MATHEMATICAL MODEL OF THE APPARENT MASS OF THE BACK OF A SEATED PERSON EXPOSED TO FORE-AND-AFT WHOLE-BODY VIBRATION

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A mathematical model has been developed to represent the fore-and-aft dynamic response of the back of the seated human-body during exposure to fore-and-aft whole-body vibration. Optimum parameters for the model were derived from the measured apparent masses of the backs of ten male subjects in a previously reported study. The model provides close predictions of the measured modulus and phase of the apparent mass of the back between 0.25 and 10 Hz. The fitted mass, \( m \), spring stiffness, \( k \), and damping, \( c \), obtained by fitting the model to the median apparent mass of the backs of the ten subjects showed a similar response with the median values of \( m \), \( k \), and \( c \) obtained by fitting the individual responses using modal averaging. It is concluded that the model provides an apparent mass similar to that of the human back, although it does not seek to represent the internal movement of the body during fore-and-aft excitation.

1. Introduction

Drivers, other operators, and passengers of vehicles are exposed to vibration of the body through vehicle seats. Most seats are compliant – they modify the vibration by amplifying it at low frequencies and attenuating it at high frequencies. A seat should provide a good sitting environment for the occupant (e.g. it should be comfortable over a long period) and not prevent any activities the occupant has to perform. There have been extensive studies of the transmission of vertical vibration through seats and a few studies have investigated the transmission of fore-and-aft vibration. Most studies have involved the measurement of seat transmissibility with subjects sitting in the seat, either in laboratory studies or field measurements. Although such measurements are ‘real’ (as human subjects are involved), they are: (i) time consuming, (ii) require a suitable vibrator (in the case of laboratory measurements) to reproduce vibration that is suitable and safe for human exposure, and this can be very expensive, and (iii) there are “inherent risks that exposure to the mechanical vibration or shock which the experiment is intended to reproduce may lead to injury or ill-health, either immediately or at some time in the future” [1].

Previous studies have shown that it is possible to predict the vertical vibration transmitted through a seat without using human subjects. The alternative methods are: (i) the use of anthropodynamic dummies with appropriate mechanical impedance to replace human subjects (e.g. [2-3]), or (ii) the use of mathematical models based on separate measurements of the impedance of the body and the impedance of the seat (e.g. [4-5]). Models representing the impedance of a seated person exposed to vertical excitation have been extensively reported (e.g. [2, 6-7]). The apparent mass of a seated person during vertical excitation can, in the simplest way, be presented by a single degree-of-freedom model [5]. Many models adequately represent the apparent mass of the seated human body with no backrest during vertical excitation, although they do not represent the mechanisms involved in the internal movements of the body.
There has been little study of models of the apparent mass of seated person during fore-and-aft excitation. Mansfield and Lundström [8] suggested a three-degree-of-freedom model (three mass-spring-damper systems arranged in parallel) can adequately represent the apparent mass of the body during fore-and-aft and lateral excitation. Nawayseh [9] showed that a two-degree-of-freedom model, with a rotational capability can represent the fore-and-aft apparent mass of the body during fore-and-aft excitation. The model was also capable of representing the ‘cross-axis’ vertical apparent mass of the body during fore-and-aft excitation. However, both models (by Mansfield and Lundström [8] and Nawayseh [9]) only consider the apparent mass of the body when subjects are seated with an upright posture with no backrest. Recently, Qiu [10] proposed a multi-degree-of-freedom model of the apparent mass of the seated human body exposed to fore-and-aft excitation, with the body consisting of two sub-systems representing the upper-body and the lower-body. The parameters were derived by comparing the model with experimental data reported by Nawayseh and Griffin [11]. Although the model shows good approximation to the apparent mass of a seated person at both the back and the seat, it is a complex model (including both translational and the rotational degrees-of-freedom) and not a mechanistic model.

The objective of this paper is to develop a simple lumped parameter model that can adequately represent the apparent mass of the back during fore-and-aft excitation. The developed model is not a mechanistic model: it does not represent the mechanisms involved in body movement during the response of the body to vibration. The model was developed so that when it is combined with a dynamic model of a backrest cushion, a seat-person model can be formed to predict the fore-and-aft transmissibility of the backrest.

2. Modelling of the human body

A simple two degree-of-freedom model was developed to predict the fore-and-aft apparent mass of the back (Figure 1). The model consisted of two parallel mass-spring-damper systems.

![Figure 1: Two degree-of-freedom lumped parameter model.](image)

In the figure:

- \(m_0, m_1\) and \(m_2\): Masses of mass 0, mass 1, and mass 2. Mass 0 can be regarded as the support frame of the model.
- \(k_1\) and \(k_2\): Stiffness coefficients of the springs of sub-systems containing mass 1 and mass 2, respectively.
- \(c_1\) and \(c_2\): Damping coefficients of sub-systems containing masses 1 and 2, respectively.
The equations of motion of the model can be described as:

\[ m_1 \ddot{x}_1 + c_1 (\dot{x}_1 - \dot{x}_b) + k_1 (x_1 - x_b) = 0 \]  

(1)

\[ m_2 \ddot{x}_2 + c_2 (\dot{x}_2 - \dot{x}_b) + k_2 (x_2 - x_b) = 0 \]  

(2)

By using the Laplace Transform on the assumption that \( x(0) = 0, \dot{x}(0) = 0 \) and \( x_b(0) = 0 \), and by some mathematical manipulation, the equations of motion can be written as:

\[ (m_1 s^2 + c_1 s + k_1) X_1(s) = (c_1 s + k_1) X_b(s) \]  

(3)

\[ (m_2 s^2 + c_2 s + k_2) X_2(s) = (c_2 s + k_2) X_b(s) \]  

(4)

By Newton’s second law of motion:

\[ F(t) = m_0 \ddot{x}_b + m_1 \ddot{x}_1 + m_2 \ddot{x}_2 \]  

(5)

\[ F(s) = m_0 s^2 X_b(s) + m_1 s^2 X_1(s) + m_2 s^2 X_2(s) \]  

(6)

where \( F(t) \) is the total force acting on the model. The apparent mass of the model can be computed by dividing both sides with \( s^2 X_b(s) \), such that:

\[ M(s) = m_0 + m_1 \frac{B}{A} + m_2 \frac{D}{C} \]  

(7)

where

\[ A = m_1 s^2 + c_1 s + k_1 \]

\[ B = c_1 s + k_1 \]

\[ C = m_2 s^2 + c_2 s + k_2 \]

\[ D = c_2 s + k_2 \]  

(8)

By replacing the Laplace Transform variable, \( s \), with the angular frequency, \( \omega \), based on the relation \( s = i\omega \), the modulus, \( |M| \), and phase, \( \arctan^{-1}(M) \) of the apparent mass of the model can be calculated.

### 3. Fitting with experimental data

The parameters of the model were optimised by comparing the response of the model with the measured apparent mass of the back (whole-back in contact with the backrest) with a fore-aft vibration magnitude of 0.4 ms\(^{-2}\) r.m.s., as measured by Abdul Jalil and Griffin [12]. The experiment measured the apparent masses of 10 subjects in the frequency range 0.25 to 10 Hz. The median results, as well as the individual responses, were used in optimising the parameters of the model. A curve fitting method was used to obtain the model parameters \( m_n \), \( k_n \) and \( c_n \) \((n = 0, 1, 2)\) from the magnitude and phase of the apparent mass, \( M \). The least-square-error method with an optimisation algorithm was utilized (i.e. the constrained non-linear multivariable function in Matlab - fmincon). The parameters in the equation of each model were refined to minimize the error function over the whole frequency range, such that:
\[
error = \sum_{i=1}^{N} \left( M_P(i) - M_T(i) \right)^2 + F^* \sum_{i=1}^{N} \left( P_P(i) - P_T(i) \right)^2
\] 

where \( M_P(i) \) and \( P_P(i) \) are the corresponding apparent mass and phase from the curve fit at the \( i \) th frequency, \( M_T(i) \) and \( P_T(i) \) are the measured apparent mass and phase data, respectively. The scalar weighting, \( F \), was used to weight the relative importance of errors in the modulus and errors in the phase.

Figures 2 to 4 show the curve fitting obtained from the model with the median and individual apparent masses of the back with 10 subjects. For the optimisation process, all parameters were constrained such that the value of any parameter would be either zero, or a positive value – this was to give a physical meaning to the model, and to make it possible to construct a mechanical model. An exception was the mass \( m \) (i.e. support frame) where the minimum value was fixed at 1 kg – this was to avoid a massless frame (which would be difficult if the model were to be constructed). The optimised parameters are listed in Table 1 (median) and Table 2 (individual). Table 2 was organised so that the low frequency mode and the higher frequency mode are averaged – giving the response shown in Figure 2 (solid thin line).

![Figure 2: Median apparent mass and phase (with full back) measured with 10 subjects at 0.4 ms\(^{-2}\) r.m.s. and the optimised response of the model. Experiment (thick solid); optimised parameters with median response (---) – see Table 1; optimised parameters using median of individual responses (—) – see Table 2.](image-url)
Figure 3: Individual moduli of the apparent masses of the backs of 10 subjects (with full back) at 0.4 ms$^2$ r.m.s. with optimised response of the model. Experiment (———); model (··········).

Figure 4: Individual phases of the apparent masses of the backs of 10 subjects (with full back) at 0.4 ms$^2$ r.m.s. with optimised response of the model. Experiment (———); model (··········).
Table 1: Optimised model parameters from median apparent mass of the back with 10 subjects.

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<tr>
<th></th>
<th>m (kg)</th>
<th>m₁ (kg)</th>
<th>m₂ (kg)</th>
<th>k₁ (N/m)</th>
<th>k₂ (N/m)</th>
<th>c₁ (Ns/m)</th>
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Table 2: Optimised model parameters from the individual responses of the apparent masses of the backs of the 10 subjects.

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<th>Subject</th>
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<th>m₁ (kg)</th>
<th>m₂ (kg)</th>
<th>k₁ (N/m)</th>
<th>k₂ (N/m)</th>
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Note: Apparent mass data for subjects 1 and 2 were excluded due to errors during measurements.

The model showed a good fit with the median apparent mass and phase and was able to depict the two resonance frequencies of the apparent mass of the back around 1 Hz and 4 Hz. The model also gave a good fit to the moduli and phases of the apparent masses of individual subjects.

3.1 Sensitivity analysis

A parameter sensitivity test was carried out to determine the effect varying of parameter m, k and c on the response of the apparent mass of the back (Figure 5). The test was performed on the model in Figure 1 with parameters based on the fit to the median apparent mass. All parameters were varied by ±50% from the optimised parameters.

From the sensitivity test, it can be seen that either one of the sub-system has clear effect on the response of the apparent mass when the parameters were varied. The other sub-system only showed small effect with varying parameters. It was also observed that changing m in both of the sub-systems showed clear effect on the apparent mass at low frequency. Varying the spring stiffness, k, in only one of the sub-systems can greatly affect the apparent mass response at frequency greater than 5 Hz. Meanwhile, noticeable effect was found on the response of the apparent mass between 4 Hz and 7 Hz when the damping coefficient was varied. In addition, the damping coefficient is also sensitive on the apparent mass at resonance around 5 Hz.
4. Discussion

The justification for using the median apparent mass (thick solid line in Figure 2) to develop the human body model is that the median data have an appropriate overall shape with roughly similar resonances as the individuals (solid lines in Figure 3). The individual data in Figure 3 show evidence of two resonances, a minor resonance around 1 Hz and a major resonance around 5 Hz. The optimized response of the model based on the median data (Table 1; dotted line in Figure 2) and the median data from the individual responses (Table 2; solid thin line in Figure 2) also show two resonances: one around 1 Hz and another around 5 Hz, similar to the median data. Furthermore the model parameters obtained by fitting the model to the median apparent mass and from the median of the fitting to the individual apparent masses are seen to be similar (compare values in Table 1 and Table 2).

5. Conclusions

A simple linear two-degree-of-freedom lumped parameter model can provide good predictions of the fore-and-aft apparent mass of the human back. The three masses, $m_0$, $m_1$ and $m_2$, the two spring stiffnesses, $k_1$ and $k_2$, and the two damping values, $c_1$ and $c_2$, obtained by fitting a two degree-of-freedom model to the median measured apparent mass were similar to the median values obtained over 10 individual subjects. Although the model can estimate the apparent mass of the back and may assist the prediction of the fore-and-aft transmissibility of backrests, it does not seek to represent the way in which the body moves internally during fore-and-aft excitation.
REFERENCES


