THREE DIMENSIONAL NONLINEAR ANALYSIS OF FLUID-MEMBRANE INTERACTION IN THE RECTANGULAR TANK

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In the aerospace area, the fluid and rubberlike membrane interaction forming as the rubber bag in the solid container which has been used for the 3 axis control satellite. In this study, the rubberlike membrane together with nonlinear sloshing-like behaviour was used as the bladder tank container for interaction investigation. To overcome the moving boundary problem from the large deformation, an arbitrary Lagrangian-Eulerian (ALE) description was used to support the moving boundaries. The three dimensional analytic model of rubberlike membrane and fluid interaction was evaluated by ALE with finite element using Newmark-β and Newton Raphson method and compared by sloshing model. The nonlinear behaviours of fluid-membrane interaction in rectangular tank under the vertical vibration were examined. The flow rate and pressure distribution, strain and stress distribution, and the displacement of membrane were discussed. According to many parametric calculations, we obtained some nonlinear characteristics. Successfully, the results confirmed that the nonlinearity of fluid was important in the larger amplitude excitation.

1. Introduction

Fluid-structure interaction (FSI) problem has been mentioned in various engineering fields, such as buildings, mechanical devices, aeronautics, ocean engineering, and biomechanics. It is widely known that the fluid and rubberlike membrane interaction can be used for engineering products, such as the rubber bag in the solid container using for the 3-axis control satellite, the anti-sloshing system considering for an oil container, and the floating breakwater consisting of membrane structure on a sea-surface. In many FSI problems, a geometric nonlinear analysis was concerned according to large displacements or coupled membrane, and bending effects. The complexity and high number of calculus operations involved on FSI analysis leading to the search for an accurate prediction by computational techniques. In the recent years, there are many works which were performed the studies in sloshing rectangular tanks [1-2], The study of Bauer and Eidel found that the rectangular tanks for the free fluid surface was only partially covered with a rubber-like membrane [3]. As well, Kawakami studied the cylindrical tank model of fluid-filled and the researcher found that the surface was covered by a rubberlike membrane [4]. The effects of finite deformation of the membrane were examined by linear analysis and nonlinear finite element analysis. However, the moving boundaries of fluid and rubberlike membrane interaction causing by the large deformation of fluid region were neglected. Considering all effects, the popular method that has been used to capture the interaction between structure and fluid is an arbitrary Lagrangian-Eulerian (ALE) method. The ALE method allows an arbitrary motion of the moving boundary of fluid and rubberlike membrane interaction with respect to their frame of reference by taking the convection of these points into account as described in [5-7]. However, almost studies were interested in sloshing model, while interaction
problem was not concerned. The aim of this work was to investigate the nonlinear behaviours of fluid-membrane interaction in the horizontal rectangular tank and ALE-FEM was used for analysis. Results from this work would be benefits for researchers on the nonlinear behaviours of rubberlike membrane and fluid interaction and they can apply to more applications.

2. Analytical model and method of solution

2.1 Analytical model

Considering the rubberlike membrane and fluid interaction model, a rectangular tank of infinite with both width and length of 0.8 m is filled fluid to a height 3 m on whose upper surface is covered by membrane as shown in Fig. 1. The tank is assumed to be sufficiently long in perpendicular direction to the sheet so that three-dimensional analysis and the state of plane strain are assumed. The tank is rigid enough comparing to rubberlike membrane. The rubberlike membrane is thin enough and the assumption of plane stress can be established.

The bending stiffness and the friction with fluid are so small that they can be neglected. The membrane is simply supported at both edges of tank, and is also given the uniform and constant tension everywhere in advance. The fluid is assumed to be potential flow which is incompressible, non-viscosity and irrotational. The fluid has sufficient heat capacity, and the temperature change can be neglected.

![Figure 1: 3-D tank which upper surface is covered by membrane (1/2 model).](image)

2.2 Hyperelastic rubberlike material

In this section, the constitutive equations for the incompressible rubberlike material were discussed. Neo-Hookean (Rivlin), Mooney-Rivlin and Ogden theories are famous in constructive equation of rubber. According to Neo-Hookean and Mooney-Rivlin theory, an application of theory is limited, because the constructive equation represents nonlinear on high strain domain.

On the other hand, strain energy function suggested by Ogden presents most generally constructive equation. The constructive equation is defined as polynomial strain energy function which is used stretch rate in principle direction and it is almost accorded to experiment result of Treloar to stretch rate of 700% [8]. Ogden model is used as constitutive equation of rubberlike materials. Ogden strain energy function ($W$) is given by using stretching rate ($\lambda_r$) of principle direction as follows

$$W(\lambda_1, \lambda_2, \lambda_3) = \frac{1}{2} \sum_{r=1}^{3} \frac{\mu_r}{\lambda_r} \left[ \lambda_r^{\alpha_r} - 1 \right] + \frac{3}{2} \frac{\mu_r}{\lambda_r} - \frac{3}{2}$$

(1)

Here $\mu_r$ and $\alpha_r$ indicate material constant determined by experiment data. In this research, the values of table 1 are used. These parameters are commonly used in other numerical analyses.
Table 1: Parameters of Ogden strain energy function.

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<tr>
<td>1</td>
<td>1.3</td>
<td>62994.7</td>
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<tr>
<td>2</td>
<td>5.0</td>
<td>126.7</td>
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<td>3</td>
<td>-2.0</td>
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2.3 Variation principle of fluid-structure interaction problem

In this section, dominant equation is derived from functional of fluid-structure interaction problem based on Hamilton principle [9] when the system is excited horizontally.

Functional on structural domain $\Pi_s$ is derived by subtracting kinetic energy of membrane and gravity, potential energy by acceleration from strain energy of membrane.

$$\Pi_s = \int_0^T \int_{\Omega_s} \left\{ W - \frac{1}{2} \rho_m u \cdot \ddot{u} - \rho_m a \cdot \gamma_s \right\} d\Omega_s \, dt$$

(2)

When $\rho_m$ indicates density of membrane, $u$ indicates displacement vector, $\gamma_s$ indicates position vector of current shape, $a$ indicates acceleration vector of external force, and $\Omega_s$ indicates structural domain.

Functional on fluid domain $\Pi_f$ is derived by subtracting kinetic energy of fluid from potential energy of fluid due to assumption that fluid is incompressible and non-viscous.

$$\Pi_f = \int_0^T \int_{\Omega_f} \left\{ \rho_f a \cdot \gamma_f - \frac{1}{2} \rho_f v \cdot v \right\} d\Omega_f \, dt$$

(3)

Where $\gamma_f$ indicates position vector of present shape, $v$ indicates fluid velocity vector, $\rho_f$ indicates density of fluid, and $\Omega_f$ indicates fluid domain. Functional of fluid-structure interaction problem is derived by adding equation of continuity and geometric boundary condition to Eq. (3) by using Lagrange multiplier method and adding it to Eq. (2). In addition, Eq. (4) is derived by stationary condition.

$$\Pi_f + s = \int_0^T \left[ \int_{\Omega_s} \left\{ \delta_{\overline{E}} T \overline{S} - \rho_m \delta_{\overline{u}} T \overline{u} \right\} d\Omega_s + \int_{\Omega_f} \rho_f \overline{\delta \phi} \cdot \overline{\delta \phi} \, d\Omega_f \right] dt = 0$$

(4)

$$\pi_f = \frac{1}{2} \rho_f (\nabla \phi)^2 + \rho_f \dot{\phi} + \rho_f F_f$$

(5)

Where $\overline{E}$ and $\overline{S}$ indicate Green-Lagrange strain in principle direction and Second Piola-Kirchhoff stress, $\phi$ indicates velocity potential, $n_s$ and $n_f$ indicate normal vector on boundary of structural and fluid, and $\Gamma_{fs}$ indicates fluid-structure interactional boundary. ALE method is applied to Eq. (4) and mesh control which becomes Euler coordinate as it is close to bottom of tank and becomes Lagrange coordinate as it is close to upper of tank is defined. These are discretized by using FEM and Newmark- $\beta$ method is applied to them to deal with time derivative term and solution is derived by Newton-Rapshon method which is solution method of nonlinear equation.
3. Numerical evaluation and discussion

In the following section, the researchers concerned on the membrane behaviour in the vicinity of the natural frequency, and presented some numerical results together that related to discussion.

Before explanations, the parameters of numerical model were set up as follows. The width, length and depth of the container size were set with as 0.8 m, 0.8 m and 0.3 m respectively, and the membrane thickness was 0.001 m. Gravity is taken account. The fluid was assumed to be water, namely, the density is 1,000 kg m\(^{-3}\). The excitation was restricted to be sinusoidal horizontal excitation, and undamped free vibration continues after that. The influence of the rubberlike membrane is considered in a sloshing analysis. The tank size in the sloshing model is the same as that in the rubberlike membrane–fluid interaction model. The analysis conditions are shown in Table 2. For the interaction model, the resonant frequency at the maximum vertical displacement of 5 cycles is used as the vibration frequency. For the sloshing model, the one-degree mode natural frequency of the linear theory is used as the vibration frequency.

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<th>Table 2: Analysis condition.</th>
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<td>Initial strain</td>
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<td>(E_{x0})</td>
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<tr>
<td>Interaction model</td>
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<td>Sloshing model</td>
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3.1 Flow rate and pressure distribution

The flow rate and pressure distribution of the fluid after 15 cycles are shown in Fig. 2 and Fig. 4 for the fluid–membrane interaction and the sloshing model, respectively: Fig. 2(a) and Fig. 4(a) at \(x = 0.1\) m, Fig. 2(b) and Fig. 4(b) at \(y = 0.4\) m, and Fig. 2(c) and Fig. 4(c) for the half-model. The pressure distribution is shown with the hydrostatic pressure. It should be noted that in the sloshing model the flow in the y direction is uniform for an inviscid liquid and the free-surface pressure is zero because the atmospheric pressure is neglected.

After 15 cycles, the flow rate is almost zero in both models. The distribution of fluid pressure in the interaction model, excluding the hydrostatic fluid pressure, after 15 cycles is shown in Fig. 2(c). The pressure distribution is large compared with that shown in Fig. 3(c) for the sloshing model near the edge of the tank, owing to the prevention of fluid motion by the membrane.

3.2 Membrane displacement

The deformation of surface at symmetric plane y equal 0.4 m after 5 cycles and 15 cycles are shown in Fig. 3. The solid lines indicate rubberlike membrane interaction model and the dashed lines indicate sloshing model. The vertical axis indicates vertical displacement which is dimensionless value with depth of tank.

The deformation of membrane near centre of tank of rubberlike membrane interaction model is almost same as one of sloshing. The effect of membrane is strong only near edge of tank which is constrained by membrane. As deformation is large, the deformation of membrane becomes left-right asymmetry. This asymmetry is also appeared in sloshing model. This is due to convection term. In the other words, if a linear theory were employed, this symmetry would have been predicted [10]. In Fig. 4, the result notices that the amplitude of upward displacement is larger than the amplitude of downward displacement. After 15 cycles, the upward displacement is 5.5% greater than the downward displacement. This phenomenon is mainly observed in the study on a fluid sloshing [4] and it is strongly affected by the nonlinearity of a fluid due to large deformation.
(a) $x = 0.1\, [\text{m}]$

(b) $y = 0.4\, [\text{m}]$

(c) Pressure distribution after 15 cycles

Figure 2: The flow rate and pressure distribution of the fluid in the interaction model.

Figure 3: Surface displacement.
3.3 Strain and stress distributions

The deformation in the x–y plane after 15 cycles is shown in Fig. 5. There is large shear deformation near the corner, which is consistent with the distribution of $E_{xy}$ in Fig. 6.

$E_{xy}$ near the corner is dominated by the nonlinear term resulting from the out-of-plane deformation. In this model, the pressure is concentrated where the membrane is restrained. The shear deformation is due to a tension force generated in a direction inclined at 45° to the x-axis. Elsewhere, the fluid pressure shows a relatively uniform distribution, with no strong concentration. The Cauchy stress distribution is shown in Fig. 7. The stress distribution is almost the same as the strain distribution, for the same reasons.

Figure 5: X-Y plane deformation after vibration 15 cycles.
(Black solid line before the deformed shape, displacement 10 times)
4. Conclusions

In this section, we have analysed the dynamic characteristics of the interaction model of a rectangular tank containing fluid whose upper surface is covered by a membrane, comparing these with the results obtained from the sloshing model in the large-deformation domain. We have arrived at the following conclusions:

1. The presence of the membrane has a strong influence on the fluid pressure and deformation, because fluid motion is constrained close to the membrane. Elsewhere, the effects of the fluid are...
dominant and there is good agreement between the free-surface displacement from the sloshing model and the membrane displacement from the interaction model.

2. Nonlinearity due to left–right asymmetry of the membrane deformation is observed. At large deformation, the upward displacement is about 5.5% greater than the downward displacement and the central point moves to the left by about 1.3%. These phenomena are also found in the sloshing model owing to the presence of the convection term.

5. **Acknowledgement**

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**REFERENCES**


