For an active control of acoustic reflections, it is required to separately obtain both the incident and reflected waves. Previously, it is shown that if the acoustic characteristics of the path between the sensor and the control device are known a priori, the incident signal and the reflected signal can be separated from the signal obtained from one sensor. In this paper, we propose an optimal design method for the single sensor-based reflection controller. An optimal filter is obtained by minimizing the mean square error between the estimated reflected waves and the control signals. The proposed method shows that the optimal filter coefficients can be calculated using only the measured impulse responses of the reflection and control paths. We implemented the proposed reflection control system using a programmable DSP and performed real-time experiments in a one-dimensional duct environment. Experimental results confirm that the proposed method achieves superior performance to the previous FxLMS approach.

Keywords: Active noise control, Active reflection control, Single sensor system

1. Introduction

Reducing the reflections from an object is required in various applications. Conventional methods include passive control using a coating material. However, it is generally effective for high-frequency signals, and the thickness of the coating material needs to be increased to cope with the reflection of low-frequency signals [1]. Active reflection control schemes using adaptive filters have been widely studied since it is effective for reducing signals in a broader frequency band without adding the volume of the control system compared to passive control method [2]. In particular, the adaptive control scheme provides many benefits when the acoustic characteristics around the control signal change with time.

For active reflection control, a sensor for collecting the reflection needs to be placed close to the transducer. In such an environment, the incoming acoustic signal, the reflection signal, and the control signal are mixed and picked up by the sensor. Therefore, it is necessary to separate each signal component from the sensor signal. To this end, a beamforming technique based on a number of sensors can be considered. However, this approach requires sufficient space to accommodate the sensors being placed nearby the control device [3], resulting in excessively large control system. To solve this problem, a single sensor-based method was previously developed in [4], in which the incident and reflected signals were separated from the single sensor signal and the filtered-x LMS (FxLMS) algorithm was used to update the control filter. But major disadvantage of the approach is that immediate
control cannot be obtained due to the slow convergence speed of the FxLMS algorithm. Furthermore, abnormal operation may occur when inappropriate parameters are selected or when environmental noise changes over time [5].

In this paper, we propose an optimal design method for the single sensor-based reflection controller, under the assumption that the acoustic characteristics of the paths between the sensor and the control device are time-invariant. This assumption is easily satisfied when the sensor is placed proximity to transducer, or both the sensor and the controller are encapsulated together in a single device [6]. In this situation, the optimum filter is designed by minimizing the mean square error between the reflected signal and the control signal. In the proposed method, it is shown that the optimal filter coefficients can be calculated using the measured impulse responses of the reflection and control paths. The proposed reflection control system is implemented using a programmable DSP, and real-time experiments were conducted with a one-dimensional duct. Detailed experimental results will be provided.

2. Single sensor based active reflection control system

![Diagram of the single sensor-based active reflection control system](image)

Figure 1: Configuration of the single sensor-based active reflection control system.

Fig. 1 shows the structure of the single sensor-based active reflection control system [4, 6]. A sensor is placed proximity to the reflection surface comprising a transducer. Then, the sensor collects the incident signal $u(n)$, the reflection signal $r(n)$ and the control signal $c(n)$ all together in a mixed form, as

$$s(n) = u(n) + r(n) + c(n) + v(n), \quad (1)$$

where $v(n)$ denotes background noise. When the impulse response of the reflection path is modelled as an $L^{th}$-order FIR filter, as given by \(h_i^n, i = 0,1,\ldots,L-1\), the reflection signal can be expressed as

$$r(n) = \sum_{i=0}^{L-1} h_i^n u(n-i). \quad (2)$$

When the background noise level is sufficiently low, i.e. $v(n) \approx 0$, the reflection signal $r(n)$ and the sensor signal $s(n)$ are related [4].

$$r(n) \approx \sum_{i=0}^{L-1} h_i^n s(n-i) - \sum_{i=0}^{L-1} h_i^n r(n-i). \quad (3)$$
The above equation is unrealizable because the second term on the right-hand side contains non-causal term $r(n)$. However, by assuming that $h_i^C = 0$, Eq. (3) can be modified to

$$r(n) \approx \sum_{i=0}^{k-1} h_i^I s(n - i) - \sum_{i=1}^{k-1} h_i^C r(n - i).$$

(4)

The assumption $h_i^C = 0$ is almost always satisfied because there is an initial delay in $h_i^I$ due to the displacement between the sensor and the transducer. The control signal $c(n)$ picked up by the sensor can be expressed using the impulse response of the control path $\{h_i^C, i = 0,1,...,M - 1\}$ as

$$c(n) = \sum_{i=0}^{M-1} h_i^C y(n - i),$$

(5)

where $y(n)$ is the output signal of the control filter with coefficients $\{w_i, 0 \leq i < N\}$:

$$y(n) = \sum_{i=0}^{N-1} w_i u(n - i).$$

(6)

Finally, the incident signal can be estimated from Eq. (1) as

$$u(n) \approx s(n) - r(n) - c(n).$$

(7)

3. Optimal control filter design

Fig. 2 shows a block diagram of the active reflection control algorithm based on a single sensor. In Fig. 2, $x(n)$ is the noise source, $H_p(z)$, $H_r(z)$ and $H_c(z)$ denote the transfer functions of the primary, reflection and control paths, respectively. $H_c(z)$ denotes the transfer function of the reflection
path without the first term, i.e. \( h_i^c, 1 \leq i < L \). The symbol \(^\wedge\) denotes estimates of the corresponding signals.

The coefficients of the control filter can be adjusted to minimize the error between the estimated reflection and the control signal by the transducer for reflection control, which is defined as

\[
e(n) = \hat{r}(n) - \hat{e}(n).
\]

The FxLMS algorithm is obtained by minimizing the mean square error \( |e(n)|^2 \):

\[
w(n + 1) = w(n) + \mu H_c \hat{u}(n) e(n),
\]

where \( H_c = \begin{bmatrix} h_0^c & h_1^c & \ldots & h_{M-1}^c & 0 & \ldots & 0 \\ 0 & h_1^c & \ldots & h_{M-2}^c & h_{M-1}^c & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & h_0^c & \ldots & \ldots & h_{M-1}^c \end{bmatrix}_{N \times (N+M-1)} \)

\[
\hat{u}_A(n) = [\hat{u}(n) \hat{u}(n-1) \ldots \hat{u}(n-N-M+2)]^T,
\]

where \( w(n) = [w_0(n), \ldots, w_{N-1}(n)]^T \) is the coefficient vector of the control filter, \( \hat{u}_A(n) \) is the \((N + M - 1) \times 1\) incident signal vector, and \( \mu \) is the step-size.

The FxLMS algorithm is effective when the control path response \( \{h_i^c\} \) changes with time. However, by placing the sensor in close proximity to the control transducer, we can assume that the control path response is almost time-invariant. In this situation, it is possible to use an optimal filter instead of an adaptive one to reduce the reflections. For optimal filter design, we assume that the estimated acoustic path response is accurate and that there is no sensor noise.

An optimal filter can be obtained by satisfying the following orthogonalization condition [7]:

\[
E[\hat{u}_c(n)e(n)] = 0,
\]

where \( \hat{u}_c(n) = H_c \hat{u}_A(n) \). Since the control signal is expressed as

\[
c(n) = h_T^c y(n) = (H_c \hat{u}_A(n))w = \hat{u}_c^T(n)w,
\]

\[
y(n) = [y(n), y(n-1), \ldots, y(n-M+1)]^T.
\]

Using Eqs. (8) and (11), Eq. (10) is re-expressed as

\[
E[\hat{u}_c(n)(r(n) - \hat{u}_c^T(n)w)] = 0.
\]

Finally, the optimal coefficient vector is obtained as.

\[
w_o = \Phi^{-1} \theta,
\]

\[
\Phi = E[\hat{u}_c(n)\hat{u}_c^T(n)],
\]

\[
\theta = E[\hat{u}_c(n)r(n)].
\]

In the above equation, \( \Phi \) and \( \theta \) are autocorrelation matrix and cross-correlation vector, respectively. From the relation of \( \hat{u}_c(n) = H_c \hat{u}_A(n) \), the autocorrelation matrix \( \Phi \) can be re-expressed as

\[
\Phi = H_c E[\hat{u}_A(n)\hat{u}_A^T(n)]H_c^T = H_c R_A H_c^T.
\]
where $\mathbf{R}_{AA} = E[\tilde{\mathbf{u}}_A(n)\tilde{\mathbf{u}}_A^T(n)]$ is the $(N + M - 1) \times (N + M - 1)$ autocorrelation matrix of the estimated incident signal $\tilde{u}(n)$.

Eq. (14) implies that to have the optimal filter, it is necessary to obtain the statistical characteristics of the incident signal, i.e., $\mathbf{R}_{AA}$. If the bandwidth of the input signal is known, we can theoretically calculate the autocorrelation matrix of the corresponding signal. But since the incident signal have any frequency, it is reasonable to assume that the incident signal is not band-limited, which enables us to further assume that the input is a white noise with zero mean and unit variance. Then, the autocorrelation matrix of the incident input becomes a unit matrix: $\mathbf{R}_{AA} = \mathbf{I}_{N+M-1}$. Under this assumption, the autocorrelation matrix $\mathbf{F}$ turns to

$$
\mathbf{F} = \mathbf{H_cH_c}^T.
$$

On the other hand, the reflection signal is expressed as follows using the impulse response $\{h^c_i\}$ of the reflection path.

$$
r(n) = h^c_r(n),
$$

$$
\mathbf{h}_r = [h^c_0, h^c_1, ..., h^c_{L-1}]^T, \quad \mathbf{u}_B(n) = [u(n), u(n - 1), ..., u(n - L + 1)]^T.
$$

Using Eq. (16), the cross-correlation vector $\mathbf{\Theta}$ can be re-expressed as

$$
\mathbf{\Theta} = \mathbf{H_cE}[\tilde{\mathbf{u}}_A(n)\tilde{\mathbf{u}}_B^T(n)]\mathbf{h}_r = \mathbf{H_cR_{AB}h}_r.
$$

Under the same assumption that the incident signal is not band-limited, the matrix $\mathbf{R}_{AB}$ is given as a matrix with a dimension of $(N + M - 1) \times (L - 1)$:

$$
\mathbf{R}_{AB} = \mathbf{T} = \begin{cases} 
\begin{bmatrix}
\mathbf{I}_{L-1} \\
\mathbf{0}_{N+M-L}
\end{bmatrix}, & \text{if } N + M \geq L \\
\mathbf{I}_{N+M-1} \begin{bmatrix}
\mathbf{0}_{L-N-M}
\end{bmatrix}, & \text{if } L > N + M
\end{cases},
$$

where $\mathbf{0}_{N+M-L}$ and $\mathbf{0}_{L-N-M}$ are zero matrices with dimensions of $(N + M - L) \times (L - 1)$ and $(N + M - 1) \times (L - N - M)$, respectively. Finally, the optimal filter coefficient $\mathbf{w}_o$ in Eq. (16) is re-written as

$$
\mathbf{w}_o = (\mathbf{H_cH_c}^T + \delta \mathbf{I}_N)^{-1}(\mathbf{H_cT} \mathbf{h}_r).
$$

where $\delta$ is a regularization constant to prevent numerical problem caused by ill-condition.

Eq. (19) shows that the optimal filter can be calculated only using the impulse responses of the reflection and control paths, i.e., $\{h^c_i\}$ and $\{h^r_i\}$, in the single sensor-based reflection control system shown in Fig. 2.

4. Simulation and results

The performance of the proposed algorithm was measured in a 1-dimensional duct shown in Fig 3.
The acoustic duct used in the experiments has a dimension of $1 \times 0.14 \times 0.14m^3$ and constructed using 5mm thick acrylic. TMS320C6747 DSP was used for the real-time implementation of the proposed algorithm. The AKG’s C417 pin microphone was used as a sensor together with an amplifier and a 5.5inch loudspeaker was used as a control transducer.

To design the optimal control filter, we first measured the impulse responses of both the control path and the reflection path. Impulse responses were measured using a swept sine signal. The impulse response of the control path was measured by applying a swept signal to the control transducer. To measure the impulse response of the reflection path, a swept sine was applied to the speaker corresponding to the noise source. However, the actual measure in this case is $r_p(\tau) + r_p(\tau)h_r(\tau)$, which is a mixture of the impulse responses of the incident path and reflection path. To separate $h_r(\tau)$ from the measured impulse response, the deconvolution technique in [4] was used.

Fig. 4 shows the estimated impulse responses of the reflection and control paths with $M = L = 64$, and the optimal filter coefficients calculated using the estimated impulse response is also shown.

The performance of the proposed optimal filter-based algorithm was evaluated using one period of 1kHz sine wave as noise source, the noise source was repeatedly emitted at an interval of 0.018 second to test robustness of the control filter. The order of the optimal control filter ($N$) was 64. For comparison, we also performed the reflection control using the FxLMS algorithm with $\mu = 0.16$. 
Figure 5: The sensor signals: (a) when the control filter was off, (b) when the FxLMS algorithm was used, and (c) when the proposed optimal control filter was used.

Fig. 5 shows the sensor signal in various conditions. It is shown that the adaptive filter-based algorithm has some difficulty in reducing the reflections in the initial period. On the other hand, the proposed optimal filter reduces the reflection signal stably from the very beginning.

To compare the steady-state performance, we also measured the echo reduction (ER) improvement, as given by

\[
ER_{\text{improve}} = ER_{\text{on}} - ER_{\text{off}},
\]

\[
ER_{\text{off}} = 20 \log_{10}(U(f)) - 20 \log_{10}(R(f)), \quad ER_{\text{on}} = 20 \log_{10}(U(f)) - 20 \log_{10}(R_{\text{on}}(f)),
\]

where \(U(f)\) and \(R(f)\) denote the strengths of the incident signal and the echo (reflection), respectively. \(ER_{\text{on}}\) and \(ER_{\text{off}}\) indicate the rate of reflection decay when the echo canceller is enabled. Fig. 6 shows the change of ER over time. The ER improvement was obtained averaging the results of 100 independent trials. The proposed optimal-filter method provides robust ER from the beginning and it outperforms the FxLMS-based method.
5. Conclusion

In this paper, we proposed a method of optimal filter design for a single sensor-based active reflection control system. The proposed optimal filter is obtained using the pre-measured impulse responses of the reflection and control paths. Unlike the existing adaptive filter-based algorithm, it does not require initial convergence time and is not influenced by system parameters. Through experiments, it was confirmed that the proposed optimal control filter achieves stable reflection control and outperforms the conventional FxLMS-based method.

REFERENCES


