This paper presents a novel stiffness control strategy for magnetorheological elastomer based vibration absorber attached to a multi-degree-of-freedom system. The proposed control strategy does not rely on the relationship between the magnetic current and the resonant frequency of the vibration absorber. It uses the phase difference between the relative acceleration of the vibration absorber mass respect to the primary system and the absolute acceleration of the primary system to check whether the vibration absorber is tuned properly. Simulation results show the proposed control strategy is efficient to make the magnetorheological elastomer vibration absorber trace the exciting frequency rapidly.

Keywords: magnetorheological elastomer, vibration absorber, stiffness control

1. Introduction

Dynamic vibration absorber (DVA) is widely used to suppress the undesired vibration of a structure. According to the feature of varying its mechanical parameters, the dynamic vibration absorber can be categorized into passive vibration absorber, semi-active vibration absorber and active vibration absorber [1-3]. Generally, the passive vibration absorber can be regarded as a single-degree-of-freedom system with constant damping, stiffness and mass. Once designed, its physical parameters cannot be changed, which makes the passive vibration absorber only efficient in a very narrow frequency band. The active vibration absorber can be regarded as a passive vibration absorber with an active element. By controlling the active element, the active vibration absorber can reach excellent vibration attenuation performance. However, it has the disadvantages of large energy consumption and the stability problem. The semi-active vibration absorber can be regarded as a passive vibration absorber with variable stiffness and/or damping. By varying the stiffness and/or damping, the semi-active vibration absorber can trace the exciting frequency so as to improve its vibration attenuation performance [4-7].

The magnetorheological elastomer dynamic vibration absorber (MRE DVA) is a kind of semi-active vibration absorber [8, 9]. It uses the magnetorheological elastomer as its smart spring element, which is due to the controllable modulus of magnetorheological elastomer by the external magnetic field [10, 11]. By varying the magnetic current, the resonant frequency of the MRE DVA can be controlled continuously, rapidly and reversibly. Therefore, the MRE DVA can trace the exciting frequency by controlling the magnetic current.

The control goal for the MRE DVA is to make its resonant frequency coincide with the exciting frequency [1]. Generally, the control process for the most semi-active vibration absorbers involves in fast identification of the exciting frequency and looking up the control signal from the experimental data [12-14]. For these semi-active vibration absorbers, the relation between the control signal and the resonant frequency is clear and accurate. Therefore, the control signal can be obtained fast and accurately after the exciting frequency is identified. However, this control process is not
suitable for the MRE DVA. The MRE DVA uses magnetorheological elastomer as its spring element. Magnetorheological elastomer is a kind of viscoelastic material. Its modulus is related to not only the external magnetic field but also the strain amplitude, strain rate, time, temperature, etc. [15-17] The experimental data about the resonant frequency of the MRE DVA and the magnetic current are obtained in specific condition. It is not accurate to use the experimental data in real-time control directly. Therefore, a control strategy, which does not rely on the accurate relation between the resonant frequency of the MRE DVA and the magnetic current, needs to be investigated.

In this study, a phase based integral control strategy was proposed for the MRE DVA attached to a multi-degree-of-freedom system. The proposed control strategy does not rely on the accurate relationship between the resonant frequency and the magnetic current and can make the MRE DVA trace the exciting frequency rapidly. Following the introduction, the control strategy is discussed in section II. Section III shows the simulation results. Finally, the conclusions are summarized in section IV.

2. Control strategy

![Figure1: Schematic diagram of a multi-degree-of-freedom system.](image)

In engineering applications, most vibrating objects are multi-degree-of-freedom systems. Therefore, in this study, a multi-degree-of-freedom system is used to represent the primary system. Fig. 1 shows the schematic diagram of a multi-degree-of-freedom system, where the black part represents the primary system with n degrees of freedom and the green part represents the MRE DVA. The equations of motion for the primary system attached with the MRE DVA can be written as

\[ M \ddot{x} + C \dot{x} + Kx = F \]  

where

\[
M = \begin{bmatrix}
m_1 & 0 & 0 & ... & 0 \\
0 & m_1 & 0 & ... & 0 \\
0 & 0 & m_3 & 0 & ... \\
... & ... & ... & ... & ...
\end{bmatrix} \quad C = \begin{bmatrix}
c_1 + c_2 & -c_2 & 0 & 0 & ... & 0 \\
-c_2 & c_2 + c_3 & -c_3 & 0 & ... & 0 \\
0 & ... & ... & ... & ... \\
0 & 0 & ... & ... & -c_n & 0 \\
0 & 0 & ... & ... & -c_n & 0 \\
\end{bmatrix} \\
K = \begin{bmatrix}
k_1 + k_2 & -k_2 & 0 & 0 & ... & 0 \\
-k_2 & k_2 + k_3 & -k_3 & 0 & ... & 0 \\
0 & ... & ... & ... & ... & 0 \\
0 & 0 & ... & ... & -k_n & 0 \\
... & ... & ... & -k_n & 0 & 0 \\
0 & 0 & ... & ... & -k_n & 0 \\
\end{bmatrix} \\
F = \begin{bmatrix}
x_1 \\
x_2 \\
x_n \\
x_{n-1} \\
x_{n+1} \\
\end{bmatrix}
\]

Therefore, the relation between \( x_{n+1} \) and \( x_n \) is

\[ m_{n+1} \ddot{x}_{n+1} + c_{n+1} \dot{x}_{n+1} + k_{n+1} x_{n+1} = c_{n+1} \dot{x}_n + k_{n+1} x_n \]  

(2)
Eq (2) is converted into Fourier transformation and is written in Eq. (3), where \( X_n \) and \( X_{n+1} \) are the Fourier transformation of \( x_n \) and \( x_{n+1} \) respectively.

\[
(-m_{n+1} \omega^2 + j \omega c_{n+1} + k_{n+1})X_{n+1} = (j \omega c_{n+1} + k_{n+1})X_n
\]  

(3)

So, the relation between the relative acceleration of the MRE DVA mass respect to the \( n \)th degree of the primary system and the absolute acceleration of the \( n \)th degree of the primary system can be written as

\[
\frac{(j \omega)^2 X_{n+1} - (j \omega)^2 X_n}{(j \omega)^2 X_n} = \frac{(\Omega_{n+1}^2 - 1) - 2j \xi_{n+1} \Omega_{n+1}}{(\Omega_{n+1}^2 - 1)^2 + 4 \Omega_{n+1}^2 \xi_{n+1}^2}
\]  

(4)

where

\[
\omega_{n+1} = \sqrt{\frac{k_{n+1}}{m_{n+1}}} \quad \Omega_{n+1} = \frac{\omega_{n+1}}{\omega} \quad \xi_{n+1} = \frac{\xi_{n+1}}{2m_{n+1} \omega_{n+1}}
\]

From Eq. (4), the phase angle of the relative acceleration of the MRE DVA mass respect to the absolute acceleration of the \( n \)th degree of the primary system is

\[
\phi = \begin{cases} 
\tan^{-1}\left(\frac{2 \xi_{n+1} \Omega_{n+1}}{\Omega_{n+1}^2 - 1}\right) & \Omega_{n+1} > 1 \\
\frac{\pi}{2} & \Omega_{n+1} = 1 \\
\tan^{-1}\left(\frac{2 \xi_{n+1} \Omega_{n+1}}{\Omega_{n+1}^2 - 1}\right) + \pi & \Omega_{n+1} < 1
\end{cases}
\]  

(5)

where the range of the inverse tangent is (-\( \pi/2 \), \( \pi/2 \)). According to Eq. (5), the phase angle lies between 0 and \( \pi \). The phase angle equals \( \pi/2 \) represents the resonant frequency of the MRE DVA equals the exciting frequency. If the phase angle lies between 0 and \( \pi/2 \), the resonant frequency of the MRE DVA is bigger than the exciting frequency. In this condition, the magnetic current should be decreased so as to decrease the resonant frequency of the MRE DVA. If the phase angle is bigger than \( \pi/2 \), the resonant frequency of the MRE DVA is smaller than the exciting frequency. In this condition, the magnetic current of the MRE DVA should be increased. Therefore, the phase difference of the relative acceleration of the MRE DVA mass and the absolute acceleration of the \( n \)th degree of the primary system can be used to check whether the resonant frequency of the MRE DVA coincides with the exciting frequency. Based on the observation of the phase angle, a stiffness control law is proposed as follows.

\[
\frac{dI}{dt} = \beta \cos \phi
\]  

(6)

Where \( \beta \) is the gain, \( I \) is the magnetic current and \( \phi \) is the phase angle. From Eq. (5), the following relation can be obtained if the gain \( \beta \) in Eq. (6) is chosen to be negative.

\[
\begin{align*}
\frac{dI}{dt} & > 0 \quad \Omega_{n+1} < 1 \\
\frac{dI}{dt} & = 0 \quad \Omega_{n+1} = 1 \\
\frac{dI}{dt} & < 0 \quad \Omega_{n+1} > 1
\end{align*}
\]  

(7)

Therefore, with the help of the stiffness control law shown in Eq. (6), the magnetic current will trend to the optimal value, which makes the resonant frequency of the MRE DVA coincide with the exciting frequency. Besides, the stiffness control law is not based on the relation between the mag-
netic current and the resonant frequency of the MRE DVA. The proposed stiffness law does not rely on the accurate relation of the magnetic current and the resonant frequency of the MRE DVA.

3. Simulation

A two-degree-of-freedom system attached with a MRE DVA, which is shown in Fig. 2, is used to evaluate the proposed stiffness control strategy. In Fig. 2, the black part represents the primary system and the green part represents the MRE DVA with controllable stiffness. In order to facilitate the simulation, the relationship between the magnetic current and the resonant frequency is assumed to be \( 20 + I \), where \( I \) is the magnetic current. It needs to be mentioned that this formula is only used to model the MRE DVA. Because the proposed stiffness control law does not rely on the model of the MRE DVA, the assumed relationship between the magnetic current and the resonant frequency of the MRE DVA does not affect the evaluation of the stiffness control law. The parameters used in the simulation are shown in Table 1, where \( f = \pi/2 \), and \( f_3 \) is the initial resonant frequency of the MRE DVA with the magnetic current of 0. \( c_1, c_2, c_3, k_1, k_2 \) and \( k_3 \) can be calculated using Eq. (4). The exciting harmonic force is acted on \( m_1 \) with the amplitude of 100 N.

![Figure 2: Schematic diagram of the simulation model.]

Table 1: Parameters used in simulation

<table>
<thead>
<tr>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( m_3 )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
<th>( \xi_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20kg</td>
<td>20kg</td>
<td>2kg</td>
<td>25Hz</td>
<td>80Hz</td>
<td>20Hz</td>
<td>0.01</td>
<td>0.01</td>
<td>0.025</td>
</tr>
</tbody>
</table>

The control process is shown in Fig. 2. In order to realize the control law shown in Eq. (6), a phase detector is needed to obtain the cosine of the phase angle. Assume two harmonic signals are

\[
y_1 = A_1 \sin \omega t \quad y_2 = A_2 \sin(\omega t - \phi)
\]

So, the following formula holds

\[
\frac{y_1 \cdot y_2}{A_{1RMS}A_{2RMS}} = 2 \sin \omega t \cdot \sin(\omega t - \phi) = \cos \phi - \cos(2\omega t - \phi)
\]

where RMS means the root mean square value and

\[
A_{1RMS} = \frac{\sqrt{2}}{2} A_1 \quad A_{2RMS} = \frac{\sqrt{2}}{2} A_2
\]

From Eq. (9), the cosine of the phase angle can be obtained from the normalized product of the two harmonic signals filtered by a low pass-filter. Besides, the RMS value can also be obtained by a low pass-filter because the following formula holds
\[ y^2 = (A \sin(\omega t + \theta))^2 = \frac{A^2}{2} - \frac{A^2}{2} \cos(2\omega t + 2\theta) \]  

(10)

The simulation results are shown in Fig. 3 and Fig. 4. Fig. 3 shows the result with the gain of -0.5, while Fig. 4 shows the results with the gain of -1.0. The exciting frequency and the status of the controller are shown in Table 2. From Fig. 3 and Fig. 4, when the exciting frequency varies, the MRE DVA can change its resonant frequency to trace the exciting frequency rapidly with the help of the proposed control law. And thus, the vibration of the primary system is suppressed significantly. The simulation results demonstrate that the proposed stiffness control law is efficient.

![Simulation Results](image)

**Figure 3:** Control results with the gain of -0.5: (a) displacement of the 1st degree of the primary system; (b) displacement of the 2nd degree of the primary system; (c) resonant frequency of the MRE DVA.

**Table 2:** Exciting frequency and the status of the controller in the simulation

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0-10</th>
<th>10-30</th>
<th>30-60</th>
<th>60-90</th>
<th>90-120</th>
<th>120-150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exciting frequency (Hz)</td>
<td>25</td>
<td>25</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>Status of controller</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>on</td>
<td>on</td>
<td>on</td>
</tr>
</tbody>
</table>

Comparing Fig. 3 and Fig. 4, increasing the amplitude of the gain leads to the decreasing of the tuning time. In order to show the effect of the gain more clearly, Fig. 5 plots the control results with different amplitudes of the gain. It can be seen that increasing the amplitude of the gain will shorten the tuning time and make the MRE DVA trace the exciting frequency more quickly. However, overlarge amplitude of the gain may result in the oscillation of the magnetic current. Therefore, the amplitude of the gain should be chosen properly in order to obtain an acceptable tuning time and prevent the oscillation of the magnetic current.
Figure 4: Control results with the gain of -1.0: (a) displacement of the 1st degree of the primary system; (b) displacement of the 2nd degree of the primary system; (c) resonant frequency of the MRE DVA.

Figure 5: Influence of the gain.

The effect of the noise is also considered in the simulation. In order to investigate the effect of the noise, the exciting force is composed of a harmonic signal with the amplitude of 100 N and a uniform distributed random signal with the maximum value of 20 N. The results are shown in Fig. 6. It can be seen that when the maximum value of the noise is 20% of the amplitude of the exciting harmonic force, the proposed stiffness control law is still efficient to make the MRE DVA trace the exciting frequency rapidly.
4. Conclusion

This work presents a phase based integral control strategy for the MRE DVA to trace the exciting frequency rapidly. The proposed control strategy does not rely on the relation between the magnetic current and the resonant frequency of the MRE DVA. It uses the phase difference of the relative acceleration of the DVA mass and the absolute acceleration of the primary system to check whether the resonant frequency of the MRE DVA is tuned properly. The simulation results demonstrate that the phase based integral control strategy is efficient to make the MRE DVA trace the exciting frequency rapidly.

REFERENCES


