This paper focused on the research conducted on time reversal based signal processing techniques for ultrasound super resolution imaging in the area of industrial non-destructive evaluation (NDE). Time reversal techniques, which exploit the time reversal invariance of the wave equation, take advantage of the time-reversed fields for improved imaging and focusing. This is because time-reversed signals propagate backwards through the same medium and undergo similar reflection, refraction and multiple scattering that they underwent during the forward propagation, resulting in focusing around the initial source locations. Time reversal with multiple signal classification (TR-MUSIC), a typical time reversal based signal processing technique, is introduced to image defects in solids in this paper. Its single frequency and multi-frequency forms are tested with ultrasound array data acquired using the full matrix capture process, and their performance is estimated in terms of spatial resolution and robustness to noise, two particularly important indicators for industrial NDE. It is shown that TR-MUSIC implementing on the central frequency is capable of resolving lateral targets spaced closer than the Rayleigh limit, achieving super resolution imaging. For high noise levels, TR-MUSIC implementing on the multi-frequency is shown to provide stable ultrasound image.

Keywords: time reversal, ultrasound imaging, super resolution, NDE

1. Introduction

Ultrasound phased array imaging is now widespread in both the Non-destructive evaluation (NDE) and medical fields [1-2]. Compared to traditional single element transducers, the advantage is the ability to perform multiple inspections without the need for reconfiguration and the potential for improved sensitivity and coverage. Both fields mostly use beamforming techniques to obtain ultrasound image, such as synthetic aperture focusing technique (SAFT) [3-4], inverse wave-field extrapolation [5-6], and total focusing method (TFM) [7]. All these techniques are based on the concept of delay and sum, and work well due to their robustness as well as ease of implementation. However, their imaging resolution obeys diffraction criterion. For two closely spaced point scatterers, if their distance is smaller than the minimum resolved distance when a given array is employed, the two point scatterers cannot be distinguished according to the ultrasound image obtained by beamforming techniques. The diffraction limit has been challenged by progress made in optical microscopy and radar where near field [8-9] and far field [10] super resolution has been demon-
strated. More recently, super resolution has been demonstrated in ultrasonic imaging via the time reversal based signal processing techniques [11-12].

Ultrasound time reversal techniques have been widely applied to yield highly focused beams in applications such as the destruction of kidney stones [13], medical hyperthermia and brain therapy [14], as well as super resolution imaging [15]. One typical technique is the time reversal with multiple signal classification (TR-MUSIC) developed by Devaney [16-17]. It combines time reversal focusing with the MUSIC signal-subspace method together to yield a pseudo-spectrum that peaks at the locations of the point scatterers. This paper focused on the application of TR-MUSIC in imaging the defects in solids. Its single frequency and multi-frequency forms are considered. In addition, its ability is experimentally demonstrated in terms of spatial resolution and robustness to noise.

2. The concept of ultrasound time reversal

This section shows the concept of ultrasound time reversal, which relies on the property of time reversal invariance of the ultrasound wave propagation in a lossless medium. The time reversal focusing process is shown in Fig. 1.

An element of array is selected to generate an ultrasound pulse to illuminate the region of interest in the media. If the region contains a scatterer, the scattered wavefront is detected by the array that now works in the receive mode, and converted into electrical signals that are recorded. Then a temporal window is used to select the portion of signals from the received wavefront that are time reversed and stored in electronic memories. The time-reversed signals are finally transmitted to the transducers, thus resulting in an ultrasound wavefront that refocused on the scatterer through the interfaces [18].

3. Time reversal based signal processing technique

This section describes the detailed process of time reversal based signal processing technique utilized to obtain ultrasound image. Firstly, a forward model used to simulate the ultrasound array data is presented. The array and scatterers in the model are shown in Fig. 2, where Cartesian coordinates are employed.

As shown in Fig. 2, a uniform linear array of \( N \) elements, located at \( R_l \) (\( l \) ranging from 1 to \( N \)) is used. The array elements are assumed to be long in the \( y \) direction and so it is reasonable to model into two-dimensions, with propagation of energy in the \( x-z \) plane only. Each array element radiates ultrasound into free space \( z>0 \), in which are embedded \( M \) isotropic scatterers, centred at \( r_j \) (\( j \) ranging from 1 to \( M \)). The output of each element was a five cycle, Gaussian windowed tone burst with a centre frequency of 5MHz and a -6dB bandwidth of 50%. In this paper, \( M=2 \), meaning that two closely spaced scatterers are considered. To include possible interactions between the two scatterers, multiple scattering between them is introduced to the model.
For each transmitter-receiver pair, the resulting spectrum $H_{tx,rx}(\omega)$ in frequency domain is given by:

$$H_{tx,rx}(\omega) = D_{tx}D_{rx}F(\omega)\sum_{j=1}^{M} G(R_{tx}, r_j) f_j G(r_j, R_{rx})$$

$$+ D_{tx}D_{rx} \sum_{j=1}^{M} \sum_{j'=1}^{M} G(R_{tx}, r_j) f_j G(r_j, R_{rx}) f_{j'} \phi_{ex}'(r_{j'}) \left(1 - \delta_{j,j'} \right).$$  (1)

where $D_{tx}$ is the directivity function of the array element $R_{tx}$ and $D_{rx}$ is the directivity function of the array element $R_{rx}$. $F(\omega)$ is the frequency spectrum of array element output signal. $G$ is the free space Green function. $f$ is the scattering coefficient for the scatterer. $\phi_{ex}$ is the exciting field [19].

### 3.1 Ultrasound array data

The ultrasound array data are collected using full matrix capture (FMC) process [7]. According to the forward model, the raw time-domain data, $h_{tx,rx}(t)$, recorded from a receiving element at $R_{rx}$ when an element at $R_{tx}$ is transmitter, is the inverse Fourier transform of $H_{tx,rx}(\omega)$. This process, when carried out for each possible transmitter-receiver pair, results in $N^2$ time-domain signals which make up the ultrasound array data.

### 3.2 Multistatic response matrix

In ultrasound NDE, an array of $N$ elements acting in transmit-receive mode is usually used due to the test object. Each array element is excited sequentially and the backscattered signals are measured in parallel by all elements, yielding the multistatic response matrix $K(\omega)$ of the array at the angular frequency $\omega$. In practice, the $K(\omega)$ can be built via FFT of each time-domain signal belonging to the ultrasound array data, and the transform process is shown in Fig. 3. The Fourier transform of the each time-domain signal $h_{tx,rx}(t)$ is taken, and then the complex value of the Fourier transform at a predetermined frequency $\omega$ is extracted as the entry of the matrix $K(\omega)$. Note that if it is assumed that the performance of each array element is equal, the $K(\omega)$ is symmetrical.
3.3 Singular Value Decomposition

The time reversal based signal processing techniques depend on the singular value decomposition (SVD) of the multistatic response matrix $K(\omega)$ or the eigenvalue decomposition (ED) of the time reversal matrix $T(\omega)$ ($T(\omega) = K^*(\omega) K(\omega)$, in which the superscript ‘*’ represents the conjugate of the complex matrix).

The SVD of $K(\omega)$ at the angular frequency $\omega$ can be expressed as [16]:

$$K(\omega) = U(\omega) \Sigma(\omega) V(\omega)^\dagger = U(\omega) \Sigma(\omega) U(\omega)^T.$$  \hspace{1cm} (2)

where $U(\omega)$ and $V(\omega)$ are unitary matrices whose columns are the singular vectors, and $\Sigma(\omega)$ is a diagonal matrix with singular values in decreasing order. The superscript ‘$\dagger$’ represents the conjugate transpose of the complex matrix and ‘$T$’ represents the transpose of the complex matrix. Due to the symmetry of $K(\omega)$, $V(\omega) = U(\omega)^T$ holds, and $T(\omega)$ is Hermitian. So the eigenvectors of $T(\omega)$ are the singular vectors of $K(\omega)$, and the eigenvalues of $T(\omega)$ are the square of the singular values of $K(\omega)$.

The $U(\omega)$ can be divided into signal subspace $U_S(\omega)$ and noise subspace $U_N(\omega)$:

$$U_S(\omega) = [\mu_1(\omega), \mu_2(\omega), \ldots, \mu_m(\omega)].$$  \hspace{1cm} (3)

$$U_N(\omega) = [\mu_{m+1}(\omega), \mu_{m+2}(\omega), \ldots, \mu_N(\omega)].$$  \hspace{1cm} (4)

where $m$ is the dimension of signal subspace.

3.4 Imaging function

Time reversal based imaging is achieved through a steering vector $g(r,w)$, which for each image point $r$ in the imaging area, is given by:

$$g(r,w) = [G(R_1,r,\omega), G(R_2,r,\omega), \ldots, G(R_N,r,\omega)]^T.$$  \hspace{1cm} (5)
where $R_i (i = 1, 2, \cdots, N)$ are array element centre positions and $G$ is the relevant Green function of the medium. In two-dimensional free space, the Green function is given by:

$$G(R_i, r, \omega) = -i/4H_0^{(i)}(k|R_i - r|),$$

(6)

where $i$ is the imaginary unit, $k$ is the wavenumber, and $H_0^{(i)}$ is a cylindrical Hankel function.

For an array in direct contact with the medium, when the multistatic response matrix $K(\omega)$ is known over a range of frequencies ($\omega \in \Delta \omega$), the imaging function of TR-MUSIC is given by [16]:

$$I(r, \Delta \omega) = \left( \int_{\Delta \omega} \sum_{j=m+1}^{N} \left| \mu_j^*, g(r, \omega) \right|^2 \right)^{-1}.$$

(7)

where the angle brackets $\langle \rangle$ represent the standard inner product. The superscript ‘$*$’ represents the conjugate of the complex matrix.

Especially, when TR-MUSIC is only implemented on the central frequency of the array, $\omega_c$, termed CF-TR-MUSIC in this paper, its imaging function can be given by:

$$I(r, \omega_c) = \left( \sum_{j=m+1}^{N} \left| \mu_j^*, g(r, \omega_c) \right|^2 \right)^{-1}.$$

(8)

4. Experimental demonstration

In this section, an experimental system was designed and built to capture the ultrasound array data from test samples. Two experiments are done to estimate the performance of time reversal based signal processing techniques proposed in this paper, through imaging the defects in solids.

4.1 Experimental setup

The experimental system is shown in Fig. 4. A commercial array controller was connected to a PC. The array controller has 128 independent channels, each with 16-bit digitization. A commercial ultrasound linear array was used which has 64 elements with a centre frequency of 5MHz. The array is contact with the test sample via coupling gel.

![Experimental Setup Diagram]

Figure 4: Schematic diagram of the experimental setup.

4.2 Spatial resolution

Spatial resolution is a dramatic metric to be considered in evaluating the imaging technique utilized in the field of ultrasound NDE. Here, spatial resolution means the ability of imaging technique
to resolve two closely spaced targets distributed along the direction perpendicular to the direction of the ultrasound beam.

The first experiment is realized on a block of steel with two 1mm diameter side-drilled holes (SDHs), using the experimental setup mentioned above. The two SDHs located at \((d = \lambda, z = 39\lambda)\), \(\lambda\) is the wavelength of ultrasound wave in the steel sample. According to the Rayleigh criterion \([12]\), the minimum resolvable distance \(d_r = 0.61\lambda/\sin(\theta) = 1.5\lambda\) at \(z = 39\lambda\). \(d < d_r\) holds, meaning that the two SDHs cannot be resolved by beamforming techniques. As a comparison, the total focusing method (TFM), the gold standard of beamforming techniques, is applied to deal with the same experimental ultrasound array data for imaging the two SDHs.

Figure 5 shows the 3D ultrasound images of the two SDHs, obtained from TFM (left figure) and CF-TR-MUSIC (right figure) separately. From the left one it is obvious that there is only one peak, which can be considered only one defect in the steel sample. Therefore, TFM fails to resolve the two SDHs. From the right one it can be seen that there are two peaks shown in the ultrasound image, which is consistent with the two SDHs. Therefore, the CF-TR-MUSIC can break diffraction limit, achieving super resolution imaging.

![Ultrasound images of the two SDHs in steel sample](image)

**Figure 5: Ultrasound images of the two SDHs in steel sample.**

### 4.3 Robustness to noise

Noise is the main factor limiting the ability of ultrasound defect detection. The second experiment is performed on a block of copper, which is isotropic but exhibits a high degree of material backscatter at typical ultrasound frequencies \([20]\). Two 1mm SDHs are drilled in the copper sample, and they are located at \((d = 3.3\lambda, z = 22\lambda)\). The ultrasound array data are collected using the same experiment system.

Figure 6 shows the 3D ultrasound images of the two SDHs, obtained from CF-TR-MUSIC (left figure) and TR-MUSIC (right figure) separately. Note that CF-TR-MUSIC implements on the central frequency of ultrasound array, and TR-MUSIC implements on the frequency range from 3MHz to 7MHz. It is obvious that CF-TR-MUSIC image shows many peaks, which can be considered false defects. According to the imaging result, the internal defects cannot be reflected rightly. However, TR-MUSIC can diminish the false peaks shown in the CF-TR-MUSIC image and obtain stable imaging result. Therefore, TR-MUSIC is suited for imaging defects under noisy cases.
5. Conclusion

The paper has described an investigation into the ultrasound time reversal based signal processing techniques for imaging internal defects in solids. CF-TR-MUSIC as well as its multi-frequency form is presented to obtain the ultrasound image by post-processing the ultrasound array data collected using FMC method. An experimental system, consisting of array controller and linear array, has been built to extract the experimental ultrasound array data from test samples.

The experiment result in a block of steel shows that CF-TR-MUSIC, implementing on the central frequency of ultrasound array, can break diffraction limit and achieve super resolution imaging. The experimental result in a block of copper shows that TR-MUSIC, implementing on the given frequency bandwidth, can obtain stable ultrasound image under the noisy cases.

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