SOUND GENERATION AND CONTROL OF THERMOACOUSTIC INSTABILITIES BY NANOSECOND PLASMA DISCHARGES

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Sound generation by nanosecond repetitively pulsed plasma discharges is investigated experimentally. High voltage pulses of 10 nanosecond duration provide rapid heating of the air. A high-frequency pulse train between 20 and 30 kHz is burst modulated to generate low-frequency components. The generation of pressure waves from the modulated discharge pulse train is characterized based on acoustic measurements in an impedance tube. Different combinations of modulation frequency, electrode gap distance, modulation duty cycle and pulse repetition frequency are studied in terms of electric energy and acoustic source amplitude. The measurement results suggest that overall, the amplitude of the pressure wave components at the modulation frequency can be well estimated based on the electrical power using an analytical expression for acoustically compact unsteady heating. As an application of sound generation by low-frequency modulated NRP discharges, feedback control is applied to suppress thermoacoustic instabilities in a Rijke tube. The pressure oscillation amplitude is reduced by more than two orders of magnitude when the plasma discharges are suitably synchronized with the self-excited fluctuations.

1. Introduction

Thermoacoustic instabilities are a major challenge in the development of modern low-emission combustion systems. Undesirable self-excited oscillations arise from a feedback cycle coupling heat and pressure fluctuations. The resulting high-amplitude pressure oscillations can damage the system and restrict the operational range. Due to the high complexity of turbulent combustion processes, a sufficiently accurate prediction of unstable conditions is presently not possible. Passive as well as active methods have been developed to mitigate these undesirable system dynamics [1,2]. However, passive methods are effective in a small, fixed frequency range only while active control systems are more flexible. A current challenge for active control applications is the identification of suitable actuator technology. Combustion instability actuators are required to provide high actuation amplitudes and high bandwidth and to be sufficiently robust to work in environments at elevated temperatures and pressures [3].

Recent work has demonstrated the possibility of using nanosecond repetitively pulsed (NRP) plasma discharges to control combustion instabilities [4–6]. Discharges employed in this context correspond to special types of non-equilibrium plasma generated by high-voltage pulses of approximately 10 ns duration with repetition frequencies (PRF) in the range of 10 to 80 kHz. This type of non-equilibrium discharge has been proven to be particularly successful in affecting fundamental combustion phenomena like ignition and blow-off [7]. For plasma-assisted stabilization of lean premixed flames, less than 1% of the thermal power of the flame is typically required. Principal
interaction mechanisms of the NRP discharges with combustion instabilities can generally be of thermal, chemical, and acoustic nature. To distinguish the effect of the individual coupling mechanisms, the present investigation focuses on the effect of low-frequency modulated NRP discharges on the acoustic field.

Unsteady heat release rate is a source of sound. NRP discharges provide unsteady heating and species dissociation on the nanosecond timescale [8, 9]. This generates pressure waves propagating from each discharge [10, 11]. However, these investigations are based on much shorter timescales than relevant for typical combustor acoustics (few hundreds of Hertz). The present investigation therefore focuses on the generation of low-frequency sound waves by square-wave modulation of the high-frequency pulse train (Sections 2 and 3). To determine the acoustic source amplitude, microphone measurements in an impedance tube are conducted using the multi-microphone method. The effect of modulation frequency, pulse repetition rate, duty cycle and electrode gap distance is investigated. The acoustic source associated with NRP discharges is then applied in a Rijke tube set-up [12] to suppress thermoacoustic instabilities by means of feedback control (Section 4).

2. Sound generation from low-frequency modulated NRP discharges

Nanosecond repetitively pulsed (NRP) spark discharges were generated by a high-voltage pulse generator (FID Technologies, FPG 10-30NM10). The pulses have 10 ns width and repetition frequencies ($f_{prf}$) between 5 and 30 kHz; the energy per pulse is approximately 2.2 mJ.

![Figure 1: Idealized pulse train burst-modulated at $f_{mod}$.](image)

The high-frequency pulse train at the PRF is modulated at low frequencies relevant for thermoacoustic instabilities ($\approx$ 100–1000 Hz), with varying gate widths (Fig. 1). For idealized pulses of delta type with constant energy, the highest amplitude at the modulation frequency $f_{mod}$ is obtained when as many pulses as possible are placed in half of the modulation period $T$ (DC = 50%). The maximum pulse repetition frequency of the high-voltage pulse generator employed in the present work is limited to 30 kHz. However, the maximum energy per pulse is generated at a $f_{prf} \leq 29$ kHz. Figure 2 shows a pressure signal measured in the impedance tube for a modulation frequency $f_{mod} = 100$ Hz with $f_{prf} = 28$ kHz and DC = 50%; the associated amplitude spectrum is shown in addition. A clear response at the modulation frequency can be observed, with higher harmonics resulting from the square-wave modulation.

In order to generate low frequency components, also the pulse density method (PDM) was investigated. The PDM is based on periodic signals with variable temporal pulse densities. With this

![Figure 2: Acoustic response to burst-modulated NRP forcing. $f_{mod} = 100$ Hz, $f_{prf} = 30$ kHz, DC = 50%. Signal was low-pass filtered at 4 kHz. Left: pressure signal; right: corresponding amplitude spectrum.](image)
method, it is possible to avoid strong harmonic components in the low-frequency regime. Due to the lower pressure amplitude of the PDM at the modulation frequency, burst modulation was chosen for the investigations reported in the following.

2.1 Measurement of the acoustic source amplitude in an impedance tube

An impedance tube facilitating multiple, axially distributed microphone measurements is employed (Fig. 3). Five 1/4-in. condenser microphones are flush-mounted upstream (us) and downstream (ds) of the acoustic source (S). The impedance tube has a length of 1.5 m and a diameter of 40 mm. The microphones (G.R.A.S. 40BP with preamplifier 26AC) with a custom-made amplifier are connected to a National Instruments data acquisition board (NI 9220 with cRIO 9074). The acoustic source considered here is a sequence of plasma discharges between two tungsten electrodes with diameter 2.4 mm, placed in the center of the tube (Fig. 3, left). Low-reflecting terminations at both ends strongly reduce modal resonances and improve the measurement accuracy. A weak purging flow of less than 5 m/s was used to continuously remove product species related to the plasma chemistry from the tube.

Below the cut-on frequency of the first azimuthal mode in the tube (5 kHz), only plane waves propagate. The pressure field can then be written as

$$\hat{p}(x, \omega) = \hat{f}(\omega) e^{-i k x} + \hat{g}(\omega) e^{i k x}$$  \hspace{1cm} (1)

where $\hat{p}(x, \omega)$ is the frequency domain acoustic pressure at an axial location $x$, $\omega$ denotes the angular frequency, and $\rho$, $c$, and $k = \omega/c$ are fluid density, speed of sound, and wavenumber, respectively. $\hat{f}$ and $\hat{g}$ are wave amplitudes associated with downstream and upstream traveling components. For $n$ axial measurement locations, the linear system of equations

$$\begin{pmatrix} \hat{p}_1 \\
\hat{p}_2 \\
\vdots \\
\hat{p}_{N_{mic}} \end{pmatrix} = \begin{pmatrix} e^{-i k x_1} & e^{i k x_1} \\
 e^{-i k x_2} & e^{i k x_2} \\
 \vdots & \vdots \\
 e^{-i k x_{N_{mic}}} & e^{i k x_{N_{mic}}} \end{pmatrix} \begin{pmatrix} \hat{f} \\
\hat{g} \end{pmatrix}$$ \hspace{1cm} (2)

can be solved by least-squares inversion for the two wave amplitudes $\hat{f}$ and $\hat{g}$.

The source element with the plasma discharges can be characterized by a scattering matrix, $S$, characterizing reflection and transmission of the incident waves, and a vector of downstream and upstream propagating waves related to the source [13]:

$$\begin{pmatrix} \hat{f}_{ds} \\
\hat{g}_{us} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\
 S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} \hat{f}_{us} \\
\hat{g}_{ds} \end{pmatrix} + \begin{pmatrix} \hat{f}_s \\
\hat{g}_s \end{pmatrix}$$ \hspace{1cm} (3)

The source components $\hat{f}_s$ and $\hat{g}_s$ are only associated with the modulated discharge pulse train and independent of the incident field. The two-source method [14] is employed to determine the scattering
matrix and the source components. Since the obstruction by the two electrodes is negligible compared to the total cross-sectional area, and because the mean temperature increase associated with the discharges (≤ 20 K) is insignificant, the scattering matrix is the identity within experimental uncertainty. The source amplitudes are then simply obtained as

$$\hat{f}_s = \hat{f}_{us} - \hat{f}_{ds},$$  
$$\hat{g}_s = \hat{g}_{us} - \hat{g}_{ds}.$$  

### 2.2 Electric power at the modulation frequency and pulse energy

To correlate the amplitudes of the low-frequency acoustic waves generated by the modulated NRP discharges with the electrical power, it is essential to consider only the component at the modulation frequency. To this end, the electrical power $w(t)$ is expressed as a Fourier series, with the fundamental period corresponding to the modulation period:

$$w(t) = c_0 + \sum_{n=1}^{\infty} |c_n| \cos(n\Omega t + \arg c_n), \quad \text{with} \quad c_n = \frac{2}{T} \int_0^T w(t) e^{-i\Omega nt} \, dt.$$  

$\Omega = 2\pi f_{\text{mod}}$ denotes the angular modulation frequency. The width of the individual high-voltage pulses is short (∼10–20 ns) compared to the modulation period (∼1–10 ms). It is then possible to approximate the pulse train as a sequence of Delta functions weighted by the corresponding pulse energy:

$$w(t) = \sum_{n=1}^{N} e_n \delta(t - t_n).$$  

Here, $e_n$ and $t_n$ are energy and time corresponding to the $n$th pulse, and $N$ is the total number of pulses in one modulation period. The amplitude of the electric power at the modulation frequency, $\hat{W}$, is then given by

$$\hat{W} = |c_1| = \left| \frac{2}{T} \sum_{n=1}^{N} e_n e^{-i\Omega t_n} \right|.$$  

Voltage and current were acquired during the acoustic measurements to determine the corresponding electrical power. At the beginning of each modulation cycle, weak initialization pulses without spark formation occur that reduce the average electrical power input at the modulation frequency. To take this effect into account, four phase-averaged measurements of 40 pulses within the modulation cycle were acquired and evaluated. In each of the four measurements, the first 25 pulses were measured, and in addition, 15 pulses sampled equidistantly over the remaining high-gate period. Figure 4 shows representative voltage and current histories for one pulse and the pulse energy over the modulation period for different duty cycles. The modulation frequency was set to $f_{\text{mod}} = 352$ Hz and the pulse repetition frequency to $f_{\text{prf}} = 25$ kHz. After the first few initialization pulses, the pulse energy remains fairly constant. This justifies to obtain the energy of the pulses that were not measured by interpolation.

![Figure 4: Left: representative voltage and current signal; right: energy per pulse for various DC. $f_{\text{mod}} = 352$ Hz, $f_{\text{prf}} = 25$ kHz.](image)

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Table 1: List of test cases for acoustic and electrical energy measurements

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
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<tbody>
<tr>
<td>d (mm)</td>
<td>4</td>
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<td>4</td>
<td>4</td>
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<td>4.5</td>
<td>4.5</td>
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<td>5</td>
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<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( f_{\text{prf}} ) (kHz)</td>
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<td>25</td>
<td>27</td>
<td>29</td>
<td>20</td>
<td>25</td>
<td>27</td>
<td>29</td>
<td>20</td>
<td>25</td>
<td>27</td>
<td>29</td>
</tr>
<tr>
<td>( E/N ) at 293K (Td)</td>
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<td>95</td>
<td>96</td>
<td>92</td>
<td>90</td>
<td>86</td>
<td>85</td>
<td>87</td>
<td>83</td>
<td>80</td>
<td>79</td>
<td>-</td>
</tr>
</tbody>
</table>

3. Results and discussion

Table 1 summarizes for the investigated test cases the gap distance \( d \), the pulse repetition frequency, and the average reduced electric field \( E/N \) ( \( E \) is the electric field and \( N \) the particle density). The latter is proportional to the mean electron energy and is the main parameter for the characterization of plasma discharges [7]. The maximum electric energy per pulse in all cases is \((2.2 \pm 0.2)\) mJ. For each case, acoustic and electrical energy measurements for various duty cycles were made.

3.1 Effect of modulation duty cycle on the acoustic source amplitude

Under the assumption that the energy per pulse is constant during the high-gate period and that a constant fraction of the electrical power is transformed into sound, the acoustic source amplitude is proportional to \( \sin(\pi DC) \). Figure 5 compares the duty-cycle dependence of the measured source amplitudes [Eqs. (4) and (5)] with that obtained from the above assumptions. The measured source amplitudes are generally in good agreement with the ideal one. Maximum measured source amplitudes are observed for duty cycles close to 50%, and the amplitude approaches zero for 0% and 100% duty cycle.

The slight shift of the measured source amplitudes to higher duty cycles can be attributed to the 3–7 non-ignited sparks at the beginning of each modulation cycle, as mentioned above. The shift to higher duty cycles becomes more pronounced with increasing modulation frequency (Fig. 5, left) and decreasing pulse repetition frequency (Fig. 5, right). If the number of non-ignited sparks remains constant, their relative contribution is reduced when the total number of pulses per period is increased. This is the case for a decrease in modulation frequency and an increase of the pulse repetition frequency.

![Figure 5: Duty cycle dependence of the acoustic source amplitude for various modulation and pulse repetition frequencies. Left: \( f_{\text{prf}} = 20 \) kHz, \( d = 5 \) mm; right: \( f_{\text{mod}} = 928 \) Hz, \( d = 4.5 \) mm.](image)

3.2 Theoretical estimate of the source amplitude and comparison with measurements

The adiabatic change equation in a fluid with chemical conversion reads

\[
\frac{dp}{dt} = c^2 \frac{d\rho}{dt} + \rho c^2 \dot{\sigma},
\]

(9)
where \( \frac{d}{dt} \) is a material derivative and diffusive effects have been omitted. For an ideal gas, the second term on the right hand side, which is sometimes called thermicity, can be written as

\[
\dot{\sigma} = \frac{1}{\rho} \sum_{s,r} \left( \frac{M}{M_s} - \frac{h_s}{c_p T} \right) \nu_{sr} w_r M_s. 
\]  
(10)

Here, \( M \) is the molar mass of the mixture, \( M_s \) and \( h_s \) are molar mass and specific enthalpy of species \( s \), \( c_p \) is the heat capacity at constant pressure, \( \nu_{sr} \) is the stoichiometric coefficient of species \( s \) in the \( r \)th reaction, \( w_r \) is the rate of the \( r \)th reaction, and the sum runs over all species \( s \) and reactions \( r \). The two terms on the right hand side of (10) are related to changes in the number density and to heat release associated with chemical reactions. Combining (9) with linearized mass and momentum balances (see, for example, [15]), one obtains a wave equation for the pressure in the frequency domain with a source term proportional to \( \dot{\sigma} \):

\[
\nabla^2 \hat{p} + \omega^2 \hat{p} = -i\omega \rho_0 c^2 \hat{\sigma},
\]  
(11)

where \( \rho_0 \) is the mean density, and the mean speed of sound \( c \) has been assumed to be constant. For acoustically compact sources in ducts below the cut-on frequency for the first non-planar mode, one can then show that

\[
\hat{f}_s = \hat{g}_s = \frac{\rho_0 c}{2A} \int \hat{\sigma} \, dV, 
\]  
(12)

where the integral runs over the source domain, and \( A \) is the cross-sectional area of the duct.

It is at this point not possible to determine a precise relation between the source integral in Eq. (12) and the electrical power due to the complex processes in a nanosecond plasma discharge. Generally both species dissociation and unsteady heating may contribute to \( \dot{\sigma} \) in spark discharges. However, if it is assumed that all of the electrical power at the modulation frequency contributes to \( \dot{\sigma} \) in terms of unsteady heating, one obtains an explicit expression for the source amplitudes:

\[
\hat{f}_s = \hat{g}_s = \frac{\gamma - 1}{2Ac} \hat{W}. 
\]  
(13)

This estimate of the acoustic source amplitude is compared with the measurement data in Figure 6. Over the whole range of test cases listed in Tab. 1 and modulation frequencies between 100 and 1000 Hz, the theoretical estimate agrees fairly well with the experimental data.
4. Control of thermoacoustic instabilities with NRP discharges

To demonstrate the feasibility of using NRP discharges as an active control actuator, a feedback control scheme is applied to a Rijke tube configuration in order to suppress thermoacoustic instabilities. Without control, self-excited pressure oscillations with amplitudes of several hundred Pascal can be observed in this set-up. The oscillations are driven by an electrically heated grid, located in the lower third of the 102 cm long tube (diameter 60 mm). The system becomes unstable for heating powers larger than 175 W. A further increase of heating power leads to larger oscillation levels until significant saturation sets in at about 300 W heating power. At this operating point, the mean power of the plasma (<35 W with 2.5 mJ per pulse at DC=50%) is small compared to the heating power of the grid. The additional steady heating by the plasma then only has a minor effect on the mean temperature.

Figure 7 shows a schematic of the control scheme. The pressure signal, measured in the lower third of the tube, is fed into a Kalman filter, which estimates oscillation amplitude and phase of the pressure field in the tube. The oscillation phase is used to generate the gate signal for the low-frequency modulation of the high-frequency pulse train. In this way, there is always a defined phase relation $\Phi$ between pressure oscillation phase and plasma discharge modulation phase. The modulation duty cycle is set to 50% at high pressure oscillation levels and then decreases with oscillation amplitude. The phase difference between pressure oscillation and plasma pulse train modulation is manually varied to find the best suppression.

Figure 8 shows the root mean square (RMS) pressure signal in the Rijke tube; the uncontrolled system oscillates with an RMS amplitude of about 500 Pa. When control by modulated NRP discharges is activated, the oscillation level decreases to essentially zero. The pressure spectrum for the uncontrolled case exhibits a peak component at 176 Hz, corresponding to the half-wave mode in the tube. An oscillation amplitude reduction of more than 40 dB by plasma control can be observed.

5. Conclusion

Generation of acoustic waves by low-frequency modulated NRP plasma discharges was studied experimentally in an impedance tube. The effect of modulation duty cycle on the acoustic source amplitude was found to be very close to ideal characteristics. The electrical power at the modulation
frequency was shown to be the dominant parameter affecting the acoustic source amplitude. Gap distance and pulse repetition frequency have only a minor influence. The acoustic source amplitude generated by low-frequency modulated discharges can be reasonably well predicted based on a model for compact acoustic sources if it is assumed that all of the electrical power contributes to unsteady heating. It was demonstrated that this type of acoustic source can be used as an actuator in a feedback control scheme to suppress thermoacoustic instabilities in a Rijke tube.

Current and future work is focused on a better understanding of the sound generation mechanism by considering the discharge process in more detail and on introducing more elaborate control approaches.

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