ADJUSTING THE NOISE AMPLIFICATION OF FEEDBACK ACTIVE NOISE CONTROL SYSTEM VIA A FREQUENCY DOMAIN ADAPTIVE ALGORITHM

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There is always some noise amplification at some frequencies in a feedback active noise control system due to the waterbed effect, in order to adjust the noise amplification in an adaptive feedback active noise control system, a frequency domain filtered-x least mean square algorithm is utilized in this paper. The algorithm is based on the fact that if the magnitude frequency response of the controller filter in the specific frequency band is deliberately limited, then the noise amplification in that frequency band can be alleviated, then a constraint that limits the gain of the controller filter at some discrete frequency is merged into the cost function as a penalty term. Simulations in an active noise control headphone application show that the introduced algorithm can tune the noise amplification more effectively than the existing algorithms.

Keywords: Active noise control, waterbed effect

1. Introduction

Active noise control (ANC) system attenuates unwanted noise by employing the outputs of the secondary sources to interfere destructively with the original primary noise [1, 2]. A feedback ANC [1-2] system differs from a feedforward system in that it has no reference sensor to provide time-advanced reference signal about the primary noise, and it is used effectively in applications where the primary noise cannot be directly observed or where there are many primary noise sources, such as the active headsets and headrests [3-5]. However, the feedback structure has its own weakness in comparison to the feedforward system: the first is that its noise attenuation bandwidth is typically...
very narrow; The second is that much efforts are required to guarantee the stability of the feedback structure; the third is the waterbed effect which states that if the noise is suppressed at some frequency range, it is necessarily increased at some other range [1].

Many design methods of the feedback system are proposed to obtain both good noise reduction performance and sufficient stability. For example, the \( H_c \) [4] and the \( H_2/H_c \) optimization [5] technique are used to synthesize the robust feedback ANC system. An approach is presented to design the feedback controller by flattening the noise amplification due to the waterbed effect in the whole disturbance amplification frequency band, the amplification peaks can be suppressed to relatively small value for the given disturbance attenuation [6]. A simplified adaptive feedback ANC system is reported in [7], which adopts the error signal directly as the reference signal in an adaptive feedforward control system, it is advantageous in computational load and ease of implementation, as well as alleviation of the coupling between the feedforward and feedback structures [8].

The methods in [4-6] design the feedback controller by formulating the performance specifications from practice into the optimization problem with various design criteria and constraints, but abundant experience and repeated attempts are often required to obtain a satisfactory controller, so these methods are mainly applied to the offline feedback ANC design. The method in [7] is based on the adaptive algorithm and then can be applied to the online feedback ANC system, but the waterbed effect is not considered because it is not easy to merge the waterbed effect related constraints into the adaptive algorithm. Sometimes in practical ANC systems, it is a common requirement that the noise amplification within one specific frequency range is strictly prohibited while outside that specific frequency range is acceptable or can be further processed. Based on the fact that if the magnitude frequency response of the controller filter in the specific frequency band is deliberately limited, then the noise amplification in that frequency band can be alleviated, a frequency domain filtered-x least mean square algorithm is used in this paper to adjust the noise amplification in an adaptive feedback active noise control system.

2. Derivation of the algorithm

As shown in Fig. 1 (a), the internal model control (IMC) based feedback ANC system is composed of one error sensor to measure the residual noise \( e(n) \), one secondary sound source to generate the canceling signal \( y(n) \) for attenuation of the primary noise \( d(n) \), the synthesized reference signal \( x(n) \) is filtered through \( \hat{S}(z) \), the so-called estimation of the secondary path \( S(z) \), and the control filter \( W(z) \) is represented as a tap weight vector of length \( L \), i.e., \( \mathbf{w}(n) = [w_0(n), w_1(n), \ldots, w_L, e(n)]^T \). Here the reference signal \( x(n) \) is synthesized on the basis of \( e(n) \) and the secondary signal \( y(n) \) filtered by \( \hat{S}(z) \).

![Figure 1: Block diagram of the adaptive feedback ANC system, (a) internal model control based and (b) the simplified.](image)

The simplified adaptive feedback system [7] is depicted in Fig. 1 (b), where the reference signal comes from the error signal directly, it is advantageous in computational load and ease of implementation because of the elimination of the convolution operation required in the IMC based system.
Because the algorithm in this paper can be applied to both IMC based and simplified feedback system directly, the two systems are not explicitly discriminated in the following derivation.

The frequency domain LMS algorithm is widely used to improve computational efficiency and convergence rate due to the use of the FFT (Fast Fourier Transform) and IFFT (Inverse Fast Fourier Transform). However, it results in a delay of one block of data which may impair the ANC system performance. Therefore, in ANC systems, the filtering is performed in the time domain to avoid the delay, whereas the adaptation of \( W(z) \) is computed in the frequency domain [9].

In Fig. 2, the sizes of FFT and IFFT are \( 2L \) to avoid circular convolution effects [9]. \( x_i(n) \) is the filtered reference signal (\( x(n) \) filtered by \( \hat{S}(z) \)), \( e(n) \) is

\[
e(n) = d(n) + w^T x_i(n)\tag{1}
\]

the update equation of \( w(n) \) is

\[
w(n+1) = w(n) - \mu \text{IFFT}\{X^*(k)E(k)\}_+ \tag{3}
\]

where \( \mu \) is the step size, \( \{ \cdot \}_+ \) denotes the causal part, \( X^*(k) \) is the conjugate of \( X(k) \), \( X(k) \) and \( E(k) \) of size \( 2L \) are calculated using FFT on the corresponding time domain vectors as

\[
X(k) = \text{FFT}\{[x_i^T(n) x_i^T(n-L)]^T\} \tag{4}
\]

\[
E(k) = \text{FFT}\{[0 \ e^T(n)]^T\} \tag{5}
\]

\( 0 \) is an \( L \) point zero vector and

\[
e(n) = [e(n) e(n-1) \ldots e(n-L+1)]^T \tag{6}
\]

In the off-line feedback system design methods [4-6], the noise amplification is expressed as a constraint, then the controller \( W(z) \) is solved by minimizing the error signal energy. Fig. 1 shows that in Z transform domain [7],

\[
E(z) = D(z) + Y'(z) \tag{7}
\]

Under the assumption of slow adaptation, \( Y'(z) \) is

\[
Y'(z) = X(z)W(z)S(z) \approx X'(z)W(z) \tag{8}
\]

It can be found from Eq. (8) that if the magnitude frequency response of \( W(z) \) in the predefined frequency band is deliberately limited, then the frequency components of \( Y'(z) \) in that frequency band will be very small and the noise amplification in that frequency band of \( E(z) \) will be alleviated, therefore, the limit on the magnitude of \( W(z) \) at any discrete frequency \( k \) is defined as a constraint.

![Figure 2: Frequency domain LMS algorithm in the ANC system.](image-url)
\[ |W(k)|^2 < \theta(k) \]  \hspace{1cm} (9)

Here
\[ W(k) = \sum_{l=0}^{L-1} w_l e^{-j2\pi k l / L} = w^T f_k \]  \hspace{1cm} (10)
\[ f_k = [0 \ 0 \ \ldots \ e^{j2\pi (l-1) k / L}]^T \]  \hspace{1cm} (11)
after some mathematical arrangements, Eq. (9) is expressed as
\[ c_k = w^T (f_k^T f_k)^{-1} w - \theta(k) = w^T F w - \theta(k) < 0 \]  \hspace{1cm} (12)

To merge the waterbed effect related constraints into the adaptive algorithm, the following cost function is defined
\[ J = E[e^T(n)e(n)] + \lambda \frac{\text{sign}(c_k) + 1}{2} c_k \]  \hspace{1cm} (13)
and 0 < \lambda < 1 is the tuning factor. The corresponding update equation of \( w(n) \) is
\[ w(n+1) = \begin{cases} w(n) - \mu \text{IFFT}\{X^T(k)E(k)\}, & \text{if } |W(k)|^2 \leq \theta(k) \\ w(n) - \mu \text{IFFT}\{X^T(k)E(k) + 4\lambda LW(k)\}, & \text{if } |W(k)|^2 > \theta(k) \end{cases} \]  \hspace{1cm} (14)

It can be seen from Eq. (13) that when \( c_k < 0 \), i.e., Eq. (12) is satisfied, \( \text{sign}(c_k) = -1 \), \( \frac{\text{sign}(c_k) + 1}{2} c_k = 0 \), Eq. (13) reduces to \( J = E[e^T(n)e(n)] \), the adaptive algorithm aims to attenuate the residual noise completely; when \( c_k > 0 \), i.e., Eq. (12) is violated, \( \text{sign}(c_k) = 1 \), \( \frac{\text{sign}(c_k) + 1}{2} c_k = c_k \), Eq. (13) reduces to \( J = E[e^T(n)e(n)] + \lambda c_k \), the adaptive algorithm must attenuate the residual noise and at the same time take into account the noise amplification.

3. Simulations

Simulations were carried out to examine the performance of the constrained frequency domain algorithm in an ANC headphone application, where the experimental setup was similar to the one described in [3]. An active headphone bought from the market was used as the prototype and mounted on a B&K Type 4128C HATS [10], but the original ANC function accompanied with the headphone was disabled. The sampling frequency of the ANC system is 16000 Hz. The experiments were carried out in an anechoic chamber. A loudspeaker which played back the primary noise is placed in the horizontal plane of the headphone and approximately 40 cm away from the HATS. The measurements were performed for 4 different incident directions of the primary noise (marked with "front", "left", "right" and "rear" in Fig. 3 (b)).

![Diagrammatic view of (a) experimental configuration in the anechoic chamber and (b) the 4 different incident directions of primary noise.](image-url)

Using the estimated secondary path and the recorded signals of the reference microphone and the error microphone, the “optimal” ANC controllers of the “IMC” feedback system (Fig. 1 (a)) and the
“simplified” feedback system (Fig. 1 (b)) were designed respectively, then the difference between the PSD (power spectral density) with and without ANC was used to evaluate the noise reduction performance. Detailed performance was tested for the 4 different incident directions respectively, but only the average results of the 4 different incident directions are presented in the remainder of the paper to avoid bothering readers with too many curves.

Here, $W(z)$ and $S(z)$ are selected to be 512 taps FIR filter and 256 taps FIR filter respectively. The frequency domain algorithm (marked with “FDxLMS”) shown in Eq. (3), the leaky FxLMS algorithm (marked with “Leaky”) used in [7] and the constrained frequency domain algorithm (marked with “cFDFxLMS”) shown in Eq. (14) were utilized to design the ANC controllers of the “IMC” and the “simplified” system respectively. The parameters setting of the three algorithms are listed in Table 1. For the FDFxLMS algorithm, the step-size $\mu$ is selected to ensure fast convergence and good stability; For the leaky FxLMS algorithm, $\mu = 0.004$ and $\gamma = 0.05$ which implies the scalar leaky factor in Eq. (3) is 0.9998; For the “cFDFxLMS” algorithm, $k=153$~256 and $\theta(k)=1.78$ means that the magnitude of $W(z)$ is set to be less than 5 dB in the 2400~4000 Hz frequency range, because in a feedback ANC headphone, while suppressing the noise at 200~1000 Hz, it is assumed that the noise amplification in the 2400~4000 Hz range is controlled while some the noise amplification at the other frequency ranges is acceptable because the residual noise in this frequency range is particularly annoying due to the specific characteristics of the primary noise and passive headphone structure.

### Table 1. Parameters setting in the FDFxLMS algorithm, the leaky FxLMS algorithm and the cFDFxLMS algorithm.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>FDFxLMS</th>
<th>Leaky</th>
<th>cFDFxLMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>$\mu = 0.003$</td>
<td>$\mu = 0.004$</td>
<td>$\mu = 0.004, \lambda = 0.01$</td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0.05$</td>
<td>$\gamma = 0.05$</td>
<td>$k=153,154…256, \theta(k) = 1.78$</td>
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The performance comparison of the FDFxLMS algorithm, the Leaky algorithm and the cFDFxLMS algorithm are shown in Fig. 4. The results of the IMC feedback system are given in Fig. 4 (a). For the FDFxLMS algorithm, it is noticed that the attenuation bandwidth is mainly below 2000 Hz, the maximum noise reduction is 23 dB located at 300 Hz. For the Leaky algorithm, the attenuation bandwidth is below 1000 Hz and the maximum active noise reduction is 8 dB located at 300 Hz. For the cFDFxLMS algorithm, the attenuation bandwidth is also below 1000 Hz and the maximum active noise reduction is 17 dB at 300 Hz frequency. The FDFxLMS algorithm achieves the best noise reduction among the three algorithms, but its noise amplification is the most serious, especially in the 2400~4200 Hz range in Fig. 4 (a) where there is 5~9 dB noise amplification due to the waterbed effect. The noise amplification of the Leaky algorithm is less than 2 dB and almost uniformly distributes from 1000 Hz to 6000 Hz frequency band. The noise amplification of the cFDFxLMS algorithm is less than 5 dB from 1000 Hz to 6000 Hz frequency band.

The results of the simplified feedback system are given in Fig. 4 (b) and it is found that the maximum noise reduction of the FDFxLMS algorithm, the Leaky algorithm and the cFDFxLMS algorithm is 21 dB, 7 dB and 13 dB respectively. Similar to the results in Fig. 4 (a), the noise amplification of the FDFxLMS algorithm is also the largest among the three algorithms, the noise amplification in the frequency range of 2400~3000 Hz is 10~15 dB, which is more serious than the one in Fig. 4 (a). The noise amplification of the Leaky algorithm is also less than 2 dB and almost uniformly distributes from 1000 Hz to 7000 Hz frequency band. The noise amplification of the cFDFxLMS algorithm is less than 5 dB from 1000 Hz to 7000 Hz frequency band.
Figure 4: The ANC performance of the (a) internal model control and (b) the simplified feedback systems based on the FDFxLMS algorithm, the Leaky algorithm and the cFDFxLMS algorithm.

From the results in Fig. 4, it is confirmed that the FDFxLMS algorithm achieves the best noise reduction in the low frequency range, but its noise amplification is the most serious because the target of the FDFxLMS algorithm is to minimize error signal without any other constraints. The Leaky algorithm adds a constraint to the cost function to limit the gain of the controller filter, but the constraint does not differentiate between different frequencies, thus the Leaky algorithm cannot tune the noise amplification into specific frequencies. The cFDFxLMS algorithm can adjust the noise amplification more effectively in specified frequency range than the Leaky algorithm. Therefore, the cFDFxLMS algorithm provides a trade-off between the FDFxLMS algorithm and the Leaky algorithm.

4. Conclusions

Based on the fact that if the magnitude frequency response of the controller filter in the specific frequency band is deliberately limited, then the noise amplification in that frequency band can be alleviated, a constrained frequency domain algorithm is used in this paper to adjust the noise amplification in an adaptive feedback ANC system, where a constraint that limits the gain of the controller filter at some discrete frequency is merged into the cost function as a penalty term. Simulation results in a real ANC headphone application demonstrate that the FDFxLMS algorithm achieves the best noise reduction, but its noise amplification is the most serious. The leaky FxLMS algorithm can limit the noise amplification, but it does not differentiate between different frequencies, thus it cannot adjust the noise amplification related to different frequencies. The constrained frequency domain algorithm can adjust the noise amplification more effectively than the leaky FxLMS algorithm, which provides more flexibility for the design of an adaptive feedback ANC system.

REFERENCES


