STABILITY ANALYSIS OF BOILERS USING FUEL BLENDS CONTAINING HYDROGEN

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Domestic and industrial burners typically use natural gas fuel and thus emit considerable quantities of carbon dioxide and carbon monoxide to the atmosphere. An up-and-coming method to reduce the carbon emissions, is to add hydrogen in various quantities (0-100 %) to the natural gas fuel. The addition of hydrogen changes the combustion characteristics of the burner and may cause thermoacoustic instabilities. In the present work, we perform an analytical study of the stability behaviour of boilers; the main parameters of interest are: the hydrogen concentration in the fuel and the equivalence ratio. We model the burner as an array of small conical flames stabilised on a perforated plate with the dynamics of each individual flame described by the non-linear G-equation. The heat release rate is calculated and expressed in terms of a flame describing function. The combustion chamber geometry is approximated by a quarter wave resonator and the mathematical modelling of the complete combustion system is based on the tailored Green’s function method. The stability of the system is given by the time evolution of the acoustic velocity at the flame location and the stability behaviour is presented as a stability map in the hydrogen fraction ($x_{H_2}$) – amplitude of velocity fluctuation ($A/\bar{u}$) plane.

Keywords: domestic boiler, natural gas - H$_2$ fuel, FDF, multiple time-lag model, Green’s function

1. Introduction

In recent years, researchers are investigating the effective use of hydrogen blended fuels to increase the burning efficiency and to reduce carbon monoxide (CO) and NO$_x$ emissions from the combustion of fuels. However, hydrogen has a very high flame speed (more than 5 times higher than natural gas) which can lead to flame flashback, thermoacoustic instabilities and aeroacoustic instabilities in a combustion system (comprising of burner and heat exchanger) of a boiler system. Our paper focuses on the thermoacoustic instabilities that can cause unwanted noise, damage to the boiler and/or shortened boiler life.

In the present paper, we describe an analytical model to study the thermoacoustic instabilities in a generic boiler system. Our model is based on the following assumptions:
1. A quasi-1D geometry, which may have irregularities (such as jumps in the cross-section or blockages).
2. A compact flame: We model this generically as an input-output system described by a flame transfer function, which depends on the velocity amplitude (relative to the mean flow velocity).
3. Non-uniform temperature inside the system, i.e. there are cold regions and hot regions.
4. Generic end conditions. These are described by frequency-dependent reflection coefficients.

Under these assumptions, we aim to examine how the thermoacoustic instabilities in a domestic boiler system is influenced by the addition of variable quantities of hydrogen to lean premixed natural
gas (NG) - air flames (present in the boiler system). To this end, the boiler system is first simplified to a one dimensional network model formed of the scattering matrices of the individual components of the system. Stability predictions of the simplified system is carried out through a tailored Green’s function approach.

The paper is structured as follows: Section 2 describes the simplified one dimensional model of the boiler system, Section 3 describes the flame modelling aspect of the network model and Section 4 describes the Green’s function approach used to compute the stability predictions and also the preliminary results and discussions of our study and Section 5 provides an outlook for future work.

2. Schematic representation of combustion system

In order to model the boiler system for the purpose of thermoacoustic analysis, we first simplify it into a one dimensional (1D) system as shown in Fig. 1. The picture on the left side shows the boiler with its different components, mainly the gas inlet, flame holder plate, flame and outlet. The picture on the right side shows the simplified 1D system consisting of the different acoustic elements that can be combined using a network model as explained in Section 2.1.

2.1 Network model

The network model for the combustion (boiler) system studied is as shown in Fig. 2. It consists of a long duct containing a perforated plate and a flame. These components divide the system into three regions A, B and C–D, and the pressure field within each region is represented through forward and backward travelling waves as shown in the figure. In the network model, each component in the combustion system is represented in terms of a scattering matrix through the corresponding transmission and reflection coefficients that relate the upstream and downstream pressure waves.

The upstream (at $x = 0$) and the downstream (at $x = L$) ends of the system are represented by the reflection coefficients $R_0$ and $R_L$ respectively. The reflection coefficient $R_0$ relates the forward travelling wave $a_+$ and backward travelling wave $a_-$ in region A and $R_L$ relates $d_+$ and $d_-$ in region D.

The next component in our network model is the flame holder on which the flame is stabilised. In a typical burner assembly, this holder will be in the form of a perforated plate. In our analysis, the perforated plate is denoted by a metallic plate of finite thickness with circular holes positioned
perpendicular to the flow, at $x = x_p$. The perforated plate is modelled in terms of its reflection and transmission coefficients ($T_{AB}$, $R_{AB}$, $T_{BA}$ and $R_{BA}$ in Fig.1), as described in [1, 2]. These coefficients depend on the thickness of the plate, radius of the perforations, number of holes per unit area, frequency of the incident wave and the speed of sound.

Even though the flame is stabilised on the flame holder, in our analysis, we assume that there is a small distance between the flame and the flame holder, with the flame located at $x = x_q$, downstream of the flame holder. In Fig. 2, the flame is depicted as a thin sheet that is compact compared to the duct length (or wavelength of pressure waves). In reality, the flame is not a continuous sheet, but rather a collection of small flames that are stabilised on each of the small perforations (see Fig. 3(a)). But the length scales of these flames are very small compared to the duct length (or wavelength) and therefore the thin sheet assumption holds for such flames. For analytically modelling these flames, we approximate the flame at each perforation as a simple conical flame (Fig. 3(b)). The description of the flame model and the relevant assumptions used in the modelling are given in Section 3.

3. Flame model

Unlike the other components in the boiler system, it is the flame and its dynamics that is important when we investigate the effect of adding $H_2$ to the fuel. In thermoacoustics, the relationship between the heat release rate and the acoustic field is crucial and this relation is described by a heat release rate law, either in the time-domain or in the frequency-domain. In general, the time-domain expression for the heat release rate law is given in terms of a functional $F$, and the corresponding frequency-domain expression as a flame transfer function (FTF) $\mathcal{T}(\omega)$ as given in Eqs. (1a and 1b) respectively.

$$\frac{Q'(t)}{Q} = F \left[ \frac{u'(t)}{\bar{u}} \right], \quad \text{and} \quad \mathcal{T}(\omega) = \frac{\dot{Q}(\omega)}{\bar{Q}} \frac{\dot{u}(\omega)}{\bar{u}}.$$  (1 a, b)
where \( Q'(t) \) is the fluctuation of the heat release rate in the time-domain, \( \hat{Q}(\omega) \) is its frequency-domain equivalent (Fourier transform), and \( \bar{Q} \) is the mean rate of heat release; the same notation is used for the acoustic velocity, \( u \) (at a chosen reference position upstream of the flame). The FTF may also depend on the amplitude of the velocity fluctuation (\( \epsilon = A/\bar{u} \)), in which case it is referred to as flame describing function (FDF).

The FTF or FDF of a given burner can be measured, and the result is a sequence of complex numbers at discrete frequencies. For analytical modelling purposes, it is necessary to convert such data into a continuous function of frequency. This is typically done by some ad hoc curve-fitting procedure. Several examples can be found in the literature: [3, 4].

### 3.1 FDF for hydrogen blended fuel

For the conical flames of interest to us, FTFs/FDFs are not readily available in literature. Hence, we use the level-set solver tool GFlame [5, 7] to evaluate the FTFs of conical flames for various input parameters like the fuel-air mixture (e.g. - Methane-Air, NG-H2-Air etc.), equivalence ratio (\( \phi \)), hydrogen fraction (\( x_{H_2} \)) for NG-H2 fuel, inlet temperature, inlet pressure, amplitude of excitation (\( \epsilon = A/\bar{u} \)) etc. One of the methods to evaluate FTF for perfectly premixed flames is by measuring the flame surface area fluctuations and relating them to upstream velocity perturbations. We use the convective incompressible velocity model [8] available in GFlame to calculate the instantaneous flame shape and the flame transfer function. In our present study, we consider fluctuations in velocity only (and no fluctuations in equivalence ratio), and we also neglect the curvature dependent fluctuations in the flame speed [9].

The heat released from the combustion of fuels occurs at the flame front and any perturbation in the upstream quantities like velocity, equivalence ratio etc. will be manifested as fluctuations in the heat released. Since the heat released is proportional to the surface area of the flame front, the fluctuations in heat release can be assumed to be proportional to the fluctuations in the flame surface area. Hence, for our analysis, we currently use area fluctuations for evaluating FTFs.

Simulations of a conical flames of radius \( R = 1 \text{mm} \) were carried out using GFlame for NG-H2 mixture with three different hydrogen volume fraction (\( x_{H_2} = 0, 10, 20 \% \)). FDF for one hydrogen fraction (\( x_{H_2} = 10\% \)) is given in Fig. 4 (represented by markers). The mean flow velocity \( \bar{u} \) used for all the cases is 1 \text{m/s}, and the equivalence ratio \( \phi \) is 0.95.

![Figure 4: FDF of a conical flame for hydrogen fraction \( x_{H_2} = 10\% \). Markers represent the FDF obtained using GFlame and the curves represent their approximation using multiple time-lag method given in Section 3.2. The symbol in the legend \( \epsilon = A/\bar{u} \).](image-url)
3.2 Multiple time-lag model

The dynamic behaviour of many flames are characterised by two or more prominent time-lags, and by a distribution of the heat release rate around these time-lags. Let us assume a generic heat release rate law with \( k \) prominent time-lags \( \tau_1, \tau_2, \ldots, \tau_k \), and with a Gaussian distribution \( D \) centred around each of them,

\[
\frac{Q'(t)}{Q} = n_1 \int_{-\infty}^{\infty} \frac{u'(t-\tau)}{\bar{u}} D(\tau - \tau_1) d\tau + \cdots + n_k \int_{-\infty}^{\infty} \frac{u'(t-\tau)}{\bar{u}} D(\tau - \tau_k) d\tau,
\]

where \( D \) is given by

\[
D(\tau - \tau_j) = \frac{1}{\sigma_j \sqrt{2\pi}} e^{-\frac{(\tau - \tau_j)^2}{2\sigma_j^2}}, \quad j = 1, 2, \ldots, k.
\]

Eq. (2) contains \( 3k \) parameters, \( \tau_1, \tau_2, \ldots, \tau_k, n_1, n_2, \ldots, n_k \) and \( \sigma_1, \sigma_2, \ldots, \sigma_k \), which are treated as fitting parameters and are assumed to be amplitude-dependent. Taking the Fourier transform of Eq. (2) and rearranging, we get the FTF for each amplitude of excitation, i.e. the FDF as

\[
T_k(A, \omega) = \frac{\bar{Q}(\omega, A)}{\bar{Q}(\omega, A)} = n_1(A) e^{-\frac{\omega^2 \sigma_1(A)^2}{2} e^{i\omega \tau_1(A)}} + \cdots + n_k(A) e^{-\frac{\omega^2 \sigma_k(A)^2}{2} e^{i\omega \tau_k(A)}}.
\]

The unknown fitting parameters in this multiple time-lag approximation are determined using the optimisation routine \textsf{lsqnonlin} in \textsc{Matlab}® by minimising the error between unit impulse functions (UIRs) of the approximated FDF and the actual FDF for all amplitudes of excitation at which measurement data are available. The FDF calculated using multiple time lag approximation (for \( x_{H_2}=10\% \)) is represented by solid/dashed curves in Fig. 4.

The amplitude dependence of each parameter in the MTL approximation is also modelled analytically by representing it by a linear function, altogether, we obtain a fully analytical expression for the FDF. The amplitude dependence for \( x_{H_2}=10\% \) is shown in Fig. 5. The amplitude dependence can be approximated using a polynomial fit and the FTF at any intermediate amplitude can be evaluated.

![Figure 5: Amplitude dependence of model parameters for \( x_{H_2}=10\% \).](image)

4. Mathematical Model

The mathematical modelling of the complete combustion system is based on the tailored Green’s function method.
4.1 Tailored Green’s function

The Green’s function $G(x, x', t - t')$ is the response observed at position $x$ and time $t$ to a point source at position $x'$ firing an impulse at time $t'$. Its governing equation is

$$\frac{1}{c^2} \frac{\partial^2 G}{\partial t^2} - \frac{\partial^2 G}{\partial x^2} = \delta(x - x') \delta(t - t').$$

(5)

The tailored Green’s function [2, 10] is the solution of Eq. (5), which satisfies the same conditions at all boundaries and interfaces as the acoustic field (here expressed in terms of the velocity potential). Naturally, this is a superposition of modes,

$$G(x, x', (t - t')) = H(t - t') \Re \left[ \sum_{n=1}^{\infty} g_n(x, x') e^{-i\omega_n(t-t')} \right].$$

(6)

$H(t - t')$ denotes the Heaviside function; it guarantees causality. The quantities $g_n$ (Green’s function amplitude of mode $n$) and $\omega_n$ (modal frequencies if thermoacoustic coupling is absent) is determined analytically for the simplified boiler system (see [2] for details on the derivation).

4.2 Stability analysis

In our case, the Green’s function is a velocity potential. The governing equation for the velocity potential in the presence of unsteady heating is given by

$$\frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = B q(x, t),$$

(7)

where $B = -(\gamma - 1)/c^2$ (abbreviation), and $\gamma$ is the specific heat ratio. The term $q(x, t)$ is the fluctuating part of the heat release rate per unit mass of air, and for a compact flame (located at $x_q$), we can put

$$q(x, t) = q(t) \delta(x - x_q).$$

(8)

Applying the boundary conditions

$$\phi(x, t)|_{t=0} = \varphi_0 \delta(x - x_q) \text{ and } \frac{\partial \phi(x, t)}{\partial t} \bigg|_{t=0} = \varphi'_0 \delta(x - x_q)$$

(9)

for the velocity potential, causality conditions

$$G(x, x', t - t') = 0 \text{ and } \frac{\partial G(x, x', t - t')}{\partial t} = 0 \quad \text{if } t \leq t'$$

(10)

for Green’s functions and following the procedure in [2, 11], we can convert Eq. (7) into an integral equation for the velocity $u'_q(t)$ at the heat source,

$$u'_q(t) = B \int_{t'=0}^{t} \frac{\partial G(x, x', t - t')}{\partial x} \bigg|_{x=x_q} q(t') \ dt' - \frac{1}{c^2} \Re \left[ \sum_{n=1}^{\infty} \varphi_0 i\omega_n \frac{\partial G(x, x', t - t')}{\partial x} e^{-i\omega_n(t-t')} \right] x, x' = x_q, t' = 0.$$  

(11)

Equation (11) gives the time evolution of acoustic velocity at the heat source/flame location. The acoustic velocity grows in amplitude for the unstable case and decrease in amplitude for a steady case during its evolution in time.
4.3 Stability Predictions

The system studied is a tube (Fig. 2) of length 1m and radius 0.035m with a rigid end on the left side \((R_0 = 0)\) and an open end on the right side \((R_L = -1)\). The flame is stabilised on a perforated plate of 3mm thickness at a distance 0.2m from the rigid end. The perforations are of 1mm radius and the perforation density is \(1.09 \times 10^5\) holes/m². The composition of the natural gas-hydrogen mixture is calculated using the expressions provided in [12]. The temperature in the cold region is 300K and the temperature in the hot region is assumed as the adiabatic flame temperature calculated using the NASA Chemical Equilibrium Application solver [13, 14].

Using the tailored Green’s function, we calculate the time evolution of acoustic velocity to predict the stability behaviour of the system. In Fig. 5(a), we give a stability map showing the stability predictions for variations in hydrogen volume fraction in the fuel \(x_{H_2}\) vs the magnitude/amplitude of velocity fluctuation \(A/\bar{u}\) in the incoming velocity of the fuel-air mixture. The red stars indicate unstable states and green dots indicate stable states. We also give time evolutions for a stable state and an unstable state in Figs. 5(b) and (c) respectively. For the stable state, the amplitude of the acoustic velocity decreases initially and then reaches a limit cycle of amplitude \(\sim 0.05\), whereas for the unstable state, the amplitude increases initially and then reaches the limit cycle of amplitude \(\sim 0.05\). This limit cycle corresponds to the boundary between stable and unstable regions.

5. Outlook

In the present work we model a domestic boiler as a 1D network model, approximating the burner as a collection of simple conical flames. The fuel used in the boiler is a mixture of natural gas and hydrogen and the FDF for each conical flame was calculated using the level-set solver GFlame. The stability of the system was studied using a tailored Green’s function approach. The stability behaviour for different hydrogen fraction-amplitude ratio concentrations were obtained from the time evolution of the acoustic velocity of the system and this gives a qualitative description of the safe operability limits of boiler systems. The present study models a generic boiler using simplifying assumptions and give the stability predictions at a few combinations of \(x_{H_2}\) and \(A/\bar{u}\). The prediction of stability behaviour for an actual boiler system and preparation of stability maps in the \(x_{H_2}-A/\bar{u}\) space are part of our work in progress.

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REFERENCES


