This paper is concerned with the compressional and shear elastic wave motions in a soil medium due to the excitation of the propagating acoustic waves in underground pipelines. In our recent work, a simplified analytical model for predicting the ground surface displacements resulting from axisymmetric wave motion in buried fluid-filled pipes has been developed. The aim of the work in this paper is to investigate how the coupling nature between the propagating acoustic waves and elastic waves influences the soil vibration. To investigate this, general expressions for the soil displacements are derived for a fluid-filled pipe surrounded by an infinite elastic medium. The degree of agreement between theoretical and finite element results confirms the interfering effects of the elastic waves on the soil vibration. Furthermore, the numerical model facilitates better understanding of the radiation characteristics of elastic waves and allows for visualisation of the interference in underground pipelines.

Keywords: underground pipelines; elastic waves; soil vibration.

1. Introduction

Global population growth and urbanisation over the coming decades will have a dramatic impact on increasing demand for underground infrastructure. This underpins the necessity for the enhancement of trenchless technologies to combat the challenges associated with the sustainability and resilience of the increasing capacity in urban underground supply networks. Majority of currently available pipe location equipment is electromagnetics based, including ground penetrating radar (GPR) and electromagnetic induction methods [1-3]. They are not equally suited to all pipe materials due to the fundamental limitation of the techniques relying on the electromagnetic prop-
erties of both the target pipe material and surrounding soil. Alternatively, advances in the recent decades have focused on the development of acoustic methods, which have been identified as more applicable to locating plastic pipes [4-6]. It has been shown that the axisymmetric fluid-borne ($s=1$) wave is mainly responsible for the transfer of vibrational energy at low frequencies [7]. In addition, this wave will propagate along the pipe and radiate outward from the pipe into the surrounding soil if appropriately excited. Gao et al. [6] carried out some theoretical predictions of ground surface displacements with experimental validation, which provides a better understanding of the coupled fluid-pipe-soil system. In the development of the theory, the radiated elastic waves induced by the propagating $s=1$ wave are treated as plane waves in the far field as they reach the ground surface. This enables the adoption of the far field approximations of Hankel functions to calculate the ground surface displacements resulting from the reflection of incident plane waves at the free surface. However, there is still limited knowledge on elastic wave motions in the soil induced by the propagating $s=1$ wave.

This investigation starts with the soil vibration induced by the $s=1$ wave in the cylindrical coordinate system based on the recent work [6]. Here, the soil medium is seen to be of infinite extent, with no free ground surface being included in the analysis. A finite element (FE) model of the coupled fluid-pipe-soil system is further developed by utilising a commercial FE package (COMSOL-Multiphysics). Some numerical results of soil displacements are presented to validate the theory, and to demonstrate the radiation characteristics of elastic waves in the soil medium.

2. Theory

Below the pipe ring frequency, four wave types are the main carrier of the vibrational energy including three axisymmetric ($n=0$) waves and a bending ($n=1$) wave [7]. Of the $n=0$ waves, two wave types are quasi-longitudinal with some radial motion, termed the $s=1$, 2 waves (corresponding to a fluid-dominated and shell-dominated waves respectively); the $s=0$ wave is a torsional wave, which is uncoupled from the fluid. Here the $n$ and $s$ represent the circumferential modal order and a secondary branch order. For a buried fluid-filled pipe, the $s=1$ wave as the main propagation mode, has been exploited to reveal the location of underground pipeline along with water leakage due to its acoustical characteristics being nearly non-dispersive with less attenuation at low frequencies. Following a similar analysis by Gao et al. [7], this section briefly introduces the $s=1$ wave and derives the resultant soil displacements in the coupled fluid-pipe-soil system.

![Fig. 1: The coupled fluid-pipe-soil system.](image)

2.1 Axisymmetric $s=1$ wave

Free vibrations of a thin-wall fluid-filled pipe have been described by the Donnell-Mushtari shell equations. Figure 1 illustrates the coupled fluid-pipe-soil system under consideration. Referring to the figure, the pipe has a radius $a$ and wall thickness $h$, and is assumed to be thin such that $h/a<1$. The $s=1$ wave propagates along the pipe and radiates outward into the soil medium which sustains...
both the compressional and shear waves with the wavespeeds of \( c_r \) and \( c_d \) respectively. The surrounding soil is assumed to be homogenous and isotropic. For the surrounding soil with physical properties \( \rho_m, \lambda_m \) and \( \mu_m \) denoting the density and Lamé coefficients, \( c_r = \sqrt{(\lambda_m + 2\mu_m)/\rho_m} \) and \( c_d = \sqrt{\mu_m/\rho_m} \).

The governing equations for the axisymmetric \( s \) waves in the coupled system are obtained by [6]

\[
\begin{bmatrix}
\Omega^2 - (k_r a)^2 - SL_{11} & -iv_P (k_r a) - SL_{12} \\
-iv_P (k_r a) - SL_{21} & 1 - \Omega^2 - FL - SL_{22}
\end{bmatrix}
\begin{bmatrix}
U_s \\
W_f
\end{bmatrix} = 0
\]

where \( \Omega \) is the non-dimensional frequency, \( \Omega = k_r a \); \( k_r \) is the shell compressional wavenumber, \( k_r^2 = \omega^2 \rho_p (1-v_p^2)/E_p \); \( \rho_p, v_p, E_p \) are the density, Poisson’s ratio and Young’s modulus of the shell; \( FL \) represents the internal fluid loading effect, and is given by [6]

\[
FL = \rho_f a \Omega^2 J_0(k_r' a) / \rho_p h k_r' a J_0(k_r' a)
\]

where \( \rho_f \) is the density of the internal fluid; \( k_r' \) is the fluid radial wavenumber related to the internal fluid wavenumber, \( k_f \), by \( (k_r')^2 = k_f^2 - k_r^2 \); \( J_0(k_r' a) \) is a Bessel function of order zero, representing the internal fluid field; \( J_0 = (\partial/\partial r) J_0(\bullet) \); the matrix, \( SL \), represents the soil loading effect, and is given by

\[
\begin{align}
SL_{11} &= -\mu_m (1-v_p^2) a / E_p h k_r a \left[ a H_0(k_r' a)/H'_0(k_r' a) + k_r^2 a^2 [H_0(k_r' a)/H'_0(k_r' a)] \right] \\
SL_{12} &= i \mu_m (1-v_p^2) a / E_p h k_r a \left[ 2 - k_r^2 a^2 [H_0(k_r' a)/H'_0(k_r' a)] + k_r^2 a^2 [H_0(k_r' a)/H'_0(k_r' a)] \right] \\
SL_{21} &= SL_{12} \\
SL_{22} &= -\mu_m (1-v_p^2) a / E_p h \left[ 2 + k_r a k_r^2 a^2 [H_0(k_r' a)/H'_0(k_r' a)] + k_r^2 a^2 [H_0(k_r' a)/H'_0(k_r' a)] \right]
\end{align}
\]

where \( H_0(k_r' a) \) and \( H'_0(k_r' a) \) are the Hankel functions denoting the conical elastic waves radiating into the surrounding soil; \( k_r' \) and \( k_r'' \) are the radial compressional and shear wavenumbers, which are related to the compressional and shear wavenumbers, \( k_d \) and \( k_r \), by \( (k_r')^2 = k_d^2 - k_r^2 \) and \( (k_r')^2 = k_f^2 - k_r^2 \); and \( H'_0 = (\partial/\partial r) H_0(\bullet) \).

For the \( s=1 \) wave, the relationship between the amplitudes of the axial and radial displacements can be derived from Eq. (1) by

\[
U_i = \frac{1 - \Omega^2 - FL - SL_{22}}{i v_P (k_r a) + SL_{21}} W_i
\]

The approximate \( s=1 \) wavenumber can thus be obtained by setting the determinant of the characteristic matrix in the coupled equations of motion to zero, which leads to

\[
k_1^2 = k_f^2 \left[ 1 + \frac{2(B_f a / E_f h)(1-v_p^2)}{1 - \Omega^2 - SL_{22}} / \left( v_p - iSL_{12} / k_r a \right)^2 / (1 + SL_{11} / k_r^2 a^2) \right]
\]
2.2 Soil displacements

The equations of motion for soil displacements in the axial and radial directions are given by [6]

\[
\begin{align*}
\mathbf{u}_x &= -ik_1 H_0(k_{di}'r) (k_{ri}')^2 H_0(k_{ri}'r) \left( A_m e^{(or-k_1x)} \right) \\
\mathbf{u}_r &= -ik_{d1}' H_0'(k_{di}'r) \left( B_m \right)
\end{align*}
\]

where \( A_m \) and \( B_m \) are the potential coefficients of the compressional and shear waves respectively. From Eq. (6), the amplitudes of the axial and radial displacements can be obtained by

\[
\begin{align*}
U_x &= -ik_1 H_0(k_{di}'r) \\
U_r &= \left( k_{d1}' H_0'(k_{di}'r) A_m + (k_{ri}')^2 H_0(k_{ri}'r) \right) B_m
\end{align*}
\]

with the first and second terms indicating the contributions of the radiated compressional and shear waves respectively.

At the pipe-soil interface (\( r=a \)), the displacement continuity conditions must be satisfied. This leads to the potential coefficients in terms of the amplitudes of the axial and radial pipe wall displacements, \( U_1 \) and \( W_1 \), by

\[
\begin{align*}
A_m &= \frac{\left[ ik_1 H_0'(k_{ri}'a) - k_{ri}' k_{d1}' H_0(k_{ri}'a) H_0(k_{ri}'a) \right] U_1}{k_{ri}' k_{d1}' H_0(k_{ri}'a) H_0(k_{ri}'a) + k_{ri}'^2 H_0(k_{ri}'a) H_0(k_{ri}'a)} \\
B_m &= \frac{\left[ k_{ri}' H_0'(k_{ri}'a) k_{d1}' H_0'(k_{ri}'a) - H_0'(k_{ri}'a) H_0(k_{ri}'a) \right] U_1}{k_{ri}' k_{d1}' H_0(k_{ri}'a) H_0(k_{ri}'a) + k_{ri}'^2 H_0(k_{ri}'a) H_0(k_{ri}'a)}
\end{align*}
\]

In order to study the characteristics of soil displacements, non-dimensional amplification factors are introduced and defined as the soil displacements relative to the radial displacement of the pipe wall, \( \tilde{U}_x = U_x / W_1 \) and \( \tilde{U}_r = U_r / W_1 \). Substituting Eqs. (4), (8) into Eq. (7) gives

\[
\begin{align*}
\tilde{U}_x &= \left[ k_{ri}' H_0'(k_{ri}'a) H_0(k_{ri}'a) + k_{ri}' k_{d1}' H_0'(k_{ri}'a) H_0(k_{ri}'a) \right] U_1 \\
\tilde{U}_r &= \left[ ik_{d1}' H_0'(k_{ri}'a) H_0(k_{ri}'a) - H_0'(k_{ri}'a) H_0(k_{ri}'a) \right] U_1
\end{align*}
\]

3. Finite element (FE) model

Fig. 2: The 2-D axisymmetric FE model of the fluid-pipe-soil system.
In the preceding section, a simplified analytical model has been derived in order to obtain the soil displacements based on some assumptions. A FE model is developed in this section to validate the theory by utilising COMSOL-Multiphysics (v5.2). The 2-D axisymmetric schematic diagram of the water-pipe-soil system is shown in Fig. 2. In the model, the length of the pipe is 30 m with a burial depth of 3 m. The outer diameter and thickness of the pipe are 180 mm and 11 mm respectively. The material properties of the pipe, soil and water are listed in Table 1. With reference to Fig. 2, perfectly matched layers (PMLs) which can absorb incident waves are applied on the boundaries to simulate an infinite pipe buried in an infinite soil medium. The fluid domain marked in Fig. 2 is an acoustic field while the remaining is set as structural mechanics. The circumferential mode number is zero as default. A monopole source is placed at the centre of the pipe to simulate the acoustic pressure wave [8]. The model is meshed by free triangular elements, but PMLs are meshed by structured quadrilateral elements. To achieve good performance of the FE model, the element size must be less than 1/6 of the smallest wavelength (at 500Hz). Indeed, setting suitable mesh size according to different wavespeeds can improve the computational efficiency.

As shown in Fig. 2, two response points, A and B, are chosen in the soil and pipe wall respectively in order to calculate the axial and radial amplification factors of soil displacements, i.e. $\tilde{U}_x$ and $\tilde{U}_r$ defined by Eq. (9). The horizontal distance between response points and monopole source in this model is 2 m and the vertical distance between point A and symmetric axis is 1 m.

<table>
<thead>
<tr>
<th>Properties</th>
<th>MDPE</th>
<th>Water</th>
<th>Soil</th>
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<td>Density(kg/m$^3$)</td>
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<td>1500</td>
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<td>Young’s modulus (GN/m$^2$)</td>
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<td>-</td>
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<td>-</td>
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<tr>
<td>Loss factor</td>
<td>0.06</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

4. Results and discussions

The axisymmetric $s=1$ wave propagates along the pipe at low frequency and radiates outward from the pipe-soil interface to the far field. In this section, a buried MDPE pipe as described in Section 3, is considered to evaluate the coupling effects between the propagating acoustic waves and elastic waves. It can be seen clearly from Eq. (7) that the soil displacements are attributed to the combination of the radiated compressional and shear waves. Therefore, the elastic properties of the soil are important in influencing wave radiation induced by the $s=1$ wave.

![Fig. 3: Wavenumber and attenuation for a buried water-filled MDPE pipe: (a) real part of the wavenumber; (b) attenuation.](image-url)
In order to verify the accuracy of the model, the propagation characteristics of the \( s=1 \) wave are first investigated. Figs. 3(a) and (b) plot the real part of the wavenumber and the attenuation. Here, the attenuation is defined by the loss in dB per unit distance (measured in pipe radii) by

\[
\text{Attenuation (dB/\text{a})} = -20 \frac{\text{Im}(k,a)}{\text{ln}(10)}
\]  

(10)

As shown in the figures, for both the real part of the wavenumber and the attenuation, the deterioration of agreement between the theory and FM method (FMM) at very low frequencies below 100 Hz is as expected due to the reflections caused by the PMLs in the FE model. In addition, the FEM gives a slight overestimation relative to the theory in the predictions of both the real part of the wavenumber and the attenuation. The discrepancies increase with increasing frequencies. This is because the axisymmetric model in the FEM calculate the \( n=0 \) mode, whereas only the \( s=1 \) wave is considered in the theoretical model. In the calculation using the FEM, it has been assumed that by exciting the internal fluid, only the \( s=1 \) wave will be excited. This assumption, however, is most likely violated, as the fluid and structure are well coupled, since even pure pressure or pipe wall excitations will to some small extent excite the \( s=2 \) wave [9].

From Fig. 3(a), the calculated wavespeed of the \( s=1 \) wave is roughly 340 m/s in the frequency range concerned, which is greater than both the compressional and shear wavespeeds in the soil. Thus both elastic waves will radiate effectively into the surrounding soil, suggesting the occurrence of interference between the elastic waves. To further illustrate the interference phenomena, Figure 4 shows the colour plots of the magnitudes of soil displacements at 200 Hz. The propagation of the \( s=1 \) wave and the radiation pattern with wave interference in the soil are clearly evident in both the radial and axial directions: the \( s=1 \) wave decays as it propagates along the pipeline; the elastic waves radiate outward from the pipe-soil interface manifesting as some interference stripes in both directions. It is noted that away from the excitation source (at \( x=0 \)), the axial soil displacement reaches the minimum magnitude in contrast with the radial soil displacement with greater strength.

![Fig. 4: Single frequency images displaced with a linear colour scale at 200 Hz: (a) magnitude of the axial displacement; (b) magnitude of the radial displacement.](image)

Figs. 5 and 6 plot the axial and radial amplification factors respectively. In general, reasonable agreement between the theoretical and FEM results is demonstrated in these figures. Below 100 Hz, fluctuations are observed in both the magnitude and phase of the axial amplification factor using the FEM, because the PMLs applied at pipe ends do not completely absorb the propagating acoustic waves, resulting in reflections at low frequencies as mentioned above. Increasing the pipe length can reduce the fluctuations at the expense of computational time. At higher frequencies, the devia-
tions of the FEM relative to the theory are more pronounced possibly because of the influence of the existence of the axisymmetric $s=2$ wave.

![Graph](image)

**Fig. 5**: The axial amplification factor: (a) magnitude; (b) unwrapped phase.

![Graph](image)

**Fig. 6**: The radial amplification factor: (a) magnitude; (b) unwrapped phase.

For the axial amplification factors, as shown in Figs. 5(a) and (b), the radiated shear wave contributions are dominant compared to those of the radiated compressional wave for both the magnitude and phase at lower frequencies below 350 Hz. The contribution of the radiated compressional wave to the magnitude increases with frequency, and thus becomes comparable to that of the radiated shear wave at higher frequencies. As a result, strong interference between the elastic waves is evident at about 390 Hz: the magnitude reaches the minimum value (close to zero in Fig. 5(a)) and the phase has an abrupt change as shown in Fig. 5(b).
In contrast, for the radial amplification factors as plotted in Figs. 6(a) and (b), the interference between the compressional and shear waves is distinguishable in the whole frequency range of interest because both the radiated elastic waves contribute significantly. Similar to the axial amplification factor, a clear phase change at 105 Hz coincident with the magnitude minimum due to the interfering effects.

5. Conclusion

In this paper, elastic wave motions in buried pipes have been investigated. Based on the coupled equations of motion for a fluid-filled pipe surrounded by an infinite elastic medium, expressions for the soil displacements have been derived with the contributions of the compressional and shear waves given under the pipe-soil coupling conditions. A subsequent 2D axisymmetric model has been developed using the commercial FE package, enabling the validation of the theoretical model and facilitating the visualisation of the inference of elastic wave motions. For the soil considered in this paper (in the case when both elastic waves radiate), it has been demonstrated that, the soil axial displacement is mainly dominated by the radiated shear wave; the interference effects on the radial soil displacement are more obvious in the whole frequency range.

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REFERENCES