Flutter control of bridge deck section using a combination of aerodynamic and mechanical measures, i.e., controllable winglets and rotating mass dampers is considered. Self-excited wind forces acting on deck and winglets, which are modeled as flat plates, are obtained using the Scanlan-Tomko model with flutter derivatives obtained from Theodorsen functions. Roger’s rational function approximation is used for time domain representation of wind forces. Measured outputs, comprising vertical and torsional displacement of deck, are fed back to the controller. Speed dependent gains from variable gain output feedback is used for winglets control input i.e., rotation of winglets relative to deck. Control using winglets enhance the critical speed to twice the uncontrollable flutter speed. Further attenuation of vertical response is obtained by using rotating mass dampers configured to provide only a resultant vertical force due to counter-rotating unbalanced masses. A constant rotor speed is considered with a start-stop criteria based on the vertical displacement of the deck and the rotor position. This generates the desired additional vertical force to be generally out of phase with the vertical velocity. The start-stop criteria provides greater attenuation when compared to the damper operated in continuous rotation mode for fixed number of cycles - especially at lower wind speeds. A maximum reduction of 15% in RMS vertical response is obtained when compared to control using winglets only.

Keywords: flutter control, winglet, rational function approximation, output feedback, rotor damper

1. Introduction

Modern long span cable supported bridges are very flexible and exhibit low damping, thus vulnerable to wind induced vibrations. Flutter is a dynamic instability due to self-excited forces resulting from wind-structure interactions. The failure of original Tacoma Narrows Bridge due to torsional flutter opens a challenging aspect in the field of flutter control of cable supported bridges. Scanlan and Tomko developed a flutter derivatives (FDs) based linear unsteady model of self-excited lift and moment acting on a deck section. These FDs can be obtained from wind tunnel tests or, for flat plate, from Theodorsen’s circulation function. Dependence of FDs on reduced frequency complicates the stability analysis and control system design in frequency domain. Thus, Roger proposed a time domain formulation using rational function approximation (RFA) of FDs.

Different mechanical and aerodynamic measures are studied for flutter suppression of cable supported bridges. Pourzeynali and Datta proposed a semi-active hydraulic tuned mass damper (TMD) with fuzzy logic control to provide the variable damping. Chen and Kareem showed that TMDs are effective in controlling soft flutter but less effective for hard flutter. Thus a combination of aerodynamic and auxiliary energy dissipating devices has been suggested. Starossek and Scheller proposed a novel active mass damper consists of a rotating rod with a mass attached to its free end. In a preferred configuration these dampers can generate resultant force and/or moment due to...
centrifugal forces induced by rotating unbalanced masses. Aerodynamic measures include additional control surfaces like winglets or flaps. Huynh and Thoft-Christensen [7] studied flutter suppression using winglets, with different combination of phase and amplitude of winglet pitching as parameters. Li et al. [8] considered a twin-winglet system for active aerodynamic control of flutter. Relative rotations of winglets are considered as control input and state feedback with pole placement observer is used for the controller design. Wilde and Fujino [9] used constant and variable gain output feedback (VGOF) control with winglet torque as input. The VGOF control law, proposed by Halyo et al. [10], is based on minimizing a global performance index defined over the entire operating range. Bera and Chandiramani [11] studied control of a deck section using winglets actuated by servomotors. Constant and wind speed dependent gains using VGOF is considered with winglet rotation as input.

In this study, flutter suppression of a bridge deck section using a combination of winglets and rotating mass dampers (RMD) is considered. Roger’s RFA is used for time domain modeling of self-excited foreshocks on deck and winglets, idealized as flat plates for their FDs. The VGOF controller is considered and the control input i.e., relative rotation of winglets are based on direct feedback of vertical and torsional displacements of deck. The RMD proposed in [6] is considered here for further attenuation of deck vertical responses. Dampers configuration that generates only vertical control force is considered. A star-stop criteria, based on vertical displacement of the deck and the rotor position, is proposed. Numerical result shows the effectiveness of the implemented control strategy.


The system consists of a deck, a pair of identical winglets and two pairs of rotating mass dampers (RMD) as shown in Fig. 1(a). The winglets, rotate along axes parallel to the deck axis, are symmetrically attached on leading (windward) and trailing (leeward) sides of the deck. Deck and winglets are modeled as flat plates and thus, only vertical and torsional degrees of freedom are considered. The winglets are sufficiently separated from the deck and hence their aerodynamic interaction is negligible. The winglet mass is neglected compared to that of deck. The main component of a single damper comprises a rotating rod with a lumped mass attached to its free end and the basic unit of RMD comprises vertical and torsional degrees of freedom. The system is based on minimizing a global performance index defined over the entire operating range. Bera and Chandiramani [11] studied control of a deck section using winglets actuated by servomotors. Constant and wind speed dependent gains using VGOF is considered with winglet rotation as input.

The aerodynamic lift and moment acting on deck/winglets, $\mathbf{F}_\text{aer}$, with $D$ and $W$ denoting deck and winglet, respectively; $\Delta R$ denotes absolute displacement of the $R$ component; Note that $W$ denotes both winglets, i.e., the L and T winglets are considered as control input and state feedback with pole placement observer is used for the controller design. Wilde and Fujino [9] used constant and variable gain output feedback (VGOF) control with winglet torque as input. The VGOF control law, proposed by Halyo et al. [10], is based on minimizing a global performance index defined over the entire operating range. Bera and Chandiramani [11] studied control of a deck section using winglets actuated by servomotors. Constant and wind speed dependent gains using VGOF is considered with winglet rotation as input.

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$$\mathbf{M}_s \mathbf{\ddot{\Delta}}_D + \mathbf{C}_s \mathbf{\dot{\Delta}}_D + \mathbf{K}_s \mathbf{\Delta}_D = \mathbf{F}_D + (\mathbf{E} - d \mathbf{S}_1) \mathbf{F}_L + (\mathbf{E} + d \mathbf{S}_1) \mathbf{F}_T + \mathbf{F}_c$$  \hspace{1cm} (1)

Here $\Delta_D = [h \alpha]^T$ comprises vertical and torsional degrees of freedom of the deck; $\mathbf{M}_s = \text{diag}[m I]$, $\mathbf{C}_s = \text{diag}[2m \omega_h \zeta_h \ 2 I \omega_h \zeta_h]$, $\mathbf{K}_s = \text{diag}[m \omega_h^2 \ I \omega_h^2]$, are mass, structural damping, and stiffness matrices, respectively, pertaining to the deck; $m$ and $I$ are the mass and mass moment of inertia, respectively, $\omega_h$ and $\zeta_h$ are natural frequencies of vertical and torsional motion, respectively, and $\zeta_h$ and $\zeta_\alpha$ are corresponding modal damping ratios, of the deck; $\mathbf{F}_R = [L_R \ M_R]^T$ comprises self-excited aerodynamic lift and moment acting on deck/winglets, $R \equiv D, L, T$, with $D$, $L$, $T$ denoting deck, leading winglet, and trailing winglet, respectively; $\mathbf{E} = \text{diag}[1 \ 1]$; $\mathbf{S}_1 = [0 \ 0 \ 1 \ 0]$; $2d$ is the spacing between winglet centroids; $\mathbf{F}_c = [F_{\text{RMD}} \ M_{\text{RMD}}]^T$ is the additional control force due to RMD, where $F_{\text{RMD}}$ is the resultant force acting at deck center in the downward direction and $M_{\text{RMD}}$ is the resultant moment in the positive clockwise direction.

The aerodynamic lift and moment on deck and winglets are obtained as [11]

$$\mathbf{F}_R = \begin{bmatrix} L_R \\ M_R \end{bmatrix} = \frac{1}{2} \rho B_J^2 \omega \begin{bmatrix} H_{1J} & H_{2J} \\ A_{1J} B_J & A_{2J} B_J^2 \end{bmatrix} \mathbf{\ddot{\Delta}}_R + \frac{1}{2} \rho B_J^2 \omega^2 \begin{bmatrix} H_{1J} & H_{3J} B_J \\ A_{1J} B_J & A_{3J} B_J^2 \end{bmatrix} \mathbf{\Delta}_R$$  \hspace{1cm} (2)

Here $J \equiv D, W$, with $D$ and $W$ denoting deck and winglet, respectively; $\mathbf{\Delta}_R$ denotes absolute displacement of the $R$ component; Note that $W$ denotes both winglets, i.e., the L and T winglets are assumed to have identical flutter derivatives and widths; $\rho$ is the air density; $B_J$ is the deck/winglet width; $\omega$ is the circular frequency of oscillation; $H_{iJ}$ and $A_{iJ}$ $(i = 1 - 4)$ are non-dimensional flutter
The aerodynamic forces can be expressed as \[ F_R = \frac{1}{2} \rho U^2 \left( b_J A_{1j} b_j \Delta R + \frac{B_J}{U} b_J A_{2j} b_j \Delta R + \frac{B_J^2}{U^2} b_J A_{3j} b_j \Delta R + \sum_{i=1}^{n_l} \Delta^a_{iR} \right) \] where \( U \) is the wind velocity and \( V_{ij} \) is the reduced velocity.

However, a time domain representation of aerodynamic forces is suitable for control applications. Roger [3] proposed a time domain formulation in which the aerodynamic force coefficients are approximated by rational functions of the Laplace variable. Using Laplace transformation and nonlinear least-squares optimization, self-excited forces can be expressed as [11]

\[ F_R = \frac{1}{2} \rho U^2 \left( b_J A_{1j} b_j \Delta R + \frac{B_J}{U} b_J A_{2j} b_j \Delta R + \frac{B_J^2}{U^2} b_J A_{3j} b_j \Delta R + \sum_{i=1}^{n_l} \Delta^a_{iR} \right) \]

in which the aerodynamic states i.e., \( \Delta^a_{iR} \) is defined as

\[ \Delta^a_{iR} = b_J A_{(i+3)j} b_j \Delta R = \frac{\lambda_{ij} U}{B_J} \Delta^a_{iR} \]

Here \( b_J = \text{diag}[B_J]; A_{1j}, A_{2j}, A_{3j} \), represents the aerodynamic- stiffness, damping, and mass, respectively; \( \lambda_{ij}(>0) \) are the frequency independent lag coefficients that model time delays and \( n_l \) are the number of lag terms considered. Details regarding RFA technique can be found in [9,11].

The free body diagram of rotor \( i (i = 1 - 4) \) is shown in Fig. 1(b). Refering to the coordinate system shown, the equations of motion of \( i \)-th rotor are [6]

\[ \begin{align*} m_i \left[ -d_i (\dot{\alpha} \sin \alpha + \dot{\phi}_i \cos \alpha - \dot{\phi}_i^2 \sin \phi_i) - r_i (\ddot{\phi}_i \cos \phi_i - \dot{\phi}_i^2 \sin \phi_i) \right] &= F_{yi} \\
( I_i + m_i r_i^2 ) \ddot{\phi}_i - m_i r_i ( \dot{h} - g ) \sin \phi_i + d_i ( \dot{\alpha} \sin (\phi_i - \alpha) - \dot{\alpha}^2 \cos (\phi_i - \alpha) ) &= M_{Mi} \end{align*} \]

where \( m_i \) and \( I_i \) are the \( i \)-th rotor mass and mass moment of inertia, respectively; \( M_{Mi} \) are the moments to be applied by the actuators. The rotor numbering is \( 1 - 4 \) from left to right, with rotor lengths \( r_1 = r_2 = r_1 \) and \( r_2 = r_4 = r_r \); \( d_1 = -d_4 = -a \); \( d_2 = -d_3 = -a - \frac{c}{2} \) and \( a \) as shown in Fig. 1(a). Each rotor having identical mass and inertia i.e., \( m_i = 0.25 m_R \) and \( I_i = 0.25 I_R \). The angular positions of the rotors 2, 3 and 4 are related to that of rotor 1 as: \( \varphi_2 = -\varphi_1; \varphi_3 = \varphi_1 + \psi; \varphi_4 = -\varphi_1 + \psi \); where constant angle \( \psi \) represent the phase shift between the rotor pair.

Thus variable \( \varphi_1 \) defines the positions of all the rotors. Now, using Eq. (5) with these assumptions, the resultant force and moment are obtained as [6]

\[ \begin{align*} F_{RMD} &= -m_R \ddot{h} + \frac{m_R}{2} r_1 (\dot{\varphi}_1 p_1 + \dot{\varphi}_1^2 p_2) \\
M_{RMD} &= -m_R \left( a^2 + \frac{c^2}{4} \right) \dot{\alpha} + \frac{m_R}{2} r_1 a \left[ -\dot{\varphi}_1 \cos \alpha p_3 + (\dot{\alpha} \sin \alpha - (\varphi_1^2 - \dot{\alpha}^2) \cos \alpha) p_4 \right] \\
&\quad - \frac{m_R}{2} r_1 \left[ -\dot{\varphi}_1 \sin \alpha p_2 + (\dot{\alpha} \cos \alpha + (\varphi_1^2 - \dot{\alpha}^2) \sin \alpha) p_1 \right] \end{align*} \]
with
\[
\begin{align*}
  p_1 &= \sin \varphi_1 + G_R \sin(\varphi_1 + \psi); \\
  p_2 &= \cos \varphi_1 + G_R \cos(\varphi_1 + \psi); \\
  p_3 &= \sin \varphi_1 - G_R \sin(\varphi_1 + \psi); \\
  p_4 &= \cos \varphi_1 - G_R \cos(\varphi_1 + \psi)
\end{align*}
\]

and $G_R = \frac{r_r}{r_l}$. Thus, there is no resultant horizontal force generated by the rotors. It can be seen that, the first term of $F_{\text{RMD}}$ and $M_{\text{RMD}}$ in Eq. (9) is going to modify the mass matrix $M_s$ of Eq. (1). So, only the remaining terms represent the rotor and deck motion dependent forces. However, a special case that produces only resultant vertical force is considered here. This requires the following conditions: (i) Length of all rotors are equal i.e., $G_R = 1$ (ii) For each rotor pair, two rotors are co-axially located i.e., $c = 0$ and (iii) No phase difference between the rotor pairs i.e., $\psi = 0$. Further, if the rotors are rotating at a constant angular velocity i.e., $\varphi_1 = \text{constant}$, then $\varphi_1 = 0$ and $F_c$ of Eq. (1) involves only resultant vertical force given as
\[
F_{\text{RMD}} = m_R \dot{r}_l \varphi_1^2 \cos \varphi_1
\]

with $M_s$ appropriately modified. Thus Eq. (1) can be written as
\[
\overline{M_s} \ddot{x} + \overline{C_s} \dot{x} + \overline{K_s} x = \overline{F} + \overline{E} \dot{x} + \overline{F}_{\text{RMD}}
\]

where $\overline{M_s} = M_s + \text{diag}[m_R \ m_R a^2]$ and $E_r = [1 \ 0]^T$. The absolute displacements of winglets and deck are related as
\[
\Delta_L = (E - d S_1^T) \Delta_D + S_1 u; \quad \Delta_T = (E + d S_1^T) \Delta_D + S_2 u
\]

where $S_2 = [0; 0; 0; 1]$, $u = \{\phi_L; \phi_T\}^T$ is the control input to winglets. The state vector, defined as $x = [\Delta_D; \Delta_D; \Delta_{1D}; \Delta_{n_D}; \Delta_{1L}; \Delta_{n_L}; \Delta_{1T}; \Delta_{n_T}]$, comprises mechanical and aerodynamic states. Thus, Eqs. (9) and (10), considered with Eqs. (10) and (11), yields state equations
\[
\dot{x} = A x + B_1 u + B_2 \dot{u} + B_3 \ddot{u} + L \dot{F}_{\text{RMD}}
\]

Derivatives of $u$ are eliminated by re-defining the state as $\hat{x} = x - (B_2 + A B_3) u - B_3 \dot{u}$. Deck displacements $h$, $\alpha$, are considered as the measured outputs. Thus, the state and output equations are,
\[
\dot{x} = A \hat{x} + B u + L \dot{F}_{\text{RMD}}; \quad y = C x + D u = \Delta_D
\]

where $B = B_1 + A B_2 + A^2 B_3$. The detail of the matrices in Eq. (12) are given in [11], with $M_s$ replaced by $\overline{M_s}$ and $L = \begin{bmatrix} 0 & \overline{M}^{-1} E_r & 0 & 0 & 0 \end{bmatrix}^T$.

3. Variable Gain Output Feedback

Controller design in which the control input (winglets relative rotation in this case) is obtained via measured output feedback instead of state feedback, is termed as Optimal Static Output Feedback Control. As the system matrices $A$, $B$ in Eq. (12) depend on wind speed $U$, it makes the controller gains speed dependent. Using a constant gain, obtained at a particular wind speed, may not provide effective control [11]. Also the conventional gain scheduling method is essentially a curve fitting approach lacking control theory basis [9]. Thus, Halyo et al. [10] formulated a variable gain output feedback (VGOF) control law based by minimizing a global performance index (PI) involving the entire operating range. Herein the controller design methodology for a deterministic continuous system is presented [9]. State, output, and control input equations for a variable parameter system are
\[
\dot{x}(p) = A(p)x(p) + B(p)u(p); \quad y(p) = C(p)x(p); \quad u(p) = -K(p)y(p)
\]

where $p = \{p_1 \cdots p_i \cdots p_r\}^T$ is the $r$-vector of parameters defining the operating point. For the present system $r = 1$, $p = \{U\}$. The variable gain is expressed as
\[
K(p) = K_0 + \sum_{i=1}^r p_i K_i = \overline{K} G(p); \quad \overline{K} = [K_0 \ K_1 \cdots K_r]; \quad G(p) = [I; \ p I; \cdots; p r I]
\]
A global PI defined as: 
\[ J_g = \int_{\mathbb{R}} f(p) J^* (K(p), \mathbb{R}) \, dp \]
considers the operating range of parameters \( \mathbb{R} \). Here \( f \) is the weightage assigned at operating point \( p \), and \( J^* = \frac{1}{2} \int_{0}^{\infty} [x^T Q x + u^T R u] \, dt \), is the local PI at the given operating point and corresponding optimal gain \( \tilde{K}(p) \). Positive semi-definite \( Q \) and positive definite \( R \) being the state weighting and control input weighting, respectively. Given the system and feedback in Eq. (13), we seek the global gain \( J(14) \), that minimizes \( R \) and positive definite \( B \).

Rotor position i.e., displacements are positive maximum and negative maximum, respectively. However, a tolerance in \( h_{\text{deck}} \) are shown in Fig. 2(c). Thus ideally the rotor should be at position-I and II when deck vertical position i.e., rotor in Zone-A then it is allowed to rotate till position-I and then stopped. The rotor is allowed to rotate again when the deck \( h \) attains its next positive maximum. On the other hand, if the rotor is leading the desired position i.e., rotor in Zone-B then it is allowed to rotate till position-II and then stopped. The rotor is allowed to rotate again when the deck \( h \) attains its next negative maximum. A similar criteria related to position-II is also implemented. Thus, the rotors are allowed to start or stop only from two horizontal positions i.e., position-I and II as shown in Fig. 2(c).

4. RMD Implementation: Start-stop Criteria

A constant rotor speed is considered with a start-stop criteria based on the vertical displacement of the deck and the rotor position. It is assumed that individual rotor can rotate in one particular direction only. For effective control and stability of the deck, the desired vertical control force i.e., \( F_{RMD} \) should be out of phase with the deck vertical velocity i.e., \( \dot{h} \), as shown in Fig. 2(a). The corresponding \( h \) vs \( F_{RMD} \) plot is shown in Fig. 2(b). The transition points of \( F_{RMD} \) plot are indicated as I and II, for which deck \( h \) are positive maximum and negative maximum, respectively. The corresponding rotor positions are shown in Fig. 2(c). Thus ideally the rotor should be at position-I and II when deck vertical displacements are positive maximum and negative maximum, respectively. However, a tolerance in rotor position i.e., \( \phi_t \) is introduced to cater for a change in the frequency of deck oscillation and also for effective control. So, when deck \( h \) is positive maximum and rotor is around position-I indicated by \( \phi_t \), it is allowed to rotate continuously without stopping. However, if it lagging behind the desired position i.e., rotor in Zone-A then it is allowed to rotate till position-I and then stopped. The rotor is allowed to rotate again when the deck \( h \) attains its next positive maximum. On the other hand, if the rotor is leading the desired position i.e., rotor in Zone-B then it is allowed to rotate till position-II and then stopped. The rotor is allowed to rotate again when the deck \( h \) attains its next negative maximum. A similar criteria related to position-II is also implemented. Thus, the rotors are allowed to start or stop only from two horizontal positions i.e., position-I and II as shown in Fig. 2(c). This generates the desired additional vertical force to be generally out of phase with the vertical velocity. Further, peak \( h \) for each cycle is measured and the rotors are permanently stopped when peak \( h \) for the last cycle becomes lower than 5% of the overall peak \( h \) till that cycle.

5. Results

Structural data for deck section is adapted from [8] as follows: \( m = 40100 \, \text{kg/m} \), \( I = 8100700 \, \text{kg} \cdot \text{m}^4/\text{m} \), \( \omega_h = 0.447 \, \text{rad/sec} \), \( \omega_a = 0.767 \, \text{rad/sec} \), \( \xi_h = \xi_a = 0.5\% \), \( B_D = 31 \, \text{m} \) and for winglet \( B_W = 0.05 B_D \) and \( d = 0.5 B_D \) are considered. Both deck and winglets are idealized as flat plate and their FDs are obtained from the analytical expressions in terms of Theodorsen’s function [2]. For RFA, the range of reduced velocities for deck and winglet are 0-40 and 200-500, respectively. These are chosen based on their widths. Two lag terms \( (m = 2) \) are considered for the RFA of deck and winglets FDs. For deck the coefficient matrices as well as the lag coefficients are optimized, while for winglets the lag coefficients are chosen apriori. Figure 3 shows results for the RFA of deck. The uncontrolled flutter speed is obtained at \( U = 47 \, \text{m/s} \) and its corresponding flutter frequency is 0.1011.
Figure 2: Desired plot for (a) $F_{RMD}$ vs $\dot{h}$ and (b) $F_{RMD}$ vs $h$, (c) rotor positions for start-stop criteria.

Figure 3: RFA of deck FDs (with lag optimization).

Hz. The divergence speed is 72.62 m/s. Here, control studies are carried out in two stages. Only winglet controlled deck is considered in the first stage and both winglet and RMD controlled deck is considered in the second stage.

For VGOF control, PI weighting matrices are as follows: $Q = F_q \text{diag}[K_s, M_s, 0]$, $R = F_r \text{diag}[1, 1]$, $F_q$ and $F_r$ are scaling factors. The 0 in $Q$ is of size of aerodynamic states. Thus $Q$ represents total mechanical energy. Here, results corresponding to $F_q = 0.001$, $F_r = 1000$ are presented. The absolute rotation of winglets are constrained within $\pm 15^\circ$ by modifying the control input as follows: $u = -KCx$ and if $|\alpha + \phi_L| > 15^\circ$ then $\phi_L \leftarrow \text{sgn}[\alpha + \phi_L]15^\circ - \alpha$, and similarly for $\phi_T$. Note that $\alpha_L \equiv \alpha + \phi_L$ and $\alpha_T \equiv \alpha + \phi_T$ denote absolute rotations of winglets. When the winglet rotations are not constrained, the critical speed is determined through eigenvalues of the closed loop state matrix, i.e., $A_c = A - BK C$. The controlled responses are obtained by solving Eqs. (12) using MATLAB ODE45, with control input $u$ obtained from Eq. (13)(c). Note that $F_{RMD} = 0$ when RMD is not considered. Equation (14) is used to determine gain $K$ at the operating speed $U$. Initial conditions are chosen arbitrarily as $h(0) = 0.5$ m, $\alpha(0) = 5^\circ$.

For a single operating parameter, $U$, as in the present case, Eq. (14) reads as: $K(U) = K_0 + U K_1$. The range of design speed considered is 40–57 and integration points are spaced @ 0.25 m/s. Note

Figure 4: VGOF controlled responses using winglet only.
that the upper limit of this range is the critical speed obtained using a constant gain for design speed 47 m/s (i.e., uncontrolled flutter speed). Global PI weight factors are considered as \( f = U \). Integrals in Eqs. (15) and (16) are evaluated using Simpson’s 3/8 rule and then \( \Delta \tilde{K} \) is obtained using solve command in MATLAB. Closed loop eigenvalue analysis yields divergence at 293.39 m/s. The response at various wind speeds, shown in Fig. 4, indicates that control is effective even at speeds above the range chosen for design. The response shows divergence at 96.21 m/s which is much lower than the divergence speed from eigenvalue analysis. This is because for \( U = 96.21 \) the leading winglet reaches the limiting rotation at the start and stays there, and the trailing winglet stays at the limiting rotation after 21 sec. So the control strategy with winglet and VGOF controller provides approximately 100% increase in critical speed over the uncontrolled flutter speed, and is thus quite effective.

Now further attenuation of responses are achieved by introducing RMD in the deck-winglet system. The parameters consider for RMD are: \( m_R = 2000 \) kg i.e., approximately 5% of the deck mass, \( r_l = r_r = 0.8 \) m and \( a = 0.5d \). A constant rotor speed of 0.760 rad/sec is considered. This is the average of frequencies of only winglet controlled responses at \( U = 47, 50, 55, 60, 65, 70, 75, 80, 85 \) and 90 m/s. Different \( \varphi_t \) values are used for rotor start-stop criteria. For each \( \varphi_t \), % reduction in RMS \( h \) responses with respect to only winglet controlled responses at different wind speeds are obtained. This is shown in Fig. 5 for \( \varphi_t = 5^\circ, 10^\circ, 15^\circ, 20^\circ, 30^\circ, 40^\circ \) and \( 85^\circ \). For \( \varphi_t = 5^\circ, 10^\circ \) the reductions are lowest and \( \varphi_t = 85^\circ \) gives substantially lower reduction till \( U = 65 \) m/s. This is due to the fact that for stricter tolerance i.e., for \( \varphi_t = 5^\circ, 10^\circ \) the rotors are more frequently stopping and thus hampering the additional control force generation. On the other hand, when tolerance is too relaxed i.e., for \( \varphi_t = 85^\circ \) the rotors are allowed to rotate continuously which produces undesirable control forces and hence deteriorating the control performance. The % reductions for \( \varphi_t = 15^\circ, 20^\circ, 30^\circ, 40^\circ \) are almost identical, but \( \varphi_t = 30^\circ, 40^\circ \) give relatively better reduction especially at \( U = 75 \) m/s. Finally, \( \varphi_t = 30^\circ \) is chosen.

For simpler implementation now the rotors are allowed to run continuously for a fixed number of cycles and then stopped. Total nine cases having number of fixed cycles 2 to 10 are considered and
for each case the % reduction of RMS $h$ and $\alpha$ are obtained. Overall best reduction is achieved for 5 cycles. The reduction of RMS $h$ and $\alpha$ for the best fixed number of cycles i.e., 5 cycles and for the proposed start-stop criteria with $\varphi_t = 30^\circ$ are compared. This is shown in Fig. Clearly start-stop method performs better for both RMS $h$ and $\alpha$ reduction, at least till 70 m/s. The maximum difference is observed at $U = 47$ m/s i.e., at uncontrolled flutter speed. However, the differences reduces gradually and become insignificant beyond 70 m/s. With start-stop method a maximum reduction of 14.72% for RMS $h$ and 5.72% for RMS $\alpha$ are achieved at $U = 47$ and 70 m/s, respectively. Therefore, the implementation of RMD with start-stop criteria give better control than continuously running for a fixed number of cycles. However, the stability limit remain unchanged with RMD providing only vertical control force.

6. Conclusions

Flutter control of a bridge deck section using controllable winglets and RMD is studied. Speed dependent gains from VGOF is used for winglets control input and a stat-stop criteria is implemented for RMD rotating at a constant speed. Control using winglet provides an increase in critical speed from 47 to 96.21 m/s. Also the responses are quite effectively controlled beyond the range considered for VGOF. Further attenuation of deck responses are achieved through RMD. The implementation of RMD with start-stop criteria, generates the additional vertical force to be out of phase with the deck vertical velocity, provides better control when compared to the damper operated in continuous rotation mode for fixed number of cycles - especially at lower wind speeds. However, substantial reduction is achieved only for RMS $h$, as the dampers are providing only vertical control force.

REFERENCES