A STUDY OF FLUID-STRUCTURAL COUPLING IN AN OIL-FILLED POWER TRANSFORMER

Yuxing Wang, Ming Jin, Jie Pan
Zhejiang University, College of Biomedical Engineering & Instrument Science, Hangzhou, China
e-mail: wangyuxing@zju.edu.cn

Chunming Pei
Wuhan Nanrui Power Engineering and Equipment Co., Ltd., Wuhan, China

Chong Tan, Yonggang Wang
Shandong Power Equipment Co., Ltd., Jinan, China

Abstract: Design and manufacturing quiet transformers have become a trend for minimizing the environmental noise pollution by the transformers. One of the approaches in reducing the transformer noise at the source is to control the vibration at the transmission paths inside the transformer. As a follow-up of the previous experiment study on the vibration transmission path on a 110-kV power transformer with and without cooling oil, numerical simulation is used in this paper to examine the acoustic resonances in the cooling oil of the transformer, and the acoustical-structural coupled modes, which could be used to explain the previous experimental results. This work demonstrates that the fluid-structural coupling could be an important transmission path for the sound radiation from a transformer and should be included in the design of quiet transformers.

Keywords: Power transformer, Fluid-Structural coupling, Acoustic resonance.

1. Introduction

The humming noise emitted by power and distribution transformers has been a concerned environmental issue for several decades. The study of the characteristics of transformer noise can be traced back as early as 1931, when George [1] described, the general mechanisms of transformer vibration and noise emission. They are associated with the working mechanisms of the ferromagnetic core to form a closed flux path and conductive windings to realize the transfer of electric power. However, with increased demand of electric energy consumption, the number, size and capacity of transformers have far exceeded that of 1930’s, so were their noise pollution issues.

To tackle those problems, manufactures and users of transformers turned to the low-noise core material and noise control techniques [2-3]. Effective implementation of these two strategies was usually resulted from the understanding of the mechanisms of transformer vibration and noise radiation. For example, studies on the tensile or bending stress on magnetic properties of electric steels yield coating designs on the SiFe surface to decrease its magnetostriction [4-5]. Stress-dependent core vibration phenomenon makes flexible bonding an important technique for the core assembling [6].
The influence of grain-orientation and domain size on the transformer noise introduces the laser domain refinement technique to electric steel manufactures [7]. Awareness of the core and tank resonance vibrations leads to the optimization of transformer core and tank with higher mechanical strength and lower sound radiation efficiency [2]. There are many other practical designs to eliminate transformer sound radiation [8-12]. However, few of them are directly relevant to the control at the pathes of vibration transmission within a transformer.

Study on the vibration transmission from core to the transformer tank could trace back to Churcher and King [13], who demonstrated experimentally that the sound energy transmitted through the two paths from the core to tank have the same order of magnitude. They examined the transformer sound radiation via the cooling oil as the transmission path by inserting a resilient support between the core and the bottom of transformer tank. Jin and Pan measured the vibration transmission merely by mechanical paths by evacuating the cooling oil [14] from the transformer tank. Both of their studies demonstrated that the vibration transmitted by oil path is important. Churcher and King even proposed a sound absorption enclosure inside the transformer oil to reduce the vibration transmission from core to the tank. However, there is still a need of study on the fluid-structure coupling in an oil-filled transformer, which is the mechanisms for energy transmission through the cooling oil.

In this paper, the vibration transmission from the core and windings to the transformer tank is investigated using the acoustic finite element (FE) method to the numerically model a 110kV power transformer (which is the same as that in the previous experimental study [14]) with and without cooling oil. The increased vibration in tank structural of the transformer with cooling oil at certain frequencies could now be readily explained by the fluid-structure coupling within the transformer.

2. Vibration modelling of the oil-filled transformer

2.1 Governing equations for the coupled transformer vibration

Figure 1 shows the vibration transmission model of an oil-filled transformer. The cooling oil in the space denoted by $\Omega$ is assumed as compressible fluid in the vibro-acoustical model. Its coupling interfaces with the tank structure ($\Sigma_T$) and the active parts ($\Sigma_A$) are respectively $\Gamma_T$ and $\Gamma_A$. To account for the reinforced ribs on the tank surface, the material properties of $\Sigma_T$ are described by the equivalent Young’s modulus, $E$, Poission ratio, $\nu$, and density $\rho$. The density and speed of sound in the cooling oil are respectively $\rho_o$ and $c_o$. The active parts (including core and windings) are modelled as an elastic structures and a point force is applied at the core to simulate the excitation of an impact hammer as in the previous experiment [14]. The core is mechanically attached to the bottom of the tank structure. Therefore, the continuity of vibration velocity and equilibrium of force are the conditions at the interface between the core and tank structure. The supporting condition of the transformer by the ground varies, depending upon the receptance of the ground and contacting locations and property of the mounting structure. For simplicity, the bottom of the tank is assumed to be fixed to the solid ground at the four corners. The effect of other supporting conditions on the vibration transmission path from the active parts to the tank will be briefly discussed later.

![Fluid-structure coupling model for transformer sound transmission analysis.](image)
The analysis of the fluid-structural coupled system can be made by using Hamilton’s principle, which requires that the action functional of the system satisfies the following variation equation [15]:

\[ \delta \int_{t_i}^{t_f} \left( T - U + W \right) dt = 0 \tag{1} \]

where \( T \) and \( U \) are respectively the total kinetic and potential energy of the system. \( W \) is the work done by any non-conservative force acting on the system. For simple harmonic motion, both \( T \) and \( U \) can be described as a displacement vector and sound pressure, which are commonly used to describe the dynamics of the elastic and fluid elements within the system, assisted by the stress/strain/displacement relationship in the structure, and relationship between the pressure/particle velocity in the fluid.

Including boundary conditions and the work done by the external force, the variation expression in Eq. (1) can be described by the general coordinate vectors (displacement vector \( U \) and sound pressure vector \( P \)) for all the elements of the system [16]:

\[
-\omega^2 \begin{bmatrix} \mathbf{M}_s & 0 \\ \rho \mathbf{R}^T & \mathbf{M}_f \end{bmatrix} + \begin{bmatrix} \mathbf{K}_s & -\mathbf{R} \\ 0 & \mathbf{K}_f \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ 0 \end{bmatrix} \tag{2}
\]

where \( \mathbf{M}_s, \mathbf{M}_f, \mathbf{K}_s, \) and \( \mathbf{K}_f \) are respectively the structural and fluid mass and stiffness matrices of the system. \( \mathbf{R} \) is the fluid-structural coupling matrix and \( \mathbf{F} \) is the generalized force vector.

The specific boundary conditions are:

1. The displacement vector is zero at the supporting points between the transformer tank and rigid ground;
2. The stress on the dry surface of the tank (assuming the sound pressure in the air will not affect the structural vibration) is assumed to be zero;
3. On the other “wet” surfaces, pressure in the fluid and the normal stress on the transformer tank and active parts satisfy the equilibrium condition;
4. The normal particle acceleration of the transformer oil associated with the pressure equals the normal acceleration of structure at the wet interface.

### 2.2 Finite element model of a 110kV power transformer

The schematic diagram of a 110kV power transformer is shown in Fig. 2(a). To focus on the coupling between the transformer oil and tank/active parts, the accessories of the power transformer (such as bushings, oil reservoir and complex base structure) are ignored in the FE model. In this model, the transformer windings were simplified as 3 cylindrical solids with identical dimensions, while the laminated transformer core was modelled as solid bars made of isotropic steel. Furthermore, the transformer tank was simplified as a plane shell without ribs. Equivalent thickness and Young’s modulus of transformer tank were used to take the reinforcement ribs into account. The equivalent values were determined by equating the 1st natural frequency of the ribbed tank wall with that of a uniform panel [17]. Such simplifications greatly reduced the degree-of-freedom of the transformer tank. At the interface between active parts and transformer tank, the continuity of the displacements is kept by nearest FEM nodes. The final structure model of the 110kV power transformer is shown in Fig. 2(b).

![Figure 2 (a) Schematic diagram and (b) finite element mesh of the 110kV power transformer.](image-url)
The void in Figure 2(b) forms an acoustic cavity where sound pressure in the cooling oil is generated by the vibration of the active parts. Table 1 lists the parameters of the transformer structural and cooling oil used for the following calculation and analysis.

<table>
<thead>
<tr>
<th></th>
<th>Tank</th>
<th>Oil</th>
<th>Active parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>4.99m</td>
<td>915 kg/m³</td>
<td>3.9m</td>
</tr>
<tr>
<td>Width</td>
<td>1.67m</td>
<td>1400 m/s²</td>
<td>1.32m</td>
</tr>
<tr>
<td>Height</td>
<td>2.88m</td>
<td>-</td>
<td>2.73m</td>
</tr>
<tr>
<td>Equivalent thickness</td>
<td>0.018 m</td>
<td>-</td>
<td>(0.0, -0.343 m, -0.085 m)</td>
</tr>
<tr>
<td>Equivalent Modulus</td>
<td>690 Gpa</td>
<td>-</td>
<td>210 Gpa</td>
</tr>
<tr>
<td>Density</td>
<td>7800 kg/m³</td>
<td>-</td>
<td>7800 kg/m³</td>
</tr>
</tbody>
</table>

3. Results and discussion

3.1 Acoustic modes inside the cooling-oil with rigid boundary conditions

Assuming the rigid boundary conditions on the transformer tank and active parts, the first four (non-zero natural frequency) modes of the sound inside the cooling oil were calculated and presented in Fig.3.

Even with the active parts inside the transformer tank, the nodal lines of the acoustical modes can still be observed and classified in terms of the number of nodal lines in each direction of the coordinates. Thus the first 4 acoustic modes can be identified as (0,1,0), (1,0,0), (0,2,0) and (0,1,1) modes. However, since the transformer active parts are positioned close to the left side of the transformer tank, the symmetry of its acoustic modes is affected. For example, the pressure distribution of (0,2,0) mode is no longer uniform in the x-z planes. Instead, the mode oscillates along the diagonal line. The isobaric surface, which holds the same sound pressure, is inclined to the left corner.
Moreover, an acoustic mode at 244.4Hz is also identified with mode shape close to (1, 0, 0) of the rectangular cavity.

![An acoustic mode (1,0,0) at 244.4Hz due to the occupation of active parts in the rectangular cavity.](image)

The natural frequencies of the acoustical modes (0,1,0), (1,0,0), (0,2,0) and (0,1,1) in the tank without the active parts can be readily estimated. They are compared with those with the active parts in Fig. 5, showing that the inclusion of the active parts decreases natural frequencies. Such phenomenon of decreased natural frequencies in rectangular enclosure when a diffuser is place inside have been observed and could be explained by the scattering effect of the active parts to the opposite traveling waves, which construct the modes in the tank [18-19].

![Comparison of the first four natural frequencies of the sound field in the tank with and without active parts.](image)

### 3.2 Acoustic modes inside the cooling-oil with elastic boundary conditions

The analysis of the dry transformer indicated that a large number of structure modes were found below 300Hz. When the transformer is filled with oil, those modes will couple with the rigid-wall acoustical modes and form new FSC modes with vibration energy distributed in the active parts, oil and tank structure. As a result, the acoustic modes with similar pressure distribution shown in Figs. 3 and 4, are affected by the transformer structure vibration.

For example, the sound field distribution of the FSC mode at 8.8Hz shows the similar distribution as the 1st acoustic mode of the rigid transformer structure (see Figure 3).
In Figure 7, a simple one-dimensional FSC model is utilized to explain the large change in the natural frequencies. In the model, the fluid in the rigid-wall duct with a cross section area $S$ and length $L$ has density $\rho_f$ and speed of sound $c_F$. A mass-spring oscillator at the right hand side of the duct represents the effect of elastic structure. The natural frequency of the oscillator is $\omega_s = \sqrt{\frac{K}{M_S}}$.

The natural frequencies of the FSC system can be obtained by solving the following characteristic equation:

$$\tan(\beta) = \frac{M_F}{M_S} \frac{\beta}{\beta^2 - \beta_s^2}$$

where $M_F = \rho LS$ is the total fluid mass in the duct and $\beta_s = \frac{\omega_s}{c_F} L$. The solution of Eq. (3) gives rise to the roots $\beta_n$ ($n = 0,1,2,3,...$), where the natural frequency of the system is $\omega_n = \frac{\beta_n}{L} c_F$. The $n^{th}$ mode shape includes the sound pressure in the duct and displacement of the oscillator mass displacement $\phi_n = \begin{bmatrix} \cos(\frac{\beta_n x}{L}) \\ \frac{S \cos(\beta_n)}{M_S (\omega_n^2 - \omega_s^2)} \end{bmatrix}$, also allowing the determination of the sound-to-structure energy ratio in the mode:

$$\gamma_n = \frac{E_{F,n}}{E_{S,n}} = \frac{\rho_f c_F}{\rho_S c_s} \left[ 1 + \frac{2\beta_n}{\omega_n} \sin(2\beta_n) \right]$$

$$\frac{S L}{\frac{2\omega_n^2 \cos^2(\beta_n) \omega_n^2}{M_S (\omega_n^2 - \omega_s^2)}}.$$
The change in natural frequencies of the FSC model is readily known if the boundary condition at \( x = L \) changes from rigid to pressure released. Specifically, they change from \( \omega_n = \frac{n\pi c_F}{L} \) to \( \omega_n = \left( \frac{1}{2} + n \right) \frac{\pi c_F}{L} \), corresponding to that from \( \beta_n = n\pi \) to \( \beta_n = \left( \frac{1}{2} + n \right)\pi \), \( n = 1, 2, 3, \ldots \). For the first mode \( (n = 0) \), \( \beta_0 \) changes from zero to \( \frac{\pi}{2} \). If the wavelength of the sound is larger than the length of the duct and the boundary is not pressure released, Eq. (3) can be simplified to give the non-zero natural frequency of the Helmholtz mode of the system:

\[
\omega_1 = \sqrt{\omega_s^2 + \frac{M_F c_F^2}{M_s L^2}}
\]  

(5)
demonstrating the combined effect of the boundary stiffness (via \( \omega_s^2 \)) and fluid modulus (via \( \frac{M_F c_F^2}{M_s L^2} \)) on the natural frequency. The significance effect of FSC on the natural frequency of Helmholtz mode is addressed by the large mass ratio \( \frac{M_F}{M_s} \) and large bulk modulus of the fluid \( (\rho_F c_F^2) \). If it were for air filled duct and the mass density of the boundary structure were more than that if water, the natural frequency of the coupled system would be dominated by \( \omega_s \). Depending the value of \( \beta_1 = \frac{\omega_1}{c_F} \), the mode shape in the fluid \( \cos\left(\frac{\beta_1 x}{L}\right) \) could vary significantly from unity especially in the area near the vibrating boundary structure.

The energy ratio in Eq. (5) indicates that the boundary structure only affects the system natural frequency when \( \beta \) is close to \( \beta_s \). For this case, the system energy may be dominated by the structural vibration. When \( \beta^2 - \beta_s^2 \) becomes large, the natural frequencies and mode shapes of the FSC modes can be approximately described by those for rigid boundary conditions.

4. Conclusion

As a follow-up work for the previous experimental study, this paper investigates the mechanism behind the effects of fluid-structure coupling in oil-filled power transformers. The strong coupling effects on the tank-controlled vibration modes and acoustic resonance are revealed by the FE method. This numerical study leads to the following conclusions:
The introduction of transformer active parts would decrease the acoustical resonance frequencies due to scattering effect of the active parts to the opposite traveling waves, which construct the modes in the tank.

For the transformer immersed in tones of cooling oil, the large mass ratio and large bulk modulus of the fluid will result in significant reduction of its acoustical resonance frequencies. In this case, the FSC modes might be dominated by the structural vibration, which emphasized the feasibility of structural optimization in its noise control.

Acknowledgement

This work is supported jointly by Natural Science Foundation of Zhejiang Provence Grant No. LQ18E050001, the Fundamental Research Funds for the Central Universities and National Natural Science Foundation of China Grant No. 11504324.

REFERENCES