A PRAGMATIC APPROACH TO ESTIMATE THE DYNAMIC LOADS OF COMPONENTS WITH NONLINEAR UNCERTAIN INTERFACES

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Predicting the aircraft’s dynamics and the vibration loads at components interfaces is a key task for ensuring a robust design and development of the product. Then, analysts rely on complex high-fidelity simulations to estimate the in-service behavior. Usually, while in case of isolate components the dynamic behavior can be predicted quite well, when several components are assembled through discontinuous junctions the predictiveness of a model decreases as much as the number of interfaces increase. The junctions, whose mechanical properties are seldom well characterized experimentally, often introduce a source of nonlinearity in the loads’ path. Additionally, their behavior is intrinsically uncertain and as consequence, it will make stochastic the dynamic response of the connected structures.

We propose a sample-based approach which aims to cope with both aspects, nonlinearities and uncertainties, and can be split in two main tasks. First, the computational cost of each deterministic simulation is minimized considering that the global nonlinear behavior depend on localized source of nonlinearities at the interfaces. The nonlinear dynamic response is obtained by updating, with a set of nonlinear modifications, the underlying linear Finite Element model, which is run just once. Second, the uncertainties are propagated through the model by a non-intrusive method based on the Sobol Low Discrepancy Design. As intermediate step, by mean of a smaller set of simulations, a meta-model based on Polynomial Chaos Expansion is created and finally used to evaluate the stochastic nonlinear response of the structure. Attention is paid to the Global Sensitivity Analysis as an effective analytical tool to quantify how the variance of the input parameters affects the variance of the outputs.

An industrial application considering an aircraft component whose dynamic behavior is affected by free-plays at its interfaces is presented.

Keywords: uncertainty, nonlinear, junctions, polynomial chaos, global sensitivity

1. Introduction

Interfaces are a key player in defining the dynamics of components and systems but considering properly their behavior can be a very complex task because are seldom well characterized experimentally. This is especially true for dynamic studies. Junctions’ behavior, often nonlinear, is intrinsically
uncertain and as consequence, it will make stochastic the dynamic response of the connected structures.

In the context Uncertainty Quantification (UQ), attention is paid to the Global Sensitivity Analysis (GSA) [1], which allow ranking the input parameters according to their influence on the variance of the stochastic Quantities of Interest (QoIs). The QoIs are those outputs which permit making conclusion and decisions about the problem. The sensitivity indices can be obtained by generating a meta-model based on a multivariate Polynomial Chaos Expansion (PCE) [2].

The generation of the PCE surrogate is based on sampling the deterministic model according some specific computer design, i.e. Montecarlo, but when the dynamic response of a structure is affected by strong nonlinearities, such as free-plays or rubber-like materials, the number of simulations required to generate a reliable meta-model can be still considerably high. The main reason is that these types of nonlinearities affect the smoothness of the high-dimensional response surface, which in most of cases will present some discontinuities.

On the light of previous observations, and considering that usually the junction are represented by lumped elements, the Structural Dynamic Modification Method (SDMM) [3] can provide the right approach to reduce the computational burden. The principle behind the modification methods is running the FE Model just once, extracting the modal information at the required degrees of freedom (dofs) and then update the equations, expressed in modal coordinates, through a set of modifications. While large literature exists in the field of the linear modifications, the works dealing with nonlinear modifications are significant fewer, i.e. [4].

2. Nonlinear Structural Dynamic Modifications

The formulation of the Nonlinear Structural Dynamic Modification Method (NL SDMM) followed in this work has been introduced by Menga and Hernández [5]. The modal matrix $\Phi$ and the eigenvalues $\lambda$ are assumed to be available from the FE model’s modal analysis and the dynamic equations are updated for taking into account nonlinear stiffness modifications. The initial stiffnesses of the spring elements used in the FE model for obtaining the modal properties is called underlying linear stiffness. The dynamic equilibrium in time-domain and in modal coordinates is expressed by:

$$I\ddot{q}(t) + \tilde{C}\dot{q}(t) + \Lambda q(t) = \tilde{F}(t),$$

where the modal matrix has been normalized to the mass. $I$, $\tilde{C}$, $\Lambda$ are hence the normalized mass, damping and stiffness matrices and $\tilde{F}$ is the vector of the modal forces.

In eq. (1) the normalized mass matrix is the unitary one and the normalized stiffness matrix is a diagonal matrix whose terms are the eigenvalues.

Considering a set of nonlinear modifications of the stiffness matrix of the type, $\Delta K_{\text{mod}}(u)$, $u$ being the relative displacements seen by the spring element, the update stiffness matrix of the structure is:

$$\Lambda_{\text{mod}} = \Lambda + \Phi^T \Delta K_{\text{mod}} \Phi,$$

and eq. (1) can be re-written as:

$$I\ddot{q}(t) + \tilde{C}\dot{q}(t) + \Lambda_{\text{mod}} q(t) = \tilde{F}(t).$$

Eq. (3) requires to be solved by an iterative method which takes into account that $k_{\text{equ}}$ and consequently the $\Lambda_{\text{mod}}$ are nonlinear functions of the relative physical displacements of the springs, which, through the modal matrix $\Phi$, depend on the modal displacements. In this work the state space form of eq. (3) has been implemented in MATLAB© and solved by the function ODE45, which uses an explicit Runge–Kutta integration procedure.
3. Global Sensitivity Analysis

GSA defines a qualitative and quantitative mapping between the input variables and the Quantities of Interest (QoIs)\[1\] and allows to select what the relevant parameters in the model are and score them according to their relevance in the QoIs. In this context, a sensitivity index, also called Sobol’s index \[6\], is a measure of the influence of an uncertain quantity on an output variable.

From a practical point of view, particular attention is paid to the first order and total order sensitivity indices, denoted respectively as $S_{Ii}$ and $S_{Ti}$, also called the main effects and the total effects.

If a model $Y$ is function of $N_p$ stochastic variables $X_i$ with $i = 1, ..., N_p$, the main effects represent the measure of the contribution the parameter $X_i$ to the output variance, or equivalently, the expected percentage reduction of the output variance $V(Y)$ obtained when the uncertainty of input parameter $X_i$ is eliminated. The total effects represent the total contribution of an input parameter considering its individual effect and its interactions with all the other factors.

3.1 Meta-model: Polynomial Chaos Expansion

The original Hermite Polynomial Chaos Expansion (PCE), also known as homogeneous chaos, was first derived by Wiener (1938) \[7\] for the spectral representation of any stochastic response in terms of Gaussian random variables. Later, Askey(1985) \[8\] studied the orthogonal properties of the Hermite polynomials with respect the Gaussian density function, providing the well-known Wiener-Askey scheme. But only in 2003 Xiu extended the method to others random distributions \[9\].

If a model is sampled at $N_s$ points of the high-dimensional space, the generic output $Y_n = Y(X_n)$ evaluated at the input point $X_n = [X_{n1}, X_{n2}, ..., X_{nP}]$, with $n \in [1, 2, ..., N_s]$ and $N_p$ the total number of input parameters considered, can be approximated by a linear combination of $N_k$ basis functions:

$$Y(X_n) \simeq \sum_{k=1}^{N_k} \beta_k \Psi_k(u_n).$$

(4)

In our case the basis function are orthogonal polynomials with respect a defined weight functions and usually in a specific domain. For example, Legendre polynomials are orthogonal with respect a uniform density function in the domain $[-1, 1]$. Then, $u_n$ is the normalized input random variables, or, in other words, $X_n$ has to be mapped in the domain where the orthogonality is ensured.

This relation can be expressed more compactly using matrix notation as in

$$Y(X) \simeq \Psi(u)\beta,$$

(5)

where $\Psi$ is the interpolation or kernel matrix, $Y$ is the matrix of the $N_{out}$ QoIs evaluated at $N_s$ sampled points and $X$ is the sample matrix.

Usually, $\Psi_k$ are multivariate polynomials that involve products of the one-dimensional polynomials $\psi_i(u_j)$, where $i$ is the order of the polynomial $\psi$ and $u_j$ is the $j$ component of the vector $u_n$. The multivariate expansion is obtained as tensor product of the one-dimensional basis and consequently presents higher-order terms. These higher order terms usually are truncated considering for the multivariate basis the same maximum order $o$ of the one-dimensional polynomials they come from.

Considering that, the total number of coefficients $N_k$ of the polynomial expansion of eq. (4) is given by:

$$N_k = \frac{(N_p + o)!}{N_p!o!}$$

(6)

From eq. (6) it can be noted that increasing the number of random variables or the order of the polynomial will cause a substantial grow in the number of terms $N_k$ of the PCE. That implies an appreciable increase in the sample size $N_s$ required to find the coefficients of the expansion and consequently, in case of complex simulations, the computational cost could be unaffordable.
In this work the least square method is employed to estimate the unknown coefficients $\beta_k$ of the PCE. Considering eq. (4) and minimizing the sum of the squares of the residuals:

$$\beta = \arg \min \| \Psi \beta - Y \|^2,$$

the least square fitting gives a closed-form solution:

$$\beta = (\Psi^T \Psi)^{-1} \Psi^T Y,$$  

where $\Psi$ is assumed to have full column rank.

The number of input points $N_s$ must be higher than the number of unknown coefficients $N_k$, and depending on the complexity of the response the advisable number of samples may fluctuate. Common practice is considering $N_s \approx 2 \cdot N_k$.

### 3.2 Polynomial Chaos and Sobol’s Indices

According the PCE theory, the mean and the variance of the QoIs can be estimated directly by mean of the PCE coefficients. In fact, assuming the polynomials normalized to the variance, the following relationships hold:

$$E(Y(X)) \approx \beta_0,$$  
$$V(Y(X)) \approx \sum_{k=1}^{N_k} \beta_k^2.$$  

Additionally a close similarity between the PCE and the High Dimensional Model Reduction (HDMR) proposed by Sobol exists. In fact the Sobol’s condition for the uniqueness of the decomposition, expressing the output function as the summation of a constant term plus higher dimensional zero-mean function, is satisfied thanks to the polynomials orthogonality.

For example, when a bivariate random variable is considered, the Sobol’s HDMR is:

$$Y(X) = f(X_1, X_2) = f_0 + f_1(X_1) + f_2(X_2) + f_{12}(X_1, X_2),$$

while looking at the PCE, for a second order expansion, the following equation is obtained:

$$Y(X) \approx \beta_0 + \beta_1 u_1 + \beta_2 u_2 + \beta_3 u_1^2 + \beta_4 u_2^2 + \beta_5 u_1 u_2.$$  

Comparing eq. (11) and eq. (12) is possible to see the equivalence between the two formulations, which allows an easier calculation of the partial variances and then of the sensitivity indices. It is shown in table (1).

<table>
<thead>
<tr>
<th>Sobol’s HDMR</th>
<th>PCE functions</th>
<th>Variance</th>
<th>Variance PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>$\beta_0$</td>
<td>$V(f_0)$</td>
<td>$\beta_0^2$</td>
</tr>
<tr>
<td>$f_1$</td>
<td>$\beta_1 u_1 + \beta_3 u_1^2$</td>
<td>$V(f_1)$</td>
<td>$\beta_1^2 + \beta_3^2$</td>
</tr>
<tr>
<td>$f_2$</td>
<td>$\beta_2 u_2 + \beta_4 u_2^2$</td>
<td>$V(f_2)$</td>
<td>$\beta_2^2 + \beta_4^2$</td>
</tr>
<tr>
<td>$f_3$</td>
<td>$\beta_5 u_1 u_2$</td>
<td>$V(f_3)$</td>
<td>$\beta_5^2$</td>
</tr>
</tbody>
</table>

Table 1: Equivalence between Sobol’s HDMR and PCE.

Hence, the total variance, defined by equ. (10) can be decomposed as:

$$V(Y) = V(f_0) + V(f_1) + V(f_2) + V(f_{12}) = \beta_0^2 + (\beta_1^2 + \beta_3^2) + (\beta_2^2 + \beta_4^2) + \beta_5^2,$$  

(13)
and the Sobol’s indices can be obtained by the ratio between the partial variances and the total one.

For example the first and total effects with respect the first input variable $X_1$ are:

\[
S_{I1} = \frac{V(f_1)}{V(Y)} \approx \frac{\beta_1^2}{\sum_{m=1}^{p} \beta_m^2},
\]

\[
S_{T1} = \frac{V(f_1) + V(f_{12})}{V(Y)} \approx \frac{\beta_1^2 + \beta_3^2}{\sum_{m=1}^{p} \beta_m^2}.
\]

Hence the calculation of the PCE coefficients provides a straightforward and computational inexpensive way to estimate the mean, the variance and the Sobol’s indices. It is worth highlighting that the Sobol’s HDMR is an exact decomposition, while the polynomial expansion would be exact only if the summation were extended to infinite terms. In practice the PCE is truncated to an order that in most of application does not exceed the third degree.

4. Industrial application: A380 RAM Air Inlet free-play nonlinear vibrations

In this section the UQ is considered in the frame of an industrial case of study: A380 RAM Air Inlet nonlinear vibrations. The RAM is defined as a moving part in a machine which puts pressure or force on something. The RAM Air Inlet(RAI) - RAM Air Outlet(RAO) modules provides a forced ventilation system to control air temperature around electronic systems. With reference to fig. 1(a), we can see the left hand A380 air intake of the RAI and two outtakes of the RAO. Fig. 1(b) shows the RAI system. The air flow, coming from the RAI intake, wets the forward and progressively the reward flaps, passes towards the heat exchange and finally, goes out through the RAO outtakes.

During flights, for a fixed actuator position, due to the flow acting on the flaps, particular attention is required in order to evaluate the forces explicated on the two rods, which connect, the RWD flap to the shaft. Large flight test campaign has facilitate the individuation of the worst case in term of flaps positions and unsteady pressure distributions, which can be mapped on the structural mesh in order to evaluate the rod’s reaction forces. Anyway a reliable prediction should take into account the free-plays between the arms and the rods at their interfaces, the uncertainties about the gaps sizes and the rods stiffnesses and, the uncertainties about the magnitude of the resultant forces on the flaps and their dominant frequency.

(a) A380 RAI intake and RAO outtakes (b) A380 RAI System flaps

Figure 1: A380 - RAM Air Inlet.
The root mean square (RMS) of the nonlinear rods’ reaction forces, over a time period of 10 seconds, i.e. $F_{1\text{rms}}$ and $F_{2\text{rms}}$, are the selected QoIs. The deterministic model is sampled according a Sobol’s LDD of size $N_{\text{run}} = 400$, which could be already a very expensive computational cost considering the time required by the model due to the nonlinearity. This initial cost is considerably reduced by the NL SDMM discussed in section 2.

The modal matrix is obtained in NASTRAN\textsuperscript{®} from the FE Model with the rods disconnected from their arms, then the underlying linear stiffness of the junctions is set to zero. First, a set of linear modifications has been considered to validate the capability of the SDMM to consider a disruptive change of the local stiffness, which pass from $K_0 = 0$ to a value $K_1 = 1.5e^4$. Then, interfaces whose behavior is nonlinear and uncertain are considered. The uncertain free-play stiffness force-displacement curve is shown in fig. 2(a). Both the size of the gap and the stiffness are considered as random variable with prescribed normal distribution according the values in table 2. Additionally, uniform distributions are prescribed on the magnitude and the frequency of the force exciting the RWD Flap. Fig. 2(b) shows the comparison between the linear force and the nonlinear at the interfaces when the model is at its nominal conditions.

![Figure 2: Nonlinear interfaces behavior.](image)

<table>
<thead>
<tr>
<th>Random Variable $x$</th>
<th>Distribution</th>
<th>nominal</th>
<th>lower bound</th>
<th>upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force $[N]$</td>
<td>Uniform</td>
<td>500</td>
<td>350</td>
<td>650</td>
</tr>
<tr>
<td>Frequency $[Hz]$</td>
<td>Uniform</td>
<td>35</td>
<td>29.5</td>
<td>40.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random Variable $x$</th>
<th>Distribution</th>
<th>mean value</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1[N/mm]$</td>
<td>Normal</td>
<td>$1.5e^4$</td>
<td>$1.5e^3$</td>
</tr>
<tr>
<td>$K_2[N/mm]$</td>
<td>Normal</td>
<td>$1.5e^4$</td>
<td>$1.5e^3$</td>
</tr>
<tr>
<td>Gap$_1[mm]$</td>
<td>Normal</td>
<td>1.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Gap$_2[mm]$</td>
<td>Normal</td>
<td>1.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 2: Statistical distributions of the random variables.

The least square method is used to find the approximation’s coefficients, 84 according the eq. (6), of a 3\textsuperscript{rd} order PCE over a sample of $N_s = 250$ training points. The remaining $N_{ev} = N_{\text{run}} - N_s = 150$ are reserved exclusively for the validation of the PCE surrogate and the error on this unseen set of
data is:

\[ \text{Err}_{ev} = \frac{\|Y_{ev} - \Psi_{ev}\beta\|}{N_{ev}\|\Psi_{ev}\beta\|}. \] (16)

Legendre and Hermite polynomials are respectively employed for the uniform and normal probability distributions according to table 2. The independence of the random variable preserves the orthogonality of the multivariate PCE when different types of polynomials are used. The approximation error is of order $10e^{-3}$. Usually, PCE shows lower performance in case of strong nonlinearities, and increasing the degree of polynomials not always is a guarantee of improvement because of over-fitting: the meta-model trusts the data on the training set too much and performs poorly on unknown sets.

4.1 Results

Table 3 shows the mean, the variance, the error on the evaluation set and the Sobol’s sensitivity indices. The frequency is the stochastic parameter which affects the outputs variance the most. It is in agreement with the fact that both intact and disconnect structures present amplification peaks in the prescribed range of variability. The nonlinear interfaces behavior is shown in fig. 2: the uncertainty on the size of the gaps has a much more relevant effect on the variance of reaction loads than the uncertainty on the stiffnesses. According to the results in table 3, difference exist between the first and total order effects, which means that the interaction among parameters is significant. It is particularly true for the total indices regarding the gap parameters. It makes sense because some interactions are expected at least between the applied force and the gap, then for example the $ST_5 = S_5 + S_{15} + S_{25} + S_{35} + S_{45} + S_{65}$ is expected to have at least the term $S_{15}$ different from zero.

It is worth noting that when the interactions among parameters are significant the sum of the first order indices is less than one, hence the partial variances does not cover the 100% of the total output variance. On the contrary the sum of the total effects can be greater than one, because for example the factor $S_{15}$ contributes to both $ST_5$ and $ST_1$, hence the percentages of partial variances in terms of total effects are redistributed according the value of their sum.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Results</th>
<th>$F_{1\text{rms}}$</th>
<th>$F_{2\text{rms}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean($Y$)</td>
<td>140.10</td>
<td>207.49</td>
<td></td>
</tr>
<tr>
<td>V($Y$)</td>
<td>3286.55</td>
<td>8145.43</td>
<td></td>
</tr>
<tr>
<td>error on $N_{ev}$</td>
<td>0.0015</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>Force</td>
<td>$S_1$</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>Frequency</td>
<td>$S_2$</td>
<td>0.43</td>
<td>0.30</td>
</tr>
<tr>
<td>$K_1$</td>
<td>$S_3$</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>$K_2$</td>
<td>$S_4$</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Gap$_1$</td>
<td>$S_5$</td>
<td>0.12</td>
<td>0.17</td>
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<tr>
<td>Gap$_2$</td>
<td>$S_6$</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td>Force</td>
<td>$ST_1$</td>
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<td>0.28</td>
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<tr>
<td>Frequency</td>
<td>$ST_2$</td>
<td>0.48</td>
<td>0.38</td>
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<td>$K_1$</td>
<td>$ST_3$</td>
<td>0.06</td>
<td>0.10</td>
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<tr>
<td>$K_2$</td>
<td>$ST_4$</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>Gap$_1$</td>
<td>$ST_5$</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>Gap$_2$</td>
<td>$ST_6$</td>
<td>0.22</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 3: RAI vibrations - statistical moments and Sobol’s Indices.
5. Conclusions and future work

This research presents a pragmatic approach for estimating the stochastic dynamic response of assembled structures whose interfaces are nonlinear and uncertain. Non-intrusive, or sample-based, UQ methods have received more and more attention in the engineering community during the last decade and is a very wide and active field of research. The novelty of this work is in incorporating in the UQ process the NL SDMM, which is presented as a reliable and efficient method to avoid the computational burden due to nonlinearities when they can be considered localized and then modeled through lumped elements. The feasibility of the proposed approach is tested on an industrial application. An aircraft component, whose dynamic behavior is affected by uncertain free play junctions, is considered and its nonlinear time-domain response is evaluated when it is loaded by a stochastic force.

The efficiency and predictiveness of meta-models for nonlinear models is an open field of research and further investigations will compare the PCE with other surrogates, such as the Radial Basis Functions (RBFs).

References


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