Blast vibration signal is non-linear and can easily interfered by noise. An improved wavelet threshold function is proposed for blast vibration signal denoising. Compared to traditional threshold functions, improved threshold function can overcome the discontinuity of hard threshold function. It can also evidently depress the error between estimated value and true value in soft threshold. The experimental results show that, the signal after improved threshold function denoising process have higher SNR and lower RMSE. Improved wavelet threshold function can get better denoising effect, and weaken Gibbs phenomenon near the singularities. Apply the improved threshold function into blast vibration signal denoising, the signal after denoising can primely keep the characteristic of original signal, which can also validate the superiority of improved threshold function compared to traditional ones.

Keywords: wavelet denoising; improved threshold function; blast vibration signal

1. Introduction

The blast vibration signal is a typical non-stationary signal. Its time-frequency analysis can show the characteristics of the signal, so as to provide references for the safety risk assessment of the explosion of large amounts of explosives in accidents. In the process of actual blast impact monitoring, it is easy to be influenced by external environment and testing system. Inevitably, the test signal contains noise, which affects the feature extraction of blast vibration signal. Therefore, how to remove the noise in the blast vibration signal and improve the reliability and accuracy of the test data is the basis of the analysis of the blast vibration signal.

In 1995, Donoho put forward a threshold denoising method based on wavelet analysis. This method can get the best estimation value in Besov space. Its computation is small and its implementation is simple. Therefore, it has been widely applied in the field of signal denoising\cite{1-2}. Soft threshold function and hard threshold function are the basic wavelet threshold functions. The basic principle is to set the non-important coefficients to zero, maintain the important coefficients in the hard threshold function, and shrink them in the soft threshold function. These two threshold functions are useful in signal denoising, but there are some shortcomings in continuity and approximation of the original signal\cite{3}. The hard threshold function can restore the local characteristics of the original signal very well, but due to its discontinuity, signal reconstruction will cause some oscillation and cause distortion. Although the soft threshold function is continuous, it can avoid the defect of hard threshold function, but because of the constant deviation between the estimated value and the true value, the reconstructed signal can not approach the real signal very well. In order to overcome the shortcomings of the classical threshold function, some new threshold functions have been proposed. Document \cite{4-9} put forward different improved threshold functions respectively. Simulation
results show that the threshold functions can improve the shortcomings of traditional soft and hard threshold functions to some extent.

In order to solve the problem of blast vibration signal denoising better, based on the existing research work, an improved wavelet threshold function is proposed and applied to the blasting vibration signal. Improved threshold function is continuous and high order derivative, it can effectively overcome the defects of soft and hard threshold function. In this paper, the performance of the improved threshold function is analyzed, and the performance of the simulated signal simulation is compared with the classical threshold function. Finally, the proposed threshold function is applied to the project to solve the problem of the actual explosion shock signal denoising. Simulation data and experimental studies show that the threshold function proposed in this paper is effective in denoising, and it can retain the characteristics of the original signal well, which is superior to the classical threshold function.

2. The denoising based on improved wavelet threshold function

2.1 Traditional wavelet threshold function

The wavelet threshold denoising first transforms the signal into the wavelet domain, and performs threshold processing in the wavelet domain to suppress the wavelet coefficients which contain random noise. Finally, the signal after denoising is obtained by the reconstruction of the wavelet coefficients. Threshold processing method includes the hard thresholding and soft thresholding methods, wavelet coefficients above the threshold in hard threshold method is maintained unchanged, the wavelet coefficients below the threshold in the sub space is set to be zero. The wavelet coefficients in soft thresholding method is shrunk to zero according to a fixed amount, the signal after denoising is attained through reconstruction of new wavelet coefficients. The hard threshold function expression is as follows:

$$\eta(x, \lambda) = \begin{cases} 0 & |x| < \lambda \\ x & |x| \geq \lambda \end{cases}$$

(1)

The soft threshold function expression is as follows:

$$\eta(x, \lambda) = \begin{cases} 0 & |x| < \lambda \\ \text{sign}(x)(|x| - \lambda) & |x| \geq \lambda \end{cases}$$

(2)

In the upper two expressions, $x$ is the wavelet coefficient, $\lambda$ is the threshold. In this paper, the threshold value is estimated using the VisuShrink threshold proposed by Donoho and Johostone. $\lambda = \sigma \times \sqrt{2 \log N}$, $\sigma$ is the standard deviation of the noise, $N$ is the signal length. The standard deviation of noise is $\sigma$, $\sigma = \text{Median}(|x|)/0.6745$, $\text{Median}(|x|)$ is the median of the wavelet multiresolution factorization coefficient$^{[10]}$. As two classical threshold functions, soft and hard thresholding functions have been widely used in many fields, but these two threshold functions still have defects. The hard threshold function is discontinuous at the threshold $\pm \lambda$, resulting in the oscillation when reconstructing the wavelet coefficients, and the pseudo Gibbs phenomenon is easily generated. Although the soft thresholding function avoids the discontinuity, it can be easily seen from the expression that a constant deviation is introduced when $|x| \geq \lambda$, which makes the accuracy of the method being affected when reconstructing the signal.

2.2 Traditional wavelet threshold function

In order to solve the defects of the soft and hard threshold function, an improved wavelet threshold function is proposed on the basis of the existing research. The expression is as follows:

$$\eta(x, \lambda) = \begin{cases} 0 & |x| < \lambda \\ \text{sign}(x)[|x| - \lambda] \exp(\frac{|x|^2 - \lambda^2}{C}) & |x| \geq \lambda \end{cases}$$

(3)
In the upper form $C$ is the normal number, and the performance of the threshold function can be changed by adjusting the value of $C$. In order to study the properties of the new threshold function, set $f(x) = \text{sign}(x)[|x| - \lambda]/\exp[\sqrt{\frac{|x|^2 - \lambda^2}{C}}]$. It is noted that $\lim_{C \to +\infty} f(x) = \text{sign}(x)[|x| - \lambda]$, that is to say when the value of $C$ is large, the new threshold function is equivalent to the traditional soft threshold function. Similarly, $\lim_{C \to 0} f(x) = \text{sign}(x) \cdot |x| = x$, when the value of $C$ is small, the new threshold function approaches the traditional hard threshold function. In the actual signal processing, the flexible change of the value of $C$ can achieve better denoising effect.

When $x > 0$, $f(x) = x - \lambda/\exp[\sqrt{\frac{|x|^2 - \lambda^2}{C}}]$, set $x \to +\infty$, then $f(x) \to x$. When $x < 0$, $f(x) = x + \lambda/\exp[\sqrt{\frac{|x|^2 - \lambda^2}{C}}]$, set $x \to -\infty$, then $f(x) \to x$. It can be seen from the above analysis that when $x$ tends to infinity, the threshold function $f(x)$ tends to the wavelet coefficients $x$, that is to say the $f(x) = x$ is the asymptote of the new threshold function. When the wavelet coefficient $x$ is small, the coefficients $\eta(x, \lambda)$ after denoising and wavelet coefficients vary greatly, but with the larger of $|x|$, and the value difference between $\eta(x, \lambda)$ and $x$ is more and more small. Finally when $|x| \to +\infty$, $\eta(x, \lambda)$ approaches $x$ with probability 1, which overcomes the constant deviation between the estimated wavelet coefficients of soft threshold function and original wavelet coefficient.

### 2.3 Comparison between improved and traditional threshold function

![Figure 1: The schematic diagram of three threshold functions.](image)

In order to compare the characteristics of the three threshold functions more intuitively, a schematic diagram of the three ones is given as Fig 1, in which the values of $C$ in the improved threshold functions are 0.5, 3, and 10, respectively.

It can be seen from the diagram that the hard threshold function is discontinuous when $x = \pm \lambda$, so there will be a certain oscillation when the signal is reconstructed. Although the soft threshold function is continuous, when $|x| > \lambda$, there is a constant deviation between the estimated wavelet coefficients $\eta(x, \lambda)$ and the original wavelet coefficients $x$, which will directly affect the reconstruction accuracy of the wavelet soft threshold function. The improved threshold function proposed in this paper not only solves the discontinuity at threshold value of hard threshold function, but also reduces the error value significantly compared with the constant error of soft threshold function, which also decreases with the increase of $|x|$. Therefore, the new threshold function is a better choice.

Change the value of $C$ in improved threshold function, it can be seen that the convergence rate and the convergence degree of the curve are different. The smaller the value of $C$, the faster the
convergence rate of the curve. With the increase of the value of $C$, the speed of convergence rate is reduced. Therefore, by changing the value of $C$, the performance of the function can be changed, and the improved threshold function is also more flexible.

3. Results and analysis of simulation experiment

In order to verify the performance of the improved threshold denoising method, a non-stationary signal model is established, and the signal model is superimposed by useful signals and Gauss white noise. The useful signal selects the rectangular signal with typical characteristics as the sample. The signal is denoised by wavelet threshold. The wavelet function selects the dB5 wavelet, and the decomposition level is set to 5 layers. The original rectangular signal and signal with noise are shown in Fig 2, in which the signal to noise ratio of the denoised signal is 15dB. The signal is denoised with the soft threshold function, hard threshold function and improved threshold function, the simulation results are shown in Fig 3 (a) (b) (c) respectively, the value of $C$ is 3.

We can see from Figure 3 that the denoising effect of the improved threshold denoising algorithm is obviously better than the traditional soft threshold denoising algorithm and hard threshold denoising algorithm. The denoised signal of improved threshold algorithm is more smoother and less burrs. It can also better preserve the useful signal peak and mutation. The simulation results show that the improved wavelet threshold denoising can better filtrate the useless signal components, the denoised signal waveform can highlight the original signal information.

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \hat{x}_i)^2}
\]  

Figure 2: Original signal and signal with noise.

Figure 3: denoised signal with different threshold functions.

In order to quantitatively evaluate the denoising performance of the improved threshold algorithm, the concept of Root-Mean-Square Error (RMSE) and Signal to Noise Ratio (SNR) are introduced,
\[
SNR = 10 \log \left( \frac{\sum_{i=1}^{N} \hat{x}_i^2}{\sum_{i=1}^{N} (x_i - \hat{x}_i)^2} \right)
\] (5)

In the upper two expressions, \(\{x_i\} (i = 1, 2, \ldots, N)\) is original signal, \(\{\hat{x}_i\} (i = 1, 2, \ldots, N)\) is signal with noise, and \(N\) is the number of sampling points. \(RMSE\) indicates the proximity of the denoising signal to the original signal, and \(SNR\) is the ratio of the signal with noise to the noise. The two parameters are all important indexes to measure the denoising algorithm. The larger of \(SNR\) and smaller of \(RMSE\), the better the performance of denoising algorithm.

The signal-to-noise ratio of the signal model is set as 0dB, 5dB, 10dB, 15dB and 20dB respectively. The denoising indexes of three threshold functions are calculated respectively, and the results are shown in Table 1. It can be seen that compared to the soft and hard threshold function denoising method, \(RMSE\) of the signal after the improved threshold function denoising is significantly reduced, and \(SNR\) improved accordingly. When the signal to noise ratio of signal model is 15dB, the \(SNR\) of improved threshold function is about 1.95dB higher that the soft threshold function, and is increased by about 3.11 dB compared with the hard threshold function.

<table>
<thead>
<tr>
<th>Methods</th>
<th>SNR/RMSE(5dB)</th>
<th>SNR/RMSE(10dB)</th>
<th>SNR/RMSE(15dB)</th>
<th>SNR/RMSE(20dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>hard threshold</td>
<td>12.8836/0.9504</td>
<td>18.0352/0.5252</td>
<td>21.0450/0.3714</td>
<td>23.0855/0.2936</td>
</tr>
<tr>
<td>soft threshold</td>
<td>14.3403/0.8036</td>
<td>18.6837/0.4874</td>
<td>22.2069/0.3249</td>
<td>23.8701/0.2683</td>
</tr>
<tr>
<td>improved threshold</td>
<td>15.5755/0.6971</td>
<td>20.3141/0.4040</td>
<td>24.1550/0.2642</td>
<td>25.6764/0.2179</td>
</tr>
</tbody>
</table>

4. Denoising analysis of measured explosion vibration signal

An explosive piece weighs about 450Kg, TNT equivalent is about 640Kg, due to internal defects, we decided the blasting destruction. Sensors were installed in the destruction process to measure the data. Blast vibration signal was studied in order to understand the characteristics of the signal. It can also provide reference for the security risks of explosion accidents.

Blast test diagram is as shown in Figure 4, There sensors were installed in a line, the distance is respectively 85m, 135m, 185m with explosive piece. There are stakes deep into the ground as fixed piles, sensors were stuck on the fixing piles. In engineering experiments, the vertical vibration data (Z direction) of blast waves are usually used as the basis for analysis\(^{[11]}\). Therefore, only experimental data in the direction of Z is concerned in this experiment. Experiments were carried out in a park, the testing process is normal, complete test data was acquired.

![Figure 4: The sketch map of sensor setting](image)

The test data consists of 9 paths, and one path in the direction of Z is selected randomly as an analysis object, the original signal is shown in Fig 5. It can be seen from the map that the background noise is very strong, and the explosion vibration signal is almost drowned in the noise.
Figure 5: Original blast vibration signal

Figure 6 shows the results of denoising using three threshold functions. The dB5 wavelet is selected and the number of decomposition layers is 5. Denoised signal is shown as Fig 6 (a), (b) and (c) respectively with the soft threshold function, the hard threshold function and the improved threshold function. From Fig 6, we can see that compared with the original noisy signal, the three kinds of threshold functions have achieved remarkable results, and the noise has been significantly reduced, the characteristics of the blast vibration signal have been restored.

The results show that the effect of the hard threshold function is obvious, but there are still many signal burr after denoising, and the effect of noise reduction is the worst. The signal burr is relatively less when the soft threshold function is used, and the characteristics of the shock signal are better restored. The improved threshold function is superior to the above two traditional thresholding functions in denoising, and the number of signal burrs is the least. It’s also the best way to preserve the detail features of the original signal. Therefore, the improved threshold function is a better denoising method than the traditional threshold functions.

5. Conclusion

An improved wavelet threshold function which is continuous and high order derivative is proposed in this paper. Compared with the traditional soft and hard threshold function, this function solves the shortcomings of the hard threshold function and the soft threshold function, which is not continuous at the threshold and has constant deviation separately. Through the simulation of the noisy rectangular signal, the improved threshold function proposed in this paper can better avoid the noise compared with the traditional soft and hard threshold function. It can effectively improve the signal-to-noise ratio and reduce the root mean square value of the signal after denoising. Finally, the improved wavelet threshold function is applied to the denoising of blast vibration signal. It can be
seen that the signal after denoising can well maintain the original signal characteristics, and the de-
oising effect is remarkable. It also verifies the superiority of the threshold function in denoising.

REFERENCES

6 T.Zhang, J.Zhang, Y.Zhang. Ring laser gyro drift signal denoising based on improved wavelet thresh-
10 S. Vafaei, H. Rahnejat. Indicated repeatable runout with wavelet decomposition ( IRR-WD ) for effec-