A chiral-lattice based polyurethane viscoelastic metamaterial was proposed to serve as underwater coating, which will achieve broadband vibration suppression without sacrificing its hydrostatic pressure-bearing capacity. The combined effects of damping as well as embedded local resonators on the band gaps and vibration transmission were investigated. The underlying mechanism of such a metadamping was investigated. Theoretical modelling of damped Bloch waves by utilizing traditional finite element method software was suggested. Numerical simulations of band structure as well as frequency response functions were performed in finite element modelling. Phenomena such as branch overtaking, branch cut-on or cut-off were observed and a broadband gap is obtained in the high frequency due to the presence of material damping. The internal resonators are beneficial to increase vibration attenuation in both low and high frequency range. Chiral-lattice based polyurethane coatings were manufactured by moulding and distributed rubber-coated steel cylinder inclusions were vulcanized into the nodal circles. Experimental tests of the proposed samples were conducted to validate the theoretical findings.

Keywords: chiral-lattice, polyurethane coating, local resonators, Metadamping, band gap

1. Introduction

Underwater coating designed for ships to control underwater acoustic radiation or shock mitigation, requires possessing abilities of hydrostatic resistance and vibro-acoustic attenuation simultaneously. However, high stiffness and strong damping capabilities are inconsistent for material in nature [1]. Metamaterials, which exploits novel dynamic properties by microstructure design, have gained much attention to serve as a coating [2]. The design of coatings relies upon the topology of periodic lattice. Periodically arranged lattices are beneficial to improve vibration/shock absorption capabilities at high frequencies thanks to Bragg scattering mechanism. To design coatings that possess wide, low-frequency band gaps, local resonance (LR) mechanism is of particular interest. A particular chiral-lattice based arrangement is popular as matrix since it possesses a number of advantages. The chiral lattice exhibits a Poisson’s ratio of -1, which will lead to increase in bending stiffness and shear resistance. The circular nodes were observed to rotate and the ligaments were observed to bend or buckle, which suggests enhanced energy dissipation. The circular nodes are natural hosts for inclusions. Besides, dynamic properties of the chiral lattice can be conveniently tuned. The vibration and acoustic behaviour as well as wave propagation properties of chiral lattice have been studied extensively [3-6]. Spadoni et al. [3,4] investigated the in-plane wave propagation of hexagonal chiral lattice in detail. Liu et al.[5] investigated wave propagation in a 2D chiral meta-composites filled with elastic metamaterial inclusions. Baravelli and Ruzzene [6] proposed a chiral-
lattice periodic beam with LR inclusions, which is proved to be able to confine energy by pumping it from host structure and dissipating it through viscoelastic damping. However, the attention was focused on low frequency; the study on the influence of LR on high frequency range is rare.

Most work on chiral lattice were conducted on metallic structures with negligible damping. Incorporation of damping in metamaterials has been developed to gain better understanding of dissipative Bloch wave propagation. Hussein has made enormous contribution in this area. The lumped parameter model, which introduces dashpot viscous dampers[7-9], as well as continuous phononic crystal[10, 11] which modelled as a dissipative elastic continuum, have been investigated. The state-space method and the Rayleigh perturbation method have been developed. It was shown that damping can lead to phenomena such as branch overtaking, branch cut-on or cut-off. Hussein and Frazier defined “metadamping” indicating damping emergence phenomena due to presence of LR for the presence of LRs and one dissipative constitute material phase[12]. It is seen from literatures above that it is not applicable to directly utilize the commercial software to calculate damped Bloch waves just as the standard procedures to predict undamped dispersion relations.

2. Theoretical models

The polyurethane based chiral-lattice coating is shown in Fig.1. The chiral lattice consists of 4 ligaments connected to each circular node. For coating with LRs, the node carries rubber-ring coated steel cylinders as metamaterial inclusions, denoted as core. The radius of the core and the node is r and R, respectively. The length of the ligaments is denoted as L1 and L2 in the horizontal and vertical direction. The thickness of the rubber coating, the node and the ligament are denoted as t_r, t_n and t_l respectively. The Young’s modulus, Poisson’s ratio and density of the polyurethane, rubber and steel are denoted as E_p, μ_p, ρ_p; E_s, μ_s, ρ_s and E_s, μ_s, ρ_s, respectively.

![Figure 1: Schematic of a chiral-lattice polyurethane coating and the unit cell.](image)

The discretized equation of motion for the unit cell is obtained with standard FE procedures

\[ \mathbf{M}\dot{\mathbf{u}} + \mathbf{C}\mathbf{u} + \mathbf{K}\mathbf{u} = \mathbf{F} \quad (1) \]

where \( \mathbf{M} \), \( \mathbf{C} \) and \( \mathbf{K} \) denotes the mass matrix, damping matrix and stiffness matrix of the unit cell, respectively. \( \mathbf{u} \) and \( \mathbf{F} \) are the vectors of generalized nodal displacements and forces, \( \mathbf{u} = \{\mathbf{u}_i, \mathbf{u}_l, \mathbf{u}_r, \mathbf{u}_b, \mathbf{u}_s\}^T \), \( \mathbf{F} = \{\mathbf{F}_i, \mathbf{F}_l, \mathbf{F}_r, \mathbf{F}_b, \mathbf{F}_s\}^T \). \( (\cdot)^T \) denotes a transpose operation. The definition of variables is shown in Fig.1. By virtue of Bloch’s theorem, we have

\[ \mathbf{u}_i = \mathbf{u}_0 e^{ik_1}, \quad \mathbf{u}_l = \mathbf{u}_0 e^{ik_2}, \quad \mathbf{F}_i = -\mathbf{F}_0 e^{ik_1}, \quad \mathbf{F}_l = -\mathbf{F}_0 e^{ik_2} \quad (2) \]

where \( k = k_1 + ik_2 \) is the wave vector for the two-dimensional problem, i.e., the constraint conditions are in complex forms. Defining the reduced base of \( \mathbf{u} \) and \( \mathbf{F} \) yields the governing equation of motion with \( \mathbf{C} \) and \( \mathbf{K} \) as functions of \( k \)

\[ \mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}(k)\dot{\mathbf{u}} + \mathbf{K}(k)\mathbf{u} = \mathbf{F} \quad (3) \]

where \( \mathbf{u}' = \{\mathbf{u}_i, \mathbf{u}_l, \mathbf{u}_b\}^T \), \( \mathbf{F}' = \{\mathbf{F}_i, \mathbf{F}_l, \mathbf{F}_b\}^T \). The superscript ‘*’ in Eq.(3) is neglected for simplicity. Since the variable in the left hand of Eq.(3) contains complex items denoting damping, nodal displacement cannot be separated as real and imaginary parts of the fields and follow the procedures suggested in Ref.[15]. However, the problem described by Eq.(3) is a system with damping matrix and complex coefficient matrix. The following gives a procedure to solve this eigenvalue problem.
Firstly, by dropping damping matrix $C$, the eigenvalue problem is solved to find real eigenvalues and corresponding eigenvectors of Eq.(3) by following the procedure proposed in Ref.[15] for a given wave vector $k_0$, i.e., considering the problem with two identical meshes and the field equation including the real and imaginary parts yields

$$
\begin{pmatrix}
\omega_0^2 M + K(k_0) \\
\omega_0^2 M + K(k_0)
\end{pmatrix}
\begin{pmatrix}
u^{re} \\
u^{im}
\end{pmatrix} = 
\begin{pmatrix}
F^{re} \\
F^{im}
\end{pmatrix}
$$

where $\omega_0$ is eigenvalue corresponding to wave vector $k_0$.

Then, the modal coordinate transformation of system matrices is performed

$$u = \Phi v$$

where $\Phi$ is the eigenvector matrix formed using a set of Bloch vectors obtained by solving the standard undamped eigenvalue problem at the current point $k_0$ in the reciprocal lattice space and normalized with respect to the mass matrix $M$; $v$ are vector of modal coordinates. Utilizing the orthogonality condition that the Bloch vectors exhibit with respect to $M$ and $K$, this expansion is employed to uncouple equations in Eq.(3). The differential equations of motion in modal space are

$$Iv + \Phi^T C(k_0) \Phi v + \Lambda^2 (k_0)v = 0$$

where $I$ is the identity matrix; $\Lambda^2$ is a diagonal matrix containing the first $n$ eigen frequencies $\omega_i$. For classical damping, e.g., the proportional damping, the modal damping matrix $\Phi^T C(k_0) \Phi$ is a diagonal matrix with the diagonal terms being $2\xi_i \omega_i$, where $\xi_i$ is the damping ratio of the $i$-th mode. For general damping, the modal damping matrix is either symmetric or unsymmetric.

After that, introducing the 2n-dimensional state variable vector approach, Eq.(6) is written as

$$\{\dot{z}\} = \begin{bmatrix} 0 & 1 \\ -\Lambda^2 (k_0) & -\Phi^T C(k_0) \Phi \end{bmatrix} \{z\}, \{\dot{z}\} = \begin{bmatrix} 0 & 1 \\ -\Lambda^2 (k_0) & 0 \end{bmatrix} + i \begin{bmatrix} 0 & 0 \\ -\Phi^T K_{mag}(k_0) \Phi & 0 \end{bmatrix} \{z\}, \{z\} = \begin{bmatrix} v \\ \dot{v} \end{bmatrix}$$

where $K_{mag}(k_0) = g K(k_0) + \sum_{j=1}^{N_m} 2m_j K_j + \sum_{k=1}^{N_e} K_k$ when a constant structural damping ratio is defined or an element generating a complex stiffness is present in a QR damped modal analysis with the complex mode shapes. $g$ is the constant structural damping coefficient; $m_j$ is constant structural damping coefficient for material $j$ and $K_j$ is portion of structural stiffness matrix based on material $j$; $N_m$ is the number of materials with constant structural damping coefficient; $K_k$ is the imaginary element stiffness matrix and $N_e$ is number of elements with specified imaginary stiffness matrix.

Although standard FE software does not have features to deal with damped system with complex boundary conditions, the procedure developed above can be implemented by the QR damped method for any damped system. The 2n eigenvalues of Eq.(7) are calculated using QR algorithm. The inverse iteration method is employed to calculate the complex modal subspace eigenvectors. Complex eigenvectors $\psi$ of the original system is recovered by using $\psi = \Phi z$. When the complex eigenvalues are complex conjugate pairs, only the positive imaginary solution (positive frequency) is retained. In the case of high damping, all overdamped modes are also retained. For QR damped method, stiffness and mass matrix damping multiplier for material can be considered. Since rubber and polyurethane is viscoelastic material, stiffness matrix damping multiplier is adopted to define damping in material and denoted as $C = \eta K$. The damping frequencies can be written as

$$\lambda_i = -\xi_i \omega_i + i \omega_i \sqrt{-\xi_i^2},$$

where $\omega_i = \sqrt{\text{real}(\lambda_i)^2 + \text{imag}(\lambda_i)^2}$, $\xi_i = -\text{real}(\lambda_i) / \omega_i$.

3. Numerical simulations

3.1 Band diagrams

Numerical model and FE mesh of a unit cell is shown in Fig.2. A two-dimensional model under plane stress condition is considered. The structure with/without LR is studied. Material properties are as follows, $E_p = 6.7e8 N/m^2$, $\mu_0 = 0.48$, $\rho_0 = 1200 kg/m^3$; $E_r = 2e6$, $\mu_r = 0.48$, $\rho_r = 1200 kg/m^3$ and $E_s = 2.1e11$, $\mu_s = 0.3$, $\rho_s = 7850 kg/m^3$. Damping coefficient for polyurethane, rubber and steel is 1e-5, 1e-5 and 0, respectively. Geometrical parameters for the unit cell are as follows, $R = 12 mm$, $r = 9 mm$,
$L_1=50\text{mm}$, $L_2=52\text{mm}$; $t_f=3\text{mm}$, $t_r=2\text{mm}$, $t_n=3\text{mm}$. The wavevector path along the symmetry points $O \rightarrow A \rightarrow B \rightarrow O$, bordering the irreducible Brillouin zone (IBZ) is sampled into $91$ $k$-point steps (definition of IBZ are shown in Fig.1). Only the real wavevector is considered. The resulting dispersion curves are shown in Fig. 3. The damping ratio band diagram is also shown.

![Image](image1)

**Figure 2:** Numerical model of unit cell and finite periodic structure without and with LR.

For both the phononic crystal with/without LR, it can be seen that the damping ratio diagram resembles the frequency band structure in shape, with the locations of the curves rising with an increase of $\eta$ (the curves with damping larger than $0.02$ are not shown for clear illustration). The band structure experience increasing downward shifts with increase of $\eta$. The curves do the rising or dropping at an increasing rate as the branch number increases. We can also observe the cut-on phenomenon in some branches for the damped periodic structure with/without LR (for clarity, only a representative branch is shown and the others are omitted). On the band structure diagram, the branch cut-on means a wave that can propagate along this direction (partial bandgap). It is found that when the prescribed damping levels are relatively high, it will cause many of the dispersion branches to reach the critical damping values $\xi_i=1$ as shown in ANSYS. Under this circumstance, the complex eigenvalues only have real part, which means the wave is a decaying wave that cannot be propagated along the phononic crystal structure. As a result, the band structure will show a band gap above this critical frequency. The features of curve veering exhibited by weakly coupled branches of the dispersion curves are valid for undamped and lightly damped phononic crystal. Another noteworthy observation in Fig. 3 is that the damping ratio of the first branch substantially increases. This branch represents the propagation of shear waves. The increase of its dissipation values indicates the level of shear dissipation is raised, accordingly.

![Image](image2)

**Figure 3:** Band diagram for unit cell with/without IR.

('—': $\eta=0$ (undamped); '----': $\eta=0.0001$; '—.—': $\eta=0.0005$; '......': $\eta=0.001$)

Some of the Bloch mode shapes are shown in Fig.4. The starting of the first band gap (the region between the two dashed lines) is related to the deformation shown in Fig.4(a) and Fig.4(b) for the phononic crystal without/ with LR, respectively. The latter corresponds to the transverse movement of the LR in the matrix. The features of the unique propagating wave located in the first band gap

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for wavevector at A are clearly observed in Fig.4(a) and Fig.4(b) for the phononic crystal without/with LR, respectively. The former is the deformation of the circle; the latter corresponds to the rotation of the LR in the matrix, which means the energy is located and limited in the circles or IRs. The mode shape at the starting of the second band gap (the fifth branch) is shown in Fig.4(c), corresponding to oval mode of the circle and the rubber coating.

![Mode Shapes](image)

**Figure 4: Typical mode shapes for unit cell without damping.**

### 3.2 FRF results

FRFs of finite periodic structure are studied. As shown in Fig.2, the finite periodic structure contains 7 unit cells in length direction and 3 unit cells in height direction. The overall dimensions are 36cm(length)×10cm(width)×16cm(height). The numerical model is the model of the samples studied in Section 4. The exciting and response points are defined in Fig.2.

FRFs of the finite periodic structure without LR without/with damping are shown in Fig.5. It is seen from Fig.3(a) that the first band gap for an undamped periodic chiral structure is 1306Hz~1986Hz, as shown by the region surrounded by the two thick dashed lines. Since the damping ratio for each wave is 0, the waves will propagate along the structure. There is a partial bandgap between 393Hz~1101Hz. The band gap can be seen from the attenuation of response between point 2 and point 5. If $\eta=0.00001$, the band gap diagram does not exhibit drama change. However, the damping ratio for each wave is proportional to the branch number. These waves are evanescent waves, which will decay as they propagate along the structure. The attenuation rate is more obvious for modes with higher frequency. It can be seen that the attenuation is remarkable for frequency above 431Hz.

The FRFs for the finite periodic structure with/without LR are compared in Fig.5(b), a big deep is observed from FRFs of structure with LR around 659Hz~917Hz, which is located around the LR at 693.6Hz (Fig.4(b)). Then the transfer FRF of structure with LR is inferior to that without LR, which can be explained from the band diagram that this frequency range is the band gap for structure without LR and evanescent waves still exist for that with LR. Above 2775Hz, structure with LR regains its superiority again. This can be explained by the fact that the presence of LR condensed the modes, resulting in more branches in a limited frequency range and the branch number is larger for the periodic structure with LR than that without LR to possess a larger damping ratio and faster decaying. This conclusion will be generalized for structures with LR.

FRFs of the finite periodic structure with LR considering damping are shown in Fig.6. It is seen from Fig.3(b) that the first and the second band gap are 629Hz~1052Hz and 1914Hz~2044Hz. It can be seen that the attenuation is remarkable for frequency above 373Hz. FRFs of the finite structure with LR at other points are also given to illustrate the response distribution, as shown in Fig.6 (a). Response at point 4 is further attenuated since there are unit cells along both the length direction and height direction of the vibration transfer path. Meanwhile, attenuation for response at point 1 shows a little bit small. FRFs in the height direction are given for the points on the upper and lower surface and the centers of LRs, as shown in Fig.6 (b). It is seen that FRFs at the centers of LRs are monotonously decreased, i.e., response at LR near the exciting side (d1 or d2) shows the most moderate attenuation above a certain frequency, the attenuation is strengthened gradually as it propagates along the structure (u2>m2>d2). The response for the point on the lower surface (5 or 4) does not follow this regularity except in the LR band gap range, which is illustrated as deep sag. By this observation, it is verified that vibration attenuation results from Bragg scattering mechanism is gained by continuous reflection on the interface of the periodic structure and the attenuation is...
influenced by the boundaries; while the vibration attenuation for LR mechanism is realized by localizing the response in LRs and the attenuation is not influenced by the boundaries.

![Graphs](image)

**Figure 5:** FRF of structure without LR when excited at point 2.

(a) without damping  
(b) with damping ($\eta=0.00001$)

![Graphs](image)

**Figure 6:** FRF for structure with LR when excited at point 2.

(a) FRF between Point 2 and upper/lower face  
(b) FRF between point 2 and center of LR at middle column

4. Experimental results and discussion

![Photos](image)

**Figure 7:** Photos for test samples: A(HRC 65, 1.455kg), B(HRC 60, 1.454kg), C(HRC 60, 5.730kg with LR)

![Diagram](image)

**Figure 8:** Test set-up for three samples.
A finite chiral-lattice periodic structure made of polyurethane was manufactured by moulding. Material with two different hardnesses is chosen. Distributed rubber-coated steel cylinder inclusions were vulcanized into the core. Photos and parameters are shown in Fig.7. Experimental set-up is shown in Fig.8. The samples and exciter are hanged by the flexible rope. Two exciting points are located at point 1 and 2. Response points are chosen as points 1, 2, …, 6. White noise signals were employed. FRFs are employed to evaluate the vibration transmission.

FRFs for the finite structure with/without LR are shown in Fig.9. The responses for two exciting points are presented. For exciting at point 1, it is seen that the driving-point FRF is increased for the periodic structure with LR, the transfer FRF is also increased. It is found that vibration transmission attenuation is decreased for the periodic structure with LR in the frequency range 1100Hz~3600Hz, compared with that without LR. The periodic structure with LR exhibits superiority both in the low frequency range 185Hz~1220Hz and high frequency range above 3600Hz. The above conclusions are also valid for point 5. Point 2 shows similar discipline but with a different frequency range. The observation is more obvious for the response at point 6 when exciting at point 2. Periodic structure with LR performs better both in the low frequency range 215Hz~1085Hz and in the high frequency range above 2420Hz. These observations correlate well with those found in the theoretical study.

FRFs for finite periodic structure without LR made of material with different hardnesses for exciting at point 1 and 2 are shown in Fig.10. As hardness increases, the elastic modulus increases. As a result, the stiffness of the matrix gets larger. It is shown that the responses for both the driving-point and transfer FRFs are larger for a harder matrix than those for a soft matrix in general. Only in very limited frequency range, the harder one exhibits deep sag. This is in accordance with the well-
known theory that states ‘the softer, the better attenuation’ for vibration isolation. On the other hand, for periodic structure with damp, the softer matrix contains a larger number of waves in the same frequency range. According to the fact that the damping ratio for each branch is proportional to the branch number, the damping ratio is larger for the wave at the same frequency. In addition, the deep sag could be attributed to a band gap in this frequency range.

5. Conclusions

Theoretical derivation was conducted to demonstrate the possibility to calculate damped Bloch waves by employing QR damped modal analysis method in ANSYS. The calculating procedures are presented, which is able to consider stiffness and mass matrix proportional damping as well as frequency independent damping. By employing the proposed method, wave propagation of the polyurethane-based chiral structure with/without LR is numerically studied. The free wave motion properties are reported in the form of band diagrams, damping ratio band diagram. The band gap properties are correlated with FRFs of the finite periodic structure. Numerical simulation is conducted to study the effects of damping, LR on the wave propagation characteristics. To verify theoretical findings, an experimental is carried out to measure vibration transmission of a finite chiral lattice periodic structure made of polyurethane. It is shown that dissipation can have a major influence on dispersion phenomena, which will generate a broad band gap in the high frequency. The LR is beneficial to increase vibration attenuation in both the low and high frequency range.

REFERENCES