Theoretical Analysis of the Far-Field Directional Active Noise Control

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The traditional active control of the quiet zone is to control the noise in a relatively constrained area within the sound field. In this paper, the concept of the quiet zone has been extended to the active control of the directional far-field of noise sources. The noise field generated by those noise sources like power transformer plant, which may have complex characteristics in the near-field, would approach spherical wave or plane wave in the far-field with so-called directivity pattern. A novel active control strategy, the far-field directional active noise control method, is proposed to cancel the noise in the far-field of the noise sources in some specific directions based on the wave front synthesis method. It is shown in theoretical analysis and numerical simulation that the proposed method can improve the control performance in enlarging the quiet zone while reducing the unwanted the noise produced by the control system outside the target area.

Keywords: Active noise control, Directional control, Quiet zone, Far-field

1. Introduction

Active noise control has been applied to create local quiet zones when the noise sources and the target area are well located and separated. Nelson and Elliott have introduced a model using single secondary monopole source to control radiation of the primary source [1,2]. It has been demonstrated that a global noise control can only be achieved if the distance between two sources are less than half-wavelength based on sound energy analysis [3]. After that this method is applied to investigate the configuration of the control system comprised of two linear arrays of control sources and error microphones [4]. By studying an optimal range of spacing of sources and microphones, it is revealed that large distance between error microphones and control sources will lead to large quiet zone. It is further extended to systems with control sources and microphones in planar configuration [5]. By introducing directional array methods [6], it is shown that quiet zone can also be enlarged. Note that all these work only consider the monopole noise source case, and may cause undesired sound increase elsewhere due to "waterbed effect".

Other researches mainly focus on diffuse field model in headsets and earphones. However, the problem that the quiet zone is small and only constrained to the error microphone often annoys people [7,8]. A series of theories concentrate on surrounding the entire target zone with control sources and error microphones. In this way, creating quiet zone by active control is converted to a boundary control problem which can be solved according to Huygens principle and Kirchhoff-Helmholtz integral equation. Notable methods, including "virtual sound barrier" [9,10] and "active
sound shield" [11][12], could achieve noise reduction over the whole area by controlling boundary sound pressure and velocity. Thus there is no need to consider the characteristic of the specific noise field. Existing studies only discussed the configuration and performance of the sound barrier under the assumption of plane wave incidence [13]. Methods of sound barrier and sound shield have achieved success in relatively small control area, but they become too complicated when the area grows large.

In real-life situations it is sometimes desired to keep residential buildings or rooms from the low-frequency narrow-band noise radiated from transformer substation, factory and construction site. Therefore this paper aims at achieving active noise control in the desired direction from a practical and complex noise source. The noise field of the complex source rather than monopole approximate will be fully investigated. Through theoretical analysis, the far-field directional active noise control method is proposed in this paper. Instead of surrounding the entire target area, the control systems are expected to work in the sound propagating path far from both noise sources and target area. Signals of noise field are derived from a linear microphone array and fed to the secondary sources to synthesize the wave front of the field in the desired direction. The performance of the proposed method is demonstrated by simulations and compared with the previous method of sound pressure minimization at the error microphones.

2. General Theory and System Description

In \( \mathbb{R}^3 \) open space, the noise source is assumed to be located at the origin of the spherical coordinate, which cannot be treated as a point source. For sake of simplicity, we define noise space \( V_1 \) as a spherical space of radius \( r_0 \) which incorporates the noise source. Instead of a simple monopole approximate, the complex noise source in this paper is modeled as a distribution function

\[ \rho(r', k), r' \in V_1 \] with \( k \) denoting the wave number. The radiating sound field from the noise source will influence the target area at some direction of distance \( R \) from the source. In the same way, the target space \( V_2 \) is assumed as a spherical space of radius \( r_t \). The distance \( R \) is large enough to satisfy the far-field condition, which indicates

\[ R \gg r_0, \quad R \gg \frac{r_0}{\lambda} \] (1)

Besides, \( r_t \) meets the requirements that \( R \gg r_t \) as well. The control system is placed along the sound propagation path, where the distance between the source and the system is \( r_s \). Suppose \( r_s \) is small compared to the distance in order to satisfy the far-field condition, and the control system keeps far away from the target area. Loudspeaker and microphone arrays are used to acquire the noise field information and generate control sound field. The illustration of the physical configuration is depicted in Fig. 1.

2.1 Sound Field Analysis

Based on the description above, the noise field radiated by the source can be denoted as integrals of Green function of point sources

\[ p(r, k) = \frac{1}{4\pi} \iiint_{V_1} \frac{e^{ik|r'-r|}}{|r'-r|} \rho(r') dr' \beta, \quad r \in \mathbb{R}^3 \setminus V_1 \] (2)

When \( r \) is large enough to satisfy the far-field condition, the first-order approximation that \( |r' - r| \approx r + r' \cos \alpha \) is take so that the sound field in the far field approximates to

\begin{align*}
p(r, k) & \approx \frac{e^{ik|r|}}{4\pi |r|} \iiint_{V_1} e^{ik|r'| \cos \alpha} \rho(r') dr' \beta, \quad r \in \mathbb{R}^3 \setminus V_1 \\
& = \frac{e^{ik|r|}}{4\pi |r|} D(\alpha) \quad (3)
\end{align*}
where $\alpha$ is the angle between $r$ and $r'$. $D(\alpha)$ denotes the far-field directivity of the source. It is then shown in Eq (3) that the radiation field will always approach spherical-wave form in the far-field.

From this point of view, the field of a practical source is similar to a monopole in the far field, with its directivity determined only by the source distribution. This makes it possible to acquire information about the sound field, as the field in the error sensors of the control system can be expressed in the same way

$$p(r_s, k) = \frac{e^{ik|r_s|}}{4\pi|r_s|} D(\alpha)$$

(4)

If the size of the control system is negligible compared to $r_s$, which is equivalent to paraxial condition, the Fraunhofer approximation form of the sound field should be adopted so that

$$p(r_s, k_x, k_y) = \frac{e^{ik|r_s|}}{4\pi|r_s|} P(k_x, k_y)$$

(5)

It’s demonstrated in Eq (5) that the spatial spectrum of the field along the propagating axis remains almost unchanged in the far field. $P(k_x, k_y)$ corresponds exactly to the spatial spectrum of a plane wave field, with $k_x$ and $k_y$ denoting the spatial frequency in the related direction. This lays the foundation for the control strategy in this paper. As long as the control system reconstructs the spatial spectrum in the form of $-P(k_x, k_y)$ near the axis, a quiet zone along the whole axis could be achieved. It is much larger than the quiet zone achieved by the normal control method, which is only constrained to the error sensor.

### 2.2 Proposed Control Strategy

The key point of the control strategy is to reconstruct a spatial spectrum of opposite phase which can exactly cancel out primary noise field along the axis. There are several methods to achieve this goal. As mentioned before, the sound field will approach spherical-wave or even plane-wave form in the far-field condition. In this paper, a method by synthesizing the wave front is proposed to reconstruct the spatial spectrum, which has already been used in sound reproduction system \([14]\). In
the following discussion we will show that this method has the advantage of maintaining little sound pressure increment outside the target area.

Consider a primary source $S$ in the $xz$-plane, as drawn in Fig. 2(a), the pressure of the field is given by

$$P(r, k) = S(k)D(\phi, \theta, k)\frac{e^{ikr}}{r}$$  \hspace{1cm} (6)

According to Rayleigh I integral, the sound field at the receiver position $R$ can be synthesized by monopole sources in the $xy$-plane:

$$P_{syn} = \frac{1}{2\pi} \iint i\omega \rho_0 V_n(r, k) \left[\frac{e^{ikr}}{r}\right] dxdy$$  \hspace{1cm} (7)

with $V_n$ the velocity component perpendicular to the $xy$-plane. By evaluating the equation, it follows that

$$i\omega \rho_0 V_n(r, k) = -\frac{\partial}{\partial n} P = -S(k)\frac{\partial}{\partial z} \left[D(\phi, \theta, k)\frac{e^{ikr}}{r}\right]$$  \hspace{1cm} (8)

Under the far-field assumption that $k(r + \Delta r) \gg 1$, the integral along $y$-axis in Eq (7) can be approximated by stationary phase method and yields

$$P_{syn} = S(k)\frac{ik}{2\pi} \int_{-\infty}^{\infty} \sqrt{\frac{\Delta r_0}{r_0 + \Delta r_0}} D\cos \phi \frac{e^{-ikr_0}}{\sqrt{r_0}} \frac{e^{-ik\Delta r_0}}{\Delta r_0} dx$$  \hspace{1cm} (9)

with stationary phase point at $\theta_0 = 0$. This points out that a line source is capable to reconstruct sound field in the $xz$-plane without using area sources. It can be further approximated using the same method and stationary phase point lies at $\cos \phi = (z_0 + \Delta z_0)/(\rho + \rho_0)$, as is shown in Fig. 2(b).

It’s then demonstrated that a line source of limited length is enough to reconstruct the field for the target area. Since $S(k)$ can be derived according to sound pressure information at the control system, substitution of Eq (6) into Eq (9) then gives the driving function for discrete linear secondary sources

$$Q(x) = P(x)\sqrt{\frac{1}{2\pi}} \sqrt{\frac{\Delta z_0}{z_0 + \Delta z_0}} \cos \phi$$  \hspace{1cm} (10)

Finally the geometry of control system is illustrated in Fig. 2(b). A linear loudspeaker array consisted of sufficient numbers is able control sound field in a planar target area. Its length equals to
the projection of the target area at the control system, which is illustrated by the red line. By now we prove that the proposed strategy is a practical solution to create quiet zones actively in the target area.

3. Numerical Simulations

3.1 Set Up in simulations

For the sake of simplicity, the simulations will be carried out in 2-dimensional situations where the control strategy is the same. The noise source is comprised of 30 monopoles of random magnitude, phase and of random locations inside $V_1$ with radius $r_0 = 10$ m. All monopoles radiate sound at 100 Hz, making primary sound a pure tone. The target area $V_2$ of radius $l_2 = 50$ m is 1000 m from the source and without loss of generality the direction is assumed to be $0^\circ$. The control system is located in the propagating path 500 m from primary source. According to the discussion in Section 2.2, the length of linear array used for noise control is obvious to be 25 m. It’s assumed here the information of the sound field has already been accurately acquired by the microphones at the control system. Different numbers of secondary loudspeakers are simulated as an important parameter to test the control performance. Finally it is compared to the classical control method of the same configuration.

3.2 Results

This section analyzes the quiet zones created and the noise reduction performance by the control system described above. The geometrical configuration of the target area and control system are fixed, with the number of loudspeakers changing from 16, 4 to 2. In this case, 16 correspond to the minimum number that satisfies Nyquist theorem, while 2 is the minimum number that ensure the target area would not be disturbed due to alias. The performance of noise reduction and quiet zone is shown in Fig. 3.

Fig. 3(a), Fig. 3(c) and Fig. 3(e) show the noise reduction achieved at the evaluation line while Fig. 3(b), Fig. 3(d) and Fig. 3(f) display the quiet zone. Despite different numbers of loudspeakers used in the system, the proposed method realizes about 10dB noise reduction at the evaluation line and succeed in creating large quiet zones not only in the target area but also along the whole axis. Therefore the proposed method is proven to realize directional control in open spaces. In sound reproduction system which requires enough number of loudspeakers to satisfy the Nyquist theorem, a much smaller number of secondary sources can realize reasonable reduction effect at least in the target area. This is a pleasant result since it shows its capability to realize sound reduction in an area of 50 meters diameter using only 2 speakers without other constraints. Nevertheless the number of speakers affects the control performance mainly in the side effect realized in the neighbouring of the target area. When sufficient numbers of loudspeakers are used, there is little sound pressure increase elsewhere.

Finally the proposed control strategy is compared to the previous control method by Guo and Pan [4]. They use a set of matrix equations related to the control sources and error microphones to optimize sound control in $H^2$ control. However, method of matrix inversion sometimes suffers from ill-conditioned matrix problem and always result in unpleasant noise increase outside target area. This can be only alleviated by introducing a regulation term in the matrix inversion process, with its performance displayed in the comparison of the methods in Fig. 4. In the situation described above, we also encounter the ill matrix problem due to the fact that the distance between the control system and the error sensors is much larger than the size of the control system. This leads to the fact that every element in the matrix is highly similar to each other. In order to thoroughly avoid unpleasant influence on other area, careful design and extra constraint points are required. It inevitably makes the system configuration and calculation much more complicated, as is shown in Liu’s method [6]. Though the classical method realizes a better amount of noise reduction in target area, the proposed control strategy is superior in creating large quiet area with no obvious negative effect in the neighbouring.
Figure 3: The performance of the proposed control method. The figures in the left column show the noise reduction along the evaluation line at 1000m, and the in the right the quiet zone distribution in the space.

This is achieved by utilizing the characteristic of the primary noise field and synthesizing the spherical wave front, which has not been discussed in details before. The control system can also be placed elsewhere along the propagating path, including locations near the source or even on the opposite side according to the requirement of practical implementation.

4. Conclusion

A general illustration has been presented for the noise field generated by complex sources. The noise field will approach spherical or plane wave in the far field. If the paraxial condition is satisfied, the field at a certain direction can be modeled by a spatial spectrum of a plane wave form. Rather than concentrating on actively controlling the noise field at the error microphones in previous researches, the far-field directional active control method is proposed in this paper which can be realized by reconstructing the spatial spectrum of the opposite phase. Techniques based on wave front synthesis are
Figure 4: Compare of performance of different control strategies. The original method produced by Guo etc. now suffers from ill matrix problem (black dashed line). This can be solved by adding a regulation term (black solid line), but produces more influence than the proposed method (red line).

used so that significant improvement can be made by achieving noise reduction over much larger area than the traditional quiet zone which is constrained at the error microphones. Through theoretical analysis the proposed method is shown to be advantageous in attenuating unwanted noise influence outside the target area if enough numbers of secondary sources are used to meet the Nyquist theorem. Besides, by evaluating the control performance, it is revealed that fewer secondary sources are capable of achieving directional control as well. Future works may explore the feasibility of far-field directional active control in other configurations and control methods.

REFERENCES


