Non-reciprocal wave phenomenon happens in mediums with the space-time modulation over their material properties. In this work, we study an infinite space-time lattice composed of the time-varying mass and spring of constant stiffness. The Bloch-based theoretical method is developed for the computation of the dispersion behaviour of the modulated lattice. It is found that the asymmetric band structure emerges when the spatiotemporal modulation constitutes a travelling-wave field pattern, which behaves like a biasing field that breaks the time-reversal symmetry. The effect of modulating frequency and amplitude of time-varying mass is also analyzed. We find that the modulating frequency determines the frequency position of the asymmetric bandgap, while the modulating amplitude dominates its frequency bandwidth. The studied model is expected to offer new possibilities for the unprecedented control over sounds and vibrations.

Keywords: spatiotemporal mediums, time-varying mass, non-reciprocity
where \( \theta^{(r)}_0 \) is the periodic spatial variation of inertial mass. The wavelength of spatial modulation is \( \lambda_m = Ra \) and the modulation amplitude is \( \alpha_m \). The period of time modulation is \( T_m = 2\pi / \omega_m \) with the modulation frequency \( \omega_m \). Periodic modulation of inertial mass over the space and time describes a travelling wave that propagates with velocity \( v_m = \lambda_m / T_m \), which acts as a biasing field to induce the nonreciprocal wave behaviour. In previous studies, the Bloch-based method has been developed to evaluate the dispersion characteristics of the modulated lattices composed of springs of time-varying stiffness and constant inertial masses[4]. The method will be improved here for the dispersion computation of our models.

3. Results

Figure 1(a) shows the space-time field pattern of inertial mass in the lattice system with \( R = 3 \). The pattern has been designed as the backward travelling wave, which doesn't follow the spatial inversion symmetry. Figure 1(b) shows the fundamental branch of dispersion diagrams of the modulated lattice, exhibiting the unidirectional bandgap in the forward direction, which is unavailable in the non-modulated system [Fig. 1(c)]. The physical reason can be attributed to the scattering coupling of modes of different orders. We also study the influence of modulating frequency \( \omega_m \) and amplitude \( \alpha_m \) on the asymmetric bandgap. It is found that the modulation frequency \( \omega_m \) plays a crucial role of tuning the asymmetric gap frequency, and increasing \( \alpha_m \) has an obvious effect on widening the bandwidth of asymmetric bandgap.

![Figure 1](image)

**Figure 1:** (a) The space-time field pattern of inertial mass; (b) The fundamental branch of the modulated lattice; (c) Dispersion curves of the non-modulated lattice.

4. Conclusion

The Bloch-based method is developed for the estimation of dispersion diagrams of the modulated lattice with time-varying inertial mass and constant springs. It is found that the travelling-wave field pattern of inertial mass leads to purely unidirectional bandgaps. The modulating frequency and amplitude of time-varying inertial mass determine respectively the frequency position and bandwidth of asymmetric bandgap. The non-reciprocal wave phenomenon observed in lattice systems of time-varying mass is expected to bring new technological concepts in noise and vibration control.

REFERENCES