THE NEW SOUND SOURCE IMAGING METHOD BASED ON PHASE CONJUGATION METHOD AND EXPERIMENTAL VERIFICATION

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The phase conjugation methods would obtain the accurate imaging results for the vibro-acoustic fields of the cylindrical shell in the near field. The experimental research on the identification and location of the acoustical radiation in the hemi-anechoic chamber. The results show that the method had the good application in engineering practice.

Keywords: Phase conjugation, acoustical radiation, identification and location, imaging

1. Introduction

The identification of sound sources plays an important role in the effective noise control. Time reversal can be used to focus sound and is also called phase conjugation (PC) in the frequency domain. The equivalence between phase conjugation in the frequency domain and time reversal in the time domain is established by Jackson and Dowling [1]. Due to its focusing property, the phase conjugation arrays could be used to build the image of a noise source and for source identification. However, the spatial resolution of the focused field of a classical PC array has a half wavelength limit because of diffraction. de Rosny and Fink [2] first showed that this limitation can be overcome by an acoustic sink. Fink et al [3] also showed that the subwavelength focusing could be achieved inside a micro-structured medium. Conti et al [4] demonstrated the subwavelength focusing could be obtained without a priori knowledge of the source by a near-field time reversal procedure. de Rosny and Fink [5] investigated three species of the time reversal arrays in the near field of the initial source and concluded that only the dipole time-reversal array leads to subwavelength focusing. Liu Song [6] had improved the phase conjugation method could be used to identify and locate the complex sound source.

The finite element method (FEM) and boundary element method (BEM) has been used extensively in acoustic radiation from bodies with known velocity, pressure, or impedance distribution. Typical problem types include interior and exterior problems [7]. For an exterior problem, the objective is to solve the Helmholtz equation in an unbounded fluid domain due to the acoustic radiation from a vibrating cylinder shell.

The experiment of the cylinder shell is performed in the semi-anechoic room to confirm the validity of the phase conjugation method applying to the identification of the sound source in this paper. The radiated complex pressure of the cylinder shell is measured through the scanning array and the radiated pressure and the normal velocity is reconstructed by phase conjugation method.
2. Theory

2.1 Acoustical radiation from cylinder shell

For the vibration and sound radiation from a mechanically-excited, submerged, elastic structure such as a cylinder, the coupled technique using FEM for the structure and the BEM for the fluid is a natural choice for numerical solutions.

Consider an arbitrary, submerged, three-dimensional, elastic structure subjected to time-harmonic loads. If the structure is modelled with finite elements, the resulting matrix equation of motion for the structural degrees of freedom can be written as:

\[ [Z]v = \{f_s\} - [G][A][P] \]  

where \([Z] = (-\omega^2[M] + i\omega[C] + [K]) / i\omega\) is the impedance matrix. \([M], [C]\) and \([K]\) are the structural mass, damping, stiffness matrices. \(\omega\) is the circular frequency. \(\{v\}\) is the vector of velocity. \(\{f_s\}\) is the external load vector. \([G]\) is the transformation matrix to transform a vector of normal forces to a vector of forces to all structural degrees of freedom. Matrix \([A] = \int [N]^T[N]dS\). \(\{p\}\) is the vector for the surface radiated acoustic pressure.

The boundary element method (BEM) formulation for acoustic radiation is

\[ [E][P] = [D][v_n] \]  

where \([E]\) and \([D]\) are the assembled coefficient matrices, \(\{v_n\}\) is the vector consisting of the field values for the surface normal velocity at the nodal locations of a grid defining the surface of the structure.

By the relationship between the velocity vector and the normal velocity vector

\[ \{v_n\} = [G]^T\{v\} \]  

The coupled equation (1) then is

\[ ([Z] + [G][A][E]^T[D][G]^T)\{v\} = \{f_s\} \]

When the vector of the velocity \(\{v\}\) is solved, the vector of the normal velocity \(\{v_n\}\) and the surface pressure \(\{p\}\) can also be obtained.

2.2 Acoustic phase conjugation

The reason that time-reversed sound waves travel backwards is a direct consequence of the lossless linear wave equation for the acoustic pressure \(p(r,t)\)

\[ \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \]  

This equation is time-reversal invariant because it contains only second-order derivatives with respect to time. Equation (5) ensures that if \(p(r,t)\) is a solution then \(p(r,-t)\) is too. Thus, if \(p(r,t)\) represents sound waves expanding away from a sound source, then \(p(r,-t)\) represents sound waves converging toward the same source. In the frequency domain \(p(r,t)\) and \(p(-r,-t)\) could be replaced by \(p(r,\omega)\) and \(p^*(r,\omega)\) respectively, where \(p\) has a harmonic time dependency of \(e^{i\omega t}\). For a perfect PC array, both the original field \(p\) and its normal derivative \(\frac{\partial p}{\partial n}\) should be recorded to serve as the weighting factors for the arrays of monopole and dipole sources, the phase-conjugated field at the field point \(r\) due to a point source \(r_s\) is given by [4]:

\[ p_{PC/P}(r,r_s) = \int \left[ G(r,r') \frac{\partial p^*(r',r_s)}{\partial n} - p^*(r',r_s) \frac{\partial G(r,r')}{\partial n} \right] dS \]  

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where $G(r, r_3) = \frac{e^{-ikr}}{4\pi kr}$ is the Green’s function, $S'$ denotes the surface of the PC array, $r'$ is the array element point on the $S'$. The realistic PC arrays are discrete and have $N$ array elements, the output of discrete PCP arrays can be written as

$$P_{PC/P}(r, r_3) = \sum_{n=1}^{N} \left[ G(r, r_n) \frac{\partial p^*(r_n, r)}{\partial n} - p^*(r_n, r) \frac{\partial G(r, r_n)}{\partial n} \right] \times S_n$$

(7)

The phase-conjugated field $p_{PC/P}$ is based on both the pressure and pressure gradient measurement and made of both monopole transceivers and dipole transceivers to reverse sound backwards. The phase conjugation arrays could also be based on pressure or particle velocity measurement and be made of dipoles to reverse. The phase-conjugated field by the array made of dipole transceivers based on the pressure gradient measurement is

$$p_{PC/D}(r, r_3) = \sum_{n=1}^{N} \left[ \frac{\partial G(r, r_n)}{\partial n} \frac{\partial p^*(r_n, r)}{\partial n} \right] \times S_n$$

(8)

In the following numerical analysis, the pressures calculated at the array element based on theory in 2.1 are used as the measurement pressures in the above three equations. A double layer of array elements is used to provide an approximation to the normal derivative. That is, the pressure $p = (p_1 + p_2)/2$ and the pressure gradient $\frac{\partial p}{\partial n} = (p_2 - p_1)/\Delta$, where $\Delta$ is the separation distance of the double layer.

3. **The experimental results**

The experimental device for acoustical radiation problem is composed of cylindrical shell model, support and vibration exciter. The material property and size of the cylindrical shell model used in this experiment are as follows: Material: low carbon steel; modulus of elasticity: 2.06E11; Poisson's ratio: 0.3; density: 7850; cylindrical shell thickness: 3mm. The geometric center of the cylindrical shell is taken as the origin of the coordinate, and the relative size of the cylindrical shell is shown in Figure 1. Wood materials were used in the experimental stents, and the portal structure was used. The bracket consists of two parts, and the bracket 1 is used to lift the cylindrical shell model to meet the boundary conditions of free vibration and to fix the microphone array. The bracket 2 is used to lift the vibration exciter r. The stent 1 was arranged with a 90 degree angle of 2, and the center of the two stents coincided. In the experiment, the bottom of the stent must be fixed with iron.

![Figure 1: The scheme of the whole experiment equipment](image-url)
The vibrator used in this experiment is the 4809 type vibration exciter produced by B&K Company. The vibrator can be used to calibrate the accelerometer and to analyse the vibration of the small specimen. In the experiment, the vibrator was lifted with the support 2, and the radial direction of the cylindrical shell, that is, the positive direction of the X axis, was applied. The position of the vibrator in the cylindrical shell and the experimental device are shown in Figure 2.

The excitation position is at the 0.48M of the surface of the cylindrical shell, which is at the Z=0.162m, and the excitation frequency is 10N by adjusting the power amplifier. The receiver is composed of a microphone measurement array, an acceleration sensor and a PULSE data acquisition system.

The microphone used in this experiment is a 4189 type prepolarizing free field microphone produced by B&K Company. The measurement array in this experiment is a cylindrical array of cylindrical shells. The information of the scanning array is as follows: the radial distance between the array and the cylinder is R=0.085m=0.05 \lambda , the height is 1.3m, and the number of the circumferential array elements is 15. The array element spacing: \triangle =0.13m=0.08 \lambda , \triangle =0.11m=0.07 \lambda .

First make a measuring circle for fixing the microphone, and connect the wire rope and pulley with the stepping motor on the door support of the cylindrical shell. 15 microphones are evenly arranged on the measuring circle. In the experiment, it starts from the surface of the cylindrical shell, and uses the stepper motor to control the measurement of the circular ring downward. Each time 13cm moves, 11 measurement positions are taken. 20 groups of measurements are measured for each position of the ring, and the average value is taken as the actual measurement value. When the sound pressure gradient is measured, all the microphones are moved back to 1cm on the measuring circle, and the above measurements are repeated, and the complex pressure values for calculating the sound pressure gradient are obtained.

The surface acoustic pressure of the cylindrical shell and the amplitude distribution of the normal velocity are obtained by the FEM+BEM coupling method as shown in Figure 3, 4.
Figure 3: The surface pressure amplitude of the cylinder shell by FEM+BEM

Figure 4: The normal velocity amplitude of the cylinder shell by FEM

The radiation acoustical pressure at the position of the measurement array by using FEM+BEM method is shown in Figure 5. The measured sound pressure amplitude by the scanning array is shown in Figure 6. The results show that the amplitude distribution of the acoustic pressure measured at the measured array is basically the same as the calculated value, and the measured value is less than the calculated value but no more than 3dB.

Figure 5: The pressure amplitude for the location of the measurement array by FEM+BEM
Figure 6: The pressure amplitude of the measurement value

The calculated values of the normalized amplitude at the position of $z=-0.008\,\text{m}$ are compared with the measured values as shown in Figure 7. The results show that the measured values of the normalized sound pressure are in good agreement with the calculated values.

Figure 7: The comparative curve of the calculated and measurement value at $z=-0.008\,\text{m}$ of the normalized pressure amplitude

The conjugate value of complex sound pressure of each array element are introduced into the calculation formula of sound pressure gradient. The amplitude distribution and normal velocity amplitude distribution of the cylindrical shell surface is reconstructed by phase conjugation method based on PCD as shown in Figure 8 and Figure 9.
Figure 8: The surface pressure reconstructed by PCD

Figure 9: The normal velocity reconstructed by PCD

The results in Figure 8 and Figure 9 show that the only made of dipole transceivers in the near field by cylindrical array based on the acoustic pressure gradient measurement can get better sound field reconstruction and clearly gives the distribution trend of cylindrical shell surface pressure which break the diffraction limit of $0.5\lambda$.

Figure 10 shows the comparison curve for the normalized normal vibration velocity compared the measurement and calculation value. It can be seen from the figure that the measurement and calculation value are in good agreement using the phase conjugation method, the maximum error is 0.4. This fully illustrates that the phase conjugation method can be applied to noise source recognition and location.
Figure 10: The comparative curve of the calculated and measurement value at \( z = 0.162 \text{m} \) of the normalized normal velocity

4. Conclusion

The experiment of the cylinder shell is performed in the semi-anechoic room to confirm the validity of the phase conjugation method applying to the identification of the sound source. The radiated complex pressure of the cylinder shell is measured through the scanning array and the radiated pressure and the normal velocity is reconstructed by phase conjugation method. The test results show that: The phase conjugation arrays achieve very high resolution to reconstruct the radiated pressure and normal velocity amplitude distribution. This experiment verified the feasibility of the phase conjugation method utilized in identification and location of the sound source and has the huge utility value in engineering.

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REFERENCES