RELATIONSHIP BETWEEN SOUND RADIATION FROM
SOUND-INDUCED AND FORCE-EXCITED VIBRATION

Motoki Yairi
Kajima Technical Research Institute, Chofu, Japan
email: yairi@kajima.com

Kimihiro Sakagami and Takeshi Okuzono
Graduate School of Engineering, Kobe University, Kobe, Japan

Sound radiation from solid structures is caused by sound-induced vibration as well as force-excited vibration. The former is generally called airborne sound transmission. The insulation performance of airborne sound transmission is normally evaluated under the assumption that the source of the excitation force is random-incidence sound. On the other hand, the reduction of sound radiation from force-excited vibration is also a fundamental issue in architectural acoustics. Building facilities and floor impact sounds contribute to sound radiation from the surfaces in actual buildings. These two problems have been treated as different issues in architectural acoustics only because of different types of sources. In this paper, a fundamental relationship is elucidated through the use of a simple model, and a conversion function that relates the transmission coefficient for random incident sound and radiated sound power under point-force excitation is obtained in a simple closed form. This is composed of the specific impedance and the wavenumber, and is independent of any elastic plate parameters.

Keywords: random-incidence sound, point-force excitation, conversion function

1. Introduction

1.1 Background

Sound radiation from vibrating solid surfaces is generally caused by either airborne sound or force-excited vibration. The reduction of these two types of sound radiation is a fundamental issue in noise-control engineering and a basic requirement for many buildings, cars, ships and airplanes to realize a comfortable acoustic environment [1]. In architectural acoustics, sound radiation due to airborne sound-induced vibration of walls, floors and other solid boundaries between rooms is normally called airborne sound transmission. The study of airborne sound transmission has been separated from sound radiation occurring due to force-excited vibration of solid structures [2].

The insulation performance of airborne sound transmission is typically evaluated under the assumption of random-incidence sound since various angles of incidence are likely to occur in actual situations. Considering the recent increase in collective housing units in urban areas, the demand for higher-performance airborne sound insulation is growing. Therefore, various researchers are carrying out studies based on the considerable knowledge published. For example, Brunskog [3] and Davy [4] have presented theories for predicting the sound insulation walls.

Meanwhile, the reduction of sound radiation from force-excited vibration is also a fundamental issue in architectural acoustics. Several sources of excitation force contribute to sound radiation from the surfaces in buildings, such as building facilities, railway and road traffic vibrations. In
order to reduce the forced-excited sound radiation, practical methods have theoretically been studied [5]. Floor impact sound problems, such as footfall, are another typical source of force-excited sound radiation. Practical models have been developed to predict the impact sound insulation [6]. In most analyses of sound radiation from force-excited vibration, the radiation characteristics of the structures are generally investigated assuming a point and/or distributed excitation force as the external force(s) in underwater acoustics3 as well as architectural acoustics.

As described above, airborne sound transmission through solid boundaries and sound radiation from force-excited vibrating surfaces has been treated as two separate issues because of the different sources of external force. There were only a few studies on the particular relationship between floor impact sound level and the floor’s sound transmission loss [7].

1.2 Hypothesis

Figure 1 provides a schematic explanation of the evaluation systems for airborne sound transmission and sound radiation from force-excited vibration of solid boundaries, and a hypothesis on the relationship between the two problems. $M_a$ and $M_s$ indicate general evaluation indexes for each excitation problem. For example, the sound reduction index for random-incidence and spherical incidence [8] is applicable to $M_a$, and the radiation coefficient and radiation impedance are widely used for $M_s$. In the model, $M_a$ and $M_s$ are linked using a linear operator $\varepsilon$ as follows:

$$
\begin{align*}
  M_a &= \varepsilon [M_s] \\
  M_s &= \varepsilon^{-1} [M_a]
\end{align*}
$$

To be able to estimate the airborne sound transmission from the sound radiation characteristics, or vice versa, $\varepsilon$ should exist in a form independent of the boundaries. The chief purpose of the present work is to determine the possible existence of such a linear operator.

In this paper, the sound radiation from an infinite elastic plate driven by random-incidence sound and point force excitation is theoretically investigated. The transmission coefficient for random-incidence sound and the radiated sound power under point force excitation are introduced for $M_a$ and $M_s$, respectively, and these exact solutions are both derived. Approximate solutions of $M_a$ and $M_s$ are both analysed within the low-frequency limit under the critical frequency without acoustic loading. A conversion function that relates the two problems is derived through the approximate solutions, and its accuracy and dependence on the elastic plate are discussed. Physical meanings of the conversion function are also given. Numerical calculations are used to verify if the conversion function is also applicable to the relationship between the exact solutions.

2. Transmission coefficient for random-incidence sound

2.1 Exact solution

Consider an infinite elastic plate lying in the plane $z = 0$ (Fig. 2), which vibrates under a plane wave incident at the angle $\theta$. Suppose that the vibration of the plate is in accordance with the classic thin plate theory and the acoustic admittance of both sides of the plate is zero, i.e., no absorption. The transmitted sound field is derived by simultaneously solving the governing equations of the sound field and the equation of motion of the plate. In this case, the transmission coefficient for oblique-incidence sound $\tau(\theta, \omega)$ is defined by the ratio of an incidence sound at the angle $\theta$ and the transmitted sound at the same angle. This problem has been solved previously, and the exact solution of $\tau(\theta, \omega)$ is written as follows:

$$
\tau(\theta, \omega) = \left[ \left( 1 + \frac{\cos \theta}{2 \rho_0 c_0} \frac{\omega^3 \text{Re}[D] \sin^4 \theta}{c_0^4} \right)^2 + \left( \frac{\omega \rho_0 h \cos \theta}{2 \rho_0 c_0} \right)^2 \left( 1 - \frac{\omega^2 \text{Re}[D] \sin^4 \theta}{c_0^2 \rho_0 h} \right)^2 \right]^{-1},
$$

(2)
where $\omega$ is the angular frequency, $c_0$ is the speed of sound in air, $\rho_0$ is the air density and $D = E(1-i\eta)h^3/12(1-\nu^2)$ is the flexural rigidity of the plate with $E$ being Young’s modulus, $h$ thickness, $\eta$ loss factor, $\nu$ Poisson’s ratio and $\rho_0$ density of the plate. The angle of incidence affects the bending vibration of the plate, which is well known as the coincidence effect. $\omega_c = (c_0^4 m/Re[D])^{1/2}$ is the critical frequency. When an elastic plate is accompanied by bending vibrations due to oblique plane-wave incidence, the transmission coefficient becomes larger than that of normal plane-wave incidence above the critical frequency.

The transmission coefficient for random-incidence sound is defined as the ratio of the transmitted sound power to the incident sound power from any direction. Random incidence is obtained by averaging the transmission coefficients for oblique incidence over a hemisphere. This approach, which is based on a completely diffuse sound field, does not fully reflect the actual conditions in rooms, and so various methods of truncating the angle of incidence up to a certain limit angle have been proposed. In general, this is calculated by integrating a range of $\Theta = 0^\circ - 78^\circ$ because of the good agreement with experimental results. The transmission coefficient for random-incidence sound $\tau(\omega)$ is therefore derived from the following integration:

$$
\tau(\omega) = \int_{\Theta=0}^{78} \tau(\omega) \cos \Theta \sin \Theta d\Theta \int_{\Theta=0}^{78} \cos \Theta \sin \Theta d\Theta.
$$

(3)

The integral in Eq. (3) is evaluated analytically within the low-frequency limit when $Re[D]$ is negligible:

$$
\tau(\Theta, \omega) \approx \left[ \frac{2\rho_0 c_0}{m\omega \cos \Theta} \right]^2 \left[ 1 + \left( \frac{m\omega \cos \Theta}{2\rho_0 c_0} \right)^2 \right]^{-1}.
$$

(4)

$m=\rho_p h$ is the surface density of the plate. The approximate solution of the transmission coefficient for random-incidence sound $\tau(\omega)$ is then derived by substituting Eq. (4) into Eq. (3):
The terms in square brackets in Eq. (4) denote the effect of acoustic loading. If this is ignored, it can be further simplified:

\[
\hat{\tau}(\omega) \equiv \left[ \ln \left( 1 + \left( \frac{m \omega}{2 \rho \omega_0} \right)^2 \right) - \ln \left( 1 + 0.04 \left( \frac{m \omega}{2 \rho \omega_0} \right)^2 \right) \right] \left( \frac{m \omega}{2 \rho \omega_0} \right)^2. \tag{5}
\]

In this case, \(\hat{\tau}(\omega)\) is derived by substituting Eq. (6) into Eq. (3) as well:

\[
\hat{\tau}(\omega) = 3.28 \left( \frac{2 \rho \omega_0}{m \omega} \right)^2 \equiv 3.28 \tau_0(\omega), \tag{7}
\]

where \(\tau_0(\omega)\) is the transmission coefficient for normal-incidence sound. The transmission coefficient for random-incidence sound is known to be approximately represented in relation to the transmission coefficient for normal-incidence sound. It is usually expressed in the form of a sound transmission loss in dB scale by taking the logarithm of its reciprocal as

\[
10 \log_{10} \hat{\tau}^{-1}(\omega) \equiv 10 \log_{10} \tau_0^{-1}(\omega) - 5 \text{ dB}. \tag{8}
\]

The reduction index is proportional to both the frequency and the surface density of the plate. This is called the “mass law” for random-incidence sound. Doubling the weight of the wall or the frequency gives an increase of 6 dB. The mass law is a fundamental principle for airborne sound insulation of walls, and is widely used in actual engineering situations. Numerical examples of Eqs. (3) and (7) are shown in Fig. 3. The two characteristics are in good agreement with each other in the low-frequency range below the critical frequency. Note that they differ slightly at very low frequencies because the effect of acoustic loading is ignored.

### 3. Radiated sound power under point-force excitation

#### 3.1 Exact solution

Consider an infinite elastic plate lying in the plane \(z = 0\) (Fig. 2), which vibrates driven by point force excitation. Suppose that the vibration of the plate is in accordance with the classic thin plate theory and the acoustic admittance of both sides of the plate is zero, i.e., no absorption. The radiated sound power is derived by simultaneously solving the governing equations of the sound field and the equation of motion of the plate. The effects of acoustic loading on both sides of the plate are taken into account in this model. Considering the problem as being axissymmetrical and using the Hankel transform in cylindrical coordinates, the sound pressure on the plate \(p(r_0)\) and the angular spectrum \(W(k)\) with respect to the displacement of the plate \(w(r_0)\) are obtained as follows:

\[
p(r_0) = \rho_0 \omega^2 \int_0^\pi \frac{W(k)}{\sqrt{k^2 - k_0^2}} J_0(kr_0)dk, \tag{9}
\]

\[
W(k) = -\frac{1}{2\pi} \frac{2 \rho_0 \omega^2}{\sqrt{k^2 - k_0^2}} \frac{1}{(Dk^4 - \rho_0 h \omega^2)}, \tag{10}
\]

where \(k_0\) is the acoustic wavenumber in the air and \(w(r_0)\) and \(W(k)\) are related to the Hankel transform defined by the following equations:

\[
\begin{bmatrix}
W(k) = \int_0^\pi w(r)J_0(kr)rdr \\
w(r) = \int_0^\infty W(k)J_0(kr)dk
\end{bmatrix} \tag{11}
\]
The exact solution of the radiated sound power under point force excitation, $\Pi(\omega)$, is obtained by integrating the surface intensity of the plate over the entire surface to an infinite extent, thus,

$$
\Pi(\omega) = \int_S \text{Re} \left[ \frac{1}{2} p(r_0) \bar{v}(r_0) \right] dS_0,
$$

(12)

where $v(r_0) = -i\omega w(r_0)$ is the velocity of the plate with a complex conjugate. Substituting Eqs. (9) and (10) into Eq. (12) $\Pi(\omega)$ is finally expressed by the following integral:

$$
\Pi(\omega) = \pi \rho_0 \omega^3 \int_0^k \frac{|W(k)|^2}{\sqrt{k^2 - k_0^2}} k dk.
$$

(13)

### 3.2 Approximate solution

The radiated sound power under point force excitation is also obtained by integrating the radial intensity in the far-field. In Fig. 2, $\theta$ denotes the polar angle between the direction of the receiving point and the negative direction of the z-axis. Using Rayleigh’s integral, the approximate solution of the radiated sound pressure $p(r, \theta)$ is obtained in a closed form:

$$
p(r, \theta) \equiv -2i \frac{\rho_0 \omega^3}{\cos \theta} \cdot \frac{\epsilon^{ikr}}{4\pi r}.
$$

(14)

Integrating the radial intensity $|p(r, \theta)|/2\rho_0 c_0$ over a hemisphere of radius $r$ yields the approximate solution of the radiated sound power under point force excitation $\hat{\Pi}(\omega)$; thus,

$$
\hat{\Pi}(\omega) \equiv \frac{r^2}{2 \rho_0 c_0} \int_0^{2\pi} d\phi \int_0^{\pi/2} |p(r, \theta)|^2 \sin \theta d\theta.
$$

(15)

Here further approximations are introduced. Using the coincidence frequency $\omega_c = (c_0/4m/\text{Re}[D])^{1/2}$. The integral in Eq. (13) is evaluated analytically within the low-frequency limit when $\omega^2/\omega_c^2$ is negligible. The approximate solution of the radiated sound power $\hat{\Pi}(\omega)$ is therefore obtained in a closed form:

$$
\hat{\Pi}(\omega) = \frac{1}{4\pi \rho_0 c_0} \left( \frac{\rho_0}{m} \right)^2 \left[ 1 - \frac{2 \rho_0 c_0}{m \omega_c} \tan^{-1} \frac{m \omega}{2 \rho_0 c_0} \right].
$$

(16)

The terms in square brackets indicate the effects of acoustic loading. If these effects are negligible as in Sec. II, Eq. (16) is further simplified, finally yielding:

$$
\hat{\Pi}(\omega) \equiv \frac{1}{4\pi \rho_0 c_0} \left( \frac{\rho_0}{m} \right)^2.
$$

(17)

In this equation, the characteristics of the radiated sound power under point force excitation are different from the transmission coefficient for random-incidence sound, namely, the radiated sound power is reduced in 6 dB by doubling the surface density of the elastic plate but does not depend on the frequency. Calculated examples of Eqs. (13) and (17) are shown in Fig. 4. The two characteristics are in good agreement with each other in the low-frequency range below the critical frequency. Note that both of them differ slightly at very low frequencies because the effect of acoustic loading is ignored.

### 4. Relationship between the two different excitation problems

#### 4.1 Derivation of conversion function

From the approximate solutions of the transmission coefficient for random-incidence sound $\hat{\tau}(\omega)$ in Eq. (7) and the radiated sound power under point force excitation $\hat{\Pi}(\omega)$ in Eq. (17), the following relation is derived:
\[ \hat{\tau}(\omega) = \varepsilon(\omega) \hat{\Pi}(\omega), \]  
where the conversion function \( \varepsilon(\omega) \) is as follows:
\[ \varepsilon(\omega) \equiv \frac{52\pi \rho_c c_0}{k_0^2}. \]  
\( \varepsilon(\omega) \) is a function of the specific impedance \( \rho_c c_0 \) and the wave number \( k_0 \) of the medium, and does not include any elastic plate parameters. This fact means that the linear operator \( \varepsilon \) defined in Eq. (1), exists in a form independent of the elastic plate within the low-frequency limit below the critical frequency without acoustic loading.

![Figure 3](image1.png)  
**Figure 3** Numerical example of the transmission coefficient for random-incidence sound: Comparison of the approximate solution, \( \hat{\tau}(\omega) \), in Eq. (7) (thick line) and the exact solution, \( \tau(\omega) \) in Eq. (3) (thin line) in the case of \( \rho_p = 1000 \text{ kg/m}^3, h = 20 \text{ mm}, E = 1.8 \times 10^9 \text{ N/m}^2, \eta = 0.03 \) and \( \nu = 0.3 \).

![Figure 4](image2.png)  
**Figure 4** Numerical example of the radiated sound power driven by point force excitation: Comparison of the approximate solution, \( \hat{\Pi}(\omega) \), in Eq. (17) (thick line) and the exact solution, \( \Pi(\omega) \), in Eq. (13) (thin line) in the case of \( \rho_p = 1000 \text{ kg/m}^3, h = 20 \text{ mm}, E = 1.8 \times 10^9 \text{ N/m}^2, \eta = 0.03 \) and \( \nu = 0.3 \).

### 4.2 Theoretical considerations

Consider a physical meaning of the conversion function[9]. Introducing the driving point impedance \( z_p = 8(\text{Re}[D]m)^{1/2} \) into the relationship of Eq. (18), the equation is transformed to
\[ \hat{\tau}(\omega) = \psi \frac{\hat{\Pi}(\omega)}{(2z_p)^{1/2}}, \]  
where
\[ \psi = \frac{\varepsilon(\omega)}{2z_p} = \frac{26\pi \rho_c c_0}{k_0^2 z_p}. \]  
The right side of Eq. (20) is expressed by the ratio of the input power under point force excitation \( 1/2z_p \) and the output power \( \hat{\Pi}(\omega) \) multiplied by the factor \( \psi \). The input-output power ratio is a dimensionless evaluation index as well as the transmission coefficient for random-incidence sound. The factor \( \psi \) in Eq. (21) is also dimensionless and is composed of the driving point impedance (\( \approx \) input impedance) and the specific impedance of the air (\( \approx \) radiation impedance). \( \psi \) depends on the parameters of the elastic plate since it includes the driving point impedance \( z_p \). When the input-

![Figure 5](image3.png)  
**Figure 5** Numerical example of the input power and output power: Comparison of the approximate solution, \( \frac{\hat{\Pi}(\omega)}{(2z_p)^{1/2}} \), in Eq. (17) (thick line) and the exact solution, \( \Pi(\omega) \), in Eq. (13) (thin line) in the case of \( \rho_p = 1000 \text{ kg/m}^3, h = 20 \text{ mm}, E = 1.8 \times 10^9 \text{ N/m}^2, \eta = 0.03 \) and \( \nu = 0.3 \).
output power ratio is used as the evaluation index $M_s$ for the sound radiation from force-excited vibration in Fig. 1, the linear operator $\varepsilon$ is not determined in a form independent of the elastic plate.

### 4.3 Application of the conversion function to the exact solutions

The conversion function $\varepsilon(\omega)$ derived through the discussions so far is obtained under the relationship between the approximate solutions so it is not clear whether $\varepsilon(\omega)$ is applicable to the relationship between the exact solutions. In this section, the possibility of the application of $\varepsilon(\omega)$ to the exact solutions, namely,

$$
\tau(\omega) \cong \varepsilon(\omega) \Pi(\omega),
$$

(22)

is verified by numerical calculations. Two kinds of elastic plates used in the calculations are shown in Table 1. These are assumed to be typical building materials: gypsum board (12.5 mm in thickness) and reinforced concrete panel (150 mm in thickness). In the calculations, the reciprocal of both terms in Eq. (22) are represented and compared in dB scale; thus, $10\log_{10}\tau^{-1}(\omega)$ and $10\log_{10}[\varepsilon^{-1}(\omega)\Pi^{-1}(\omega)]$. Figure 5 shows the calculated results, which exhibit similar tendencies. The typical behaviour is characterized by a significant dip around the critical frequency, which is caused by the coincidence effect of the plate. The results are in fairly good agreement with each other at all frequencies. The differences are largest around the critical frequency, but are within 2 dB.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Gypsum board</th>
<th>Reinforced Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus, $E$, N/m²</td>
<td>$1.8 \times 10^9$</td>
<td>$2.6 \times 10^{10}$</td>
</tr>
<tr>
<td>thickness, $h$, m</td>
<td>0.013</td>
<td>0.25</td>
</tr>
<tr>
<td>loss factor, $\eta$</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Poisson's ratio, $\nu$</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td>density, $\rho_p$, kg/m³</td>
<td>650</td>
<td>2400</td>
</tr>
</tbody>
</table>

Figure 5 Example single-leaf wall construction calculations based on the conversion function between exact solutions given by Eq. (22). The calculated results represent the two considered construction cases of (a) panel + gypsum board (left) and (b) reinforced concrete (right), respectively. The dashed line represents the inverse of $\tau(\omega)$ in decibels (Reduction Index) and the solid line represents the inverse of $\varepsilon(\omega)\Pi(\omega)$ in decibels.
Thus, it was shown that the exact solutions of both problems were also related by the same conversion function $\varepsilon(\omega)$ at all frequencies including above the critical frequency. This fact suggests that the sound radiation from an infinite elastic plate with random-incidence sound and point force excitation are essentially similar phenomena, and the only difference is the gradient of those characteristics in dB scale with respect to the frequency.

5. Conclusions

To acquire fundamental insight into the relationship between airborne sound transmission and sound radiation from force-excited vibration of solid structures, theoretical studies were carried out using an infinite elastic plate model. Random-incidence sound and point force excitation were introduced into the analysis as the source of external force for each problem. Exact solutions of the transmission coefficient for random-incidence sound and radiated sound power under point force excitation were shown. Approximate solutions of those problems were both derived from the exact solutions within the low-frequency limit below the critical frequency.

A conversion function that relates the two problems was obtained in a simple closed form through the approximate solutions. Numerical calculations verified that the conversion function is applicable to the relationship between the exact solutions. It was shown that the exact solutions of both problems were also related by the conversion function at all frequencies including above the critical frequency. The conversion function is composed of only the specific impedance and the wavenumber and does not include any elastic plate parameters. This means the linear operator defined in this paper exists in a form independent of an infinite elastic plate. A physical meaning for the conversion function was also given by introducing the driving point impedance of point force excitation into the relationship.

REFERENCES