to noise and in particular the risk to hearing” (art. 1). For example, assume to have measured, according to ISO 9612 [13], a value of daily noise exposure level \( L_{\text{EX,8h}} = 84.3 \text{ dB(A)} \) with an expanded uncertainty \( U(L_{\text{EX,8h}}) = 1.5 \text{ dB(A)} \). The lower and upper limits of the coverage interval are:

\[
\begin{align*}
L_{\text{EX,8h}} - U &= 84.3 - 1.5 = 82.8 \text{ dB(A)} \\
L_{\text{EX,8h}} + U &= 84.3 + 1.5 = 85.8 \text{ dB(A)}
\end{align*}
\]

The Directive 2003/10/EC in art. 3 sets three limit values:

- The lower exposure action value 80 dB(A);
- The upper exposure action value 85 dB(A);
- The exposure limit value 87 dB(A);

Comparing the value from Eq. (4) with the lower exposure action value a non-conformity is assessed. Comparing the value from Eq. (5) with the upper exposure action value a presumed non-conformity is obtained; the probability of not exceeding 85 dB(A) is 78% and the corresponding probability of exceedance is 22%. Comparing the value from Eq. (5) with the exposure limit value a conformity is assessed. For the sake of simplicity, here we neglect that, when applying the exposure limit value, the determination of the worker’s effective exposure should take account of the attenuation provided by the individual hearing protectors worn by the worker.

In the field of environmental acoustics the Italian laws leave space for different interpretations concerning the goal of the conformity assessment. On one hand, the framework law 447/95 [14] “establishes the fundamental principles for the protection of the external environment and the living environment from noise” (art. 1); from here it may be inferred that the goal of any measurement is the protection of exposed people and thus an A-type rule should be chosen. On the other hand, according to the same law the responsible of excessive noise immissions is subject to a fine (art. 10); this allows the adoption of a B-type rule; this is the interpretation of some Italian Environmental Protection Agencies (see also the guideline [15]). This choice, more juridical than scientific, guarantees as much as possible the responsible of the noise emissions, using the measurement uncertainty in his/her favor (but against exposed people). In this way a fine is imposed only when it is absolutely certain (at the selected confidence level) and the Agency doesn’t risk a complaint with legal consequences from the responsible of the noise emissions. The only way to solve the problem should be an updated law with a clear indication on how to manage the uncertainty, but this seems beyond the capacity of most lawmakers. In the meantime, the use of a B-type rule may lead to paradoxical consequences. For example, let’s suppose that in a urban area where the immission limit value is 60 dB(A), according to Italian laws, an Agency has measured 61.5 ± 1.5 dB(A). Using a B-type rule this Agency should assess a presumed conformity (no fine), because (Section 4.2, case c):

\[
L_{\text{AQ,T}} - U = 61.5 - 1.5 = 60 \leq 60 = T_U \text{ dB(A)}
\]

If the same values were measured by an independent acoustician to check the effectiveness of the noise protection measures done by the same responsible, he/she, using an A-type rule, should assess a non-conformity (insufficient mitigation measures), because at 95% confidence level (Section 4.1, case c):
6. Conclusions

The comparison between the measured values of acoustic quantities and regulatory limit values should be done very carefully. With the growing awareness of the importance of the measurement uncertainty, also thanks to the action of the major international standard bodies, a critical point has emerged: measured values are always affected by their measurement uncertainty and this modifies the result of the comparison with the limit values respect to the prior situation when also the measured values were assumed without uncertainty. The correct way of tackling the problem is the choice of an appropriate decision rule that, in turn, depends on the goal of the conformity assessment. Too often these rules are still assumed without a full knowledge of their implications, increasing the risk of incorrect decisions. On the contrary, this choice, determinant for the outcome of the assessment, must be done with awareness. For this reason the selected decision rule must always be declared, clarifying who is responsible for the choice and what is the goal of the conformity assessment.

REFERENCES