NOISE REDUCTION USING A BI-MEMBRANE SOUND ABSORBER UNIT

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Sound absorption can be used to control noise by reducing the reflected energy of an incident sound wave striking a surface. In this work, the behaviour of a sound absorbing unit composed of two tensioned impermeable membranes and two air cavities is discussed. Analysis is performed for normal-incidence sound pressure waves. The sound absorption coefficient is determined from the average input impedance of the unit. The total impedance of the system is calculated from the solution for the axi-symmetric vibration of a forced clamped membrane and using the translation impedance method. It is shown that the sound absorbing properties of the unit can be adjusted by modifying the values of tension, density, and separation distance between the membranes.

Keywords: sound absorption, membranes, noise control, impedance

1. Introduction

Sound absorption is one of the common techniques used for controlling noise. In general, sound absorption can be provided in an application by means of using porous materials. These materials are very useful when mid and high frequencies need to be absorbed. To obtain good sound absorption at lower frequencies, the thickness of the material needs to be impractically very large. An alternative is the use of membrane absorbers. These absorbers are typically composed of flexible panels, mounted over an air space that can be either partly or filled with a porous material. The porous material helps in providing an effective frequency broadband sound absorption. When the panel is excited by an incident sound wave, several flexural resonance modes are excited and maximum absorption takes place at the lowest natural frequency of the coupled panel-cavity system. Thus, the effect of the plate’s bending stiffness must be considered to describe appropriately the physical phenomenon.

Another alternative is to use an impervious membrane to absorb low-frequency noise. This type of sound absorber has been studied by researchers in the past. Unlike a plate, a membrane does not have any flexural rigidity and membranes can only sustain tensile loads. Therefore, the wave speed of the membrane only depends on its tension and mass density.

The use of tensioned elastic membranes has been known in acoustics for many years, mostly in the field of musical percussion instruments identified as membranophones. Two membranes are commonly used in drumheads, one at the top and one at the bottom, where the top membrane is usually called the batter head since it is stroked to produce sound. The resonance effect is responsible for the sound radiated and tone of the drumhead. The sound characteristics can be modified by changing the lowest natural frequency relationship between both membranes.

In this work, a variation of a membrane sound absorber is presented. A second membrane is placed inside the air space at a fixed distance from the first one. Then, a sound absorber unit is developed.
which may be combined with others to obtain a sound absorber surface. We will consider circular membranes forming two cylindrical air cavities. The sound absorption coefficient of this unit can be easily determined from the average surface impedances of the membranes.

2. Theoretical background

To determine the basic equations of the sound absorber unit, we need to derive first the solution of the equation of motion for the flexural displacement of a circular membrane in polar coordinates, which is driven by an external harmonic sound pressure. Considering that a thin elastic circular membrane of radius \( a \) is subjected to small transverse vibrations produced by a plane sound wave incident to the surface of the membrane, the steady-state equation of motion of the membrane is

\[
T \nabla^2 \Psi(r) + \omega^2 \rho_s \Psi(r) = -p,
\]

where \( \Psi(r) \) is the membrane displacement at a radial position coordinate \( r \) on the membrane surface, \( T \) is the tension per unit length of the membrane, \( \rho_s \) is the surface density of the membrane, \( \omega \) is the circular frequency, and \( p \) is the uniform sound pressure amplitude on the membrane surface. Now, we consider that the circular membrane is fixed on its perimeter \( r = a \). Since the plate is excited by the action of a uniformly distributed pressure, only axisymmetric modes need to be considered. The solution of Eq. (1) is

\[
\Psi(r) = \frac{p}{\omega^2 \rho_s} \left[ \frac{J_0(k_m r)}{J_0(k_m a)} - 1 \right],
\]

where \( J_0 \) is the Bessel function of the first kind and of the zero order and \( k_m^2 = \omega^2 \rho_s / T \). The velocity distribution on the surface of the membrane can be determined from \( v(r) = j \omega \Psi(r) \), where \( j = \sqrt{-1} \).

2.1 Average surface impedance of a vibrating membrane

The uniform sound pressure incident on the surface of the membrane produces a variable axial distribution of the velocity \( v(r) \). Therefore, to obtain an overall surface impedance, we need to derive a spatial-average displacement amplitude, which is given by

\[
\langle \nu \rangle = \frac{j \omega}{\pi a^2} \int_S \Psi(r) dS = \frac{j p}{\omega \rho_s} \left\{ \frac{2 J_1(k_m a)}{k_m a J_0(k_m a)} \right\},
\]

where \( J_1 \) is the Bessel function of the first kind and of the first order. Thus, the spatial-average impedance at the surface of the membrane \( \langle z \rangle = p / \langle \nu \rangle \) is directly determined from Eq. (3). Using the identities of Bessel functions, we can write

\[
\langle z \rangle = -j \omega \rho_s \frac{J_0(k_m a)}{J_2(k_m a)},
\]

where \( J_2 \) is the Bessel function of the first kind and of the second order.

2.2 Sound absorption coefficient of an absorber unit

Now we will consider a sound absorber unit composed by a cylindrical cavity filled with air, of radius \( a \) and length \( L \), which is closed at one end by an impervious tensioned membrane and with a perfectly reflecting surface at its opposite end. A second impervious tensioned membrane is placed inside the air cavity at a distance \( s \) from the first membrane, as it is shown in Fig. 1. A plane sound wave coming from the left induces a vibration at the first membrane which is propagated through the air cavity of length \( L - s \), which makes the second membrane to vibrate. At low frequencies we can
consider that $kL \ll 1$ and $ka \ll 1$, where $k$ is the free field wavenumber ($\omega/c$), and $c$ is the speed of sound in the air.

![Diagram of sound absorber unit made of two tensioned membranes and two air cavities.](image)

Figure 1: Sound absorber unit made of two tensioned membranes and two air cavities.

The effects of the combined membrane–air layer–membrane–air layer are considered using the impedance translation theorem. Thus, the total impedance of the system is

$$Z_T = \langle Z \rangle_1 + Z_{C1}, \quad (5)$$

where $\langle z \rangle_i = -j\omega\rho_S\frac{J_0(k_{mi}s)}{J_2(k_{mi}s)}$, is the spatial-average impedance at the surface of the $i$th membrane, $k_{mi}^2 = \frac{\omega^2\rho_S}{T_1}$,

$$Z_{C1} = \rho c \frac{pc - j\frac{Z_A}{Z_A - j\frac{pc}{cot k_s}}}{Z_A - j\frac{pc}{cot k_s}}, \quad (6)$$

$$Z_A = \langle Z \rangle_2 + Z_{C2}, \quad (7)$$

$$Z_{C2} = -j\rho c \cot k(L - s), \quad (8)$$

and $\rho$ is the mean density of air (1.18 kg/m$^3$). The normal incidence sound absorption coefficient of the system can be determined from the total impedance as

$$\alpha = 1 - \left| \frac{Z_T^{-1}}{Z_T^{-1} + 1} \right|^2. \quad (9)$$

It is important to note that the sound absorption of the system is provided basically by the vibration energy dissipation in the membranes. Therefore, it is customary to introduce the hysteretic damping of a membrane as $T = T_0(1 + j\eta)$, where $T_0$ is the real static tension and $\eta$ is the damping loss factor of the membrane.

### 3. Numerical results

In this section, some numerical examples are presented to show the effects produced on the sound absorption by changing the parameters of the absorber. We consider a cylindrical absorber of radius $a = 50$ mm and length $L = 50$ mm closed at one end with an impervious tensioned membrane ($\rho_s = 0.25$ kg/m$^2$, tension $T_0 = 40$ N/m, and $\eta = 0.025$). The other end of the tube is closed with a perfectly
reflecting surface. Now, we consider a second membrane placed inside the cylinder at a distance \( s \) from the first membrane (see Fig. 1).

Figure 2 shows the numerical results of the sound absorption coefficient for different values of \( s \), compared with the sound absorption due to only the first membrane. The second membrane has the same values of surface density, tension, and damping loss factor than the first one.

Figure 2: Normal incidence sound absorption coefficient of an absorber made of two identical tensioned membranes and two air cavities for different distances between the membranes.

We can observe that the system without a second membrane provides significant sound absorption in narrow frequency bands which is produced by the energy dissipation in the membrane. Maxima of sound absorption occur approximately at the axisymmetric membrane resonances except at the lowest frequency, where the maximum value of sound absorption is determined by the zero-crossing of the imaginary impedance of the first membrane backed by an air cavity of length \( L \) and a rigid wall.

It is observed that the position of the second membrane affects considerably the total acoustic impedance and sound absorption of the system, even if the membrane parameters are identical. It seems that the effect of broadening the sound absorption frequency regions is more evident when the second membrane gets closer to the rigid wall for frequencies between 1000 Hz and 1300 Hz. Improvement in the sound absorption is obtained at lower frequencies, between the second and third resonances, when the second membrane approaches the first one.

Figure 3 reports the results of changing the tension of the second membrane while keeping the rest of the parameters fixed. We can see that a change in the tension of the second membrane produces additional narrow frequency bands of sound absorption, increasing the overall sound absorption of the system. It is evident that if the second membrane has twice the tension of the first membrane, a peak of sound absorption emerges at a frequency that is about 41% that of the system made with two membranes under the same tension.

Figure 4 shows the effect of changing the damping loss factor of both membranes but keeping all the rest of parameters fixed. Although the sound absorption bandwidths of the first resonance have been marginally broadened by increasing the damping of the membranes, the sound absorption is significantly improved at the second and third resonances. It is noted that the presence of damping in the membrane is what provides the energy dissipation mechanism in the system.
4. Conclusions

In this work a theoretical model for a bi-membrane cylindrical sound absorber unit which can be used for noise reduction applications was developed. Equations were derived to describe the normal-incidence sound absorption of a unit. The effects of various parameters in the total sound absorption performance were numerically studied. It is observed that modification of the membrane parameters can enhance significantly the sound absorption performance of the system. In addition, the derived Eq. (9) can be useful for determining the membrane parameters (surface density, tension, and loss factors) in an optimization procedure. Future work will include experimental verification of the theory, and adding resistive elements to the system, such as a porous material or perforated plate in the air cavities. It is expected that these elements would increase the bandwidth of the absorption peaks and the overall sound absorption of the system.
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REFERENCES


