A SEPARATION METHOD OF OBJECTIVE NOISE AND BACKGROUND NOISE USING DISCRIMINANT ANALYSIS METHOD

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In the actual situation of environmental noise, it is very often that only the resultant noise fluctuation contaminated by background noise is observed. The separation of an objective noise and a background noise is very important in the field of noise evaluation and regulation problems. In this paper, a method of separating the objective noise and the background noise in a form of $L_{eq}$ noise evaluation indices is proposed by introducing a discriminant analysis method. The proposed method is experimentally confirmed by a simulation experiment and actual environmental noise data.

Keywords: separation method, objective noise and background noise, discriminant analysis method

1. Introduction

In an actual living environment, the environmental noise level fluctuates complicatedly, owing to physical, social and psychological factors. Moreover, the environmental noises, which we encounter in our daily life, exhibit various types of probability distribution forms, apart from a standard Gaussian distribution, due to the diversified causes of fluctuation. As is well known, the equivalent noise evaluation index, $L_{eq}$, plays an important role in the field of noise evaluation and/or regulation problems. Here, $L_{eq}$ is defined as the sound pressure level in dB, equivalent to the total sound energy over a given time interval. In the previous paper, a general $L_{eq}$ estimation method generally applicable to arbitrary non-Gaussian level fluctuation was proposed by introducing the lower and higher order cumulant statistics in a form of infinite expansion series expression [1].

On the other hand, by paying attention to the actual environmental noise, it is very often that an objective noise fluctuation is contaminated by the other noise sources. That is, in the actual acoustic measurement, the acoustic signal is measured by a sound level meter under the mixture of several sound sources. From the above point of view, we pay attention to the separation problem of the objective noise and the background noise from their mixtures. From the viewpoint of the environmental noise evaluation, it is very important to evaluate the contribution ratio between the objective noise and the background noise.

From the above practical point of view, in this paper, a practical separation method of the objective and background noises is proposed by introducing a discriminant analysis method. By using the estimated cumulant statistics using the discriminant analysis method, the respective $L_{eq}$ noise levels of the objective noise and/or background noise can be estimated by using the above general $L_{eq}$ estimation method. Finally, the effectiveness of the proposed method has been experimentally confirmed by applying it to a simulation experiment and actual environmental noise data.
2. Estimation method of $L_{eq}$

According to the previous study [1], a generalized expansion type expression for estimating $L_{eq}$ is given by cumulant statistics. First, let us consider the noise level fluctuation $x$ of an arbitrary non-Gaussian distribution type. As is well-known, the relationship between the noise level fluctuation $x$ and the noise energy fluctuation $E$ is given as follows:

$$x = 10 \log_{10} \frac{E}{E_0} = M \ln \frac{E}{E_0} \quad (M \equiv 10/\ln 10),$$

where $E_0$ is the reference noise energy usually taken as $10^{-12}$ [watt/m$^2$]. Here, we introduce the moment generating function $M_\omega(\theta)$ with respect to the noise level fluctuation $x$, as follows:

$$M_\omega(\theta) = \langle \exp(\theta M \ln E/E_0) \rangle,$$

where $\langle \rangle$ denotes an averaging operation with respect to the random variable $x$. The mathematical relationship between the arbitrary order cumulant $\kappa_n$ with respect to $x$ and the moment generating function $M_\omega(\theta)$ is given by

$$M_\omega(\theta) = \exp \left( \sum_{n=1}^\infty \frac{\kappa_n}{n!} \theta^n \right).$$

By replacing the parameter $\theta$ to $1/M$ in Eqs. (2) and (3), the mean value of $E$ can be easily obtained as follows:

$$\langle E \rangle = E_0 \exp \left( \sum_{n=1}^\infty \frac{1}{n!} \frac{\kappa_n}{M^n} \right).$$

Thus, a substitution of Eq. (4) into the definition of $L_{eq}$ yields a general expansion type expression for estimating $L_{eq}$, as follows:

$$L_{eq} = 10 \log_{10} \frac{\langle E \rangle}{E_0}$$

$$= \kappa_1 + \frac{\kappa_2}{2M} + \frac{\kappa_3}{6M^2} + \frac{\kappa_4}{24M^3} + \cdots$$

$$= \mu + 0.115\sigma^2 + 8.84 \times 10^{-3} \kappa_3 + 5.09 \times 10^{-4} \kappa_4 + 2.34 \times 10^{-5} \kappa_5 + \cdots,$$

where $\mu(= \kappa_1)$ and $\sigma^2(= \kappa_2)$ denote the mean value and the variance of $x$. From Eq. (5), it is possible to generally estimate $L_{eq}$ by reflecting not only lower order cumulants but also higher order cumulants as the correction terms in a hierarchical form. It should be noted that the above estimation formula agrees completely with a well-known simplified estimation formula [2] derived under the assumption of a standard Gaussian distribution as the first approximation:

$$L_{eq} = \mu + 0.115\sigma^2,$$
since higher order cumulant $\kappa_n (n = 3, 4, \cdots)$ become zero for this special case. Thus, this estimation method shows a generalized form including the well-known simplified estimation method as a special case. The relationship between the moment statistics and cumulant statistics is given by [3]:

$$\sum_{k=1}^{\infty} \frac{m_k}{(k-1)!} S^{k-1} = \left[ \sum_{n=1}^{\infty} \frac{\kappa_n}{(n-1)!} S^{n-1} \right] \left[ \sum_{l=0}^{\infty} \frac{m_l}{l!} S^l \right].$$

(7)

After the arbitrary order moment statistics of noise level fluctuation are obtained by measured data, the resultant moment $m_n = \langle x^n \rangle (n = 1, 2, \cdots)$ can be transformed into the cumulants $\kappa_n (n = 3, 4, \cdots)$.

Based on Eq. (7), the relationships between the moment statistics and the cumulant statistics are expressed as follows:

$$\kappa_1 = m_1, \quad \kappa_2 = m_2 - m_1^2, \quad \kappa_3 = m_3 - 3m_2m_1 + 2m_1^3,$$

$$\kappa_4 = m_4 - 4m_3m_1 - 3m_2^2 + 12m_2m_1^2 - 6m_1^4$$

$$\kappa_5 = m_5 - 5m_4m_1 - 10m_3m_2 + 20m_2m_1^3 + 30m_2^2 - 60m_2m_1^3 + 24m_1^5, \cdots$$

(8)

3. Introduction of discriminant analysis method

Paying attention to the actual environmental noise, an objective noise fluctuation is very often contaminated by the other noise sources. That is, in the actual acoustic measurement, the acoustic signal is measured by a sound level meter under the mixture of several sound sources. From the above point of view, we pay attention to the separation problem of the objective noise and the background noise from their mixtures. From the viewpoint of the environmental noise evaluation, it is very important to evaluate the contribution ratio between the objective noise and the background noise. Therefore, when separating background noise and objective noise, it is necessary to divide noise data into objective noise and background noise according to some reasonable criterion. In consideration of the stability of statistical information, we consider a method of practically separating objective and background noises by using the average and variance of measured data. Here, a discriminant analysis method is practically introduced [4].

Let us consider the probability density function $P(x)$ of the noise level fluctuation $x$ of the background noise or the objective noise as shown in Fig. 1. In the usual case, it is considered that the average value $\mu_2$ of the objective noise is larger than the average value $\mu_1$ of the background noise.

![Figure 1: Probability density function $P(x)$ of the mixed noise level fluctuation $x$.](image-url)
noise (i.e., \( \mu_1 < \mu_2 \)). Based on the adequate boundary value \( C \), Group I shows the category of background noise, and Group II shows the category of objective noise. Here, the area of the shaded portion of red is the probability of erroneously discriminating as Group II, in spite of Group I. Similarly, the area of the shaded portion of blue is the probability of erroneously discriminating as Group I, in spite of Group II. In this figure, \( P_1 \) and \( P_2 \) denote the above erroneous discriminant probabilities. A specific discriminant analysis method is described below. By using the measured data of noise level fluctuation \( x \), the normalized squared distances from the respective averages of Groups I and II are as follows:

\[
D_1^2 = \frac{(x - \mu_1)^2}{\sigma_1^2}, \quad \text{(9)}
\]

\[
D_2^2 = \frac{(x - \mu_2)^2}{\sigma_2^2}. \quad \text{(10)}
\]

Here, \( D_1 \) and \( D_2 \) denote the standard deviations of the Euclidean distances from the averages \( \mu_1 \) and \( \mu_2 \) (i.e., Mahalanobis' generalized distances). The boundary value \( C \) is determined under the condition of \( D_1 = D_2 \), as follows:

\[
\frac{(C - \mu_1)^2}{\sigma_1^2} = \frac{(\mu_2 - C)^2}{\sigma_2^2}. \quad \text{(11)}
\]

Therefore, the following equation can be derived as

\[
\frac{C - \mu_1}{\sigma_1} = \frac{\mu_2 - C}{\sigma_2}. \quad \text{(12)}
\]

Solving this, the adequate boundary value \( C \) can be obtained as follows:

\[
C = \frac{\mu_1 \sigma_2 + \mu_2 \sigma_1}{\sigma_1 + \sigma_2}. \quad \text{(13)}
\]

When the usual case where \( \mu_1 < \mu_2 \), it can be determined that if \( x \geq C \), then \( x \) belongs to Group II, oppositely if \( x < C \), then \( x \) belongs to Group I. In the unusual case where \( \mu_1 > \mu_2 \), the discriminant is opposite to the above determination. Actually, the averages and variances of the noise level fluctuations of background and objective noises are unknown. Therefore, after adopting temporarily the boundary value \( \tilde{C} \), the averages and variances of the noise level fluctuations of background and objective noises can be obtained. Then, we can obtain the estimated value \( \hat{C} \) by using Eq. (13). We adopt the minimization of the difference between \( \tilde{C} \) and value \( \hat{C} \) as the adequate boundary value. That is, a numerical search method is employed as follows:

\[
|\hat{C} - \tilde{C}| \rightarrow \text{min.} \quad \text{(14)}
\]
Therefore, by calculating the moment statistics with respect to the data of the background noise and the objective noise separated in this way and transforming it to the cumulant statistics, the $L_{eq}$ noise evaluation indices of the background and objective noise can be estimated by Eq. (5).

4. Simulation experiment

In this simulation experiment, we focus on the fact that the noise level fluctuation distribution is often approximated by Gaussian distribution, and simulate the background noise and the objective noise with Gaussian random numbers with arbitrary average and variance, respectively. We generate the Gaussian random numbers with known mean and variance of noise level fluctuation of objective noise and background noise. We calculate the $L_{eq}$ noise evaluation indices of experimental value and estimated value. That is, to confirm the validity of this method, the following experiment was considered. First, we use Eq. (6) derived from a standard Gaussian distribution. In this simulation experiment, the estimated value when varying the average of background noise was obtained for a specific objective noise. In general, it is considered that the lower the average value of the background noise, the smaller the overlapping portion of the probability density function is in the boundary value in the discriminant analysis, and the problem of erroneous determination does not occur much. However, from the viewpoint of environmental noise evaluation, attention was paid to the point that the equivalent noise level of a specific objective noise cannot be evaluated when the objective noise is measured because the average value of the background noise is high. Let us consider the case where the variance of the background noise is fixed and the average value is increased.

That is, Gaussian random numbers were generated with the following settings. Let us set the average of objective noise to 80.0[dB] and its variance to 20.0[dB$^2$]. Furthermore, let us set the average values of background noise to 75.0 [dB], 80.0 [dB] and 85.0 [dB] and their variances to fixed at 10.0 [dB$^2$]. The number of generated Gaussian random numbers is respectively 6000. When the average value of the background noise is nearly equal to the average value of objective noise, the fluctuation range of objective noise after separation narrows. The average and variance of the background noise are estimated to be small. Therefore, since the whole experimental value of the mixed noise is measured, we estimate only the objective noise by this method and calculate the average value of the noise energy of objective noise. That is, the energy average value of the background noise is evaluated by subtracting the average value of the mixed noise energy to the average value of objective noise energy. We adopt a method to estimate the background noise by performing decibel transformation as the energy average value of the background noise. Figure 2 shows one of the examples of the estimated

![Figure 2: Example of the determination of boundary value ($\mu_1 = 80$[dB]).](image-url)
results of boundary values and the probability density function of the mixed Gaussian random numbers. The experimental value of the probability density function is calculated by creating a frequency distribution every difference width of 2 [dB] and dividing the obtained frequency distribution by this difference width. Table 1 shows the estimated results of \( L_{eq} \) by using the propose method. As shown in this table, the estimated values of \( L_{eq} \) agree fairly well with the experimental values.

Table 1: Estimated results of \( L_{eq} \) using the proposed method

<table>
<thead>
<tr>
<th>Average of background noise [dB]</th>
<th>( L_{eq} ) of background Noise [dB] Experiment</th>
<th>Estimate</th>
<th>( L_{eq} ) of objective noise [dB] Experiment</th>
<th>Estimate</th>
<th>( L_{eq} ) of mixed noise [dB] (Experiment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.0</td>
<td>76.1</td>
<td>78.3</td>
<td>82.2</td>
<td>81.5</td>
<td>83.2</td>
</tr>
<tr>
<td>80.0</td>
<td>81.1</td>
<td>78.6</td>
<td>82.2</td>
<td>83.5</td>
<td>84.7</td>
</tr>
<tr>
<td>85.0</td>
<td>86.2</td>
<td>86.6</td>
<td>82.2</td>
<td>81.0</td>
<td>87.7</td>
</tr>
</tbody>
</table>

5. Application to actual data

A generalized expansion expression for estimating \( L_{eq} \) is given by Eq. (5). It is necessary to select the order of expansion expression. Several methods can be considered for this order selection method. In this case, a method for experimentally determining the optimum order is adopted. In general, higher-order cumulants are not stable statistical information as compared with low-order cumulants such as the first and second order cumulants. Therefore, the order of cumulants can be selected a criterion for reducing the number of expansion terms using cumulants as low as possible. This also leads to simplification of the calculation process as a result. Therefore, the following experiment was considered. Fifty kinds of music sounds were exited in a quiet room, and their noise level fluctuations were measured by using a precision sound level meter. The music sounds were selected from various kinds of CDs including pops, rocks and classics. Table 2 shows the estimation accuracies using the expansion expression of \( L_{eq} \). From the viewpoint of the stability of statistical information of the cumulant, the order is set from 2nd to 5th orders. From this table, we employ the order of cumulant up to 4th, because of fairly small average of estimation errors, smallest variance of estimation errors and largest correlation coefficient between the experimental values and estimated values of \( L_{eq} \). Accordingly, the estimation formula with the optimum order is expressed as

\[
L_{eq} = \kappa_1 + 0.115\kappa_2 + 8.836 \times 10^{-3}\kappa_3 + 5.087 \times 10^{-4}\kappa_4.
\] (15)

Since the probability phenomena in which background noise and objective noise occur are mutually exclusive events according to the basic concept of discriminant analysis, it is necessary to calculate

Table 2: Estimation accuracies using the expansion expression of \( L_{eq} \)

<table>
<thead>
<tr>
<th>Order of cumulant statistics</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of estimation errors [dB]</td>
<td>-0.63</td>
<td>0.41</td>
<td>-0.13</td>
<td>-0.03</td>
</tr>
<tr>
<td>Variance of estimation errors [dB^2]</td>
<td>0.953</td>
<td>0.414</td>
<td>0.371</td>
<td>0.551</td>
</tr>
<tr>
<td>Correlation coefficient between the experimental values and estimated ones</td>
<td>0.941</td>
<td>0.971</td>
<td>0.976</td>
<td>0.965</td>
</tr>
</tbody>
</table>
the existence probability $\alpha$ of objective noise from the sample numbers. The existence probability $\beta$ of the background noise is estimated in the same manner ($\alpha + \beta = 1$). It is necessary to pay attention that the estimated value is transformed into the noise energy average, and the value obtained by multiplying the noise energy average by the existence probabilities $\alpha$ and $\beta$. The estimated value of $L_{eq}$ can be calculated by transforming the above noise energy average into the decibel scale.

In the actual noise measurement, when the background noise and the objective noise are mixed, only the background noise or only the objective noise cannot be measured. For this reason, we employ the experimental consideration as follows. First, road traffic noise was recorded in advance as the background noise. Experimental data was respectively measured indoors with the road traffic noise as background noise and music sounds as objective noise. Then, the mixed experimental data of the background and objective noises was measured. At this time, these noise level fluctuation data was measured for 5 [min] by using a precision sound level meter. Here, the sampling interval was set to 0.1 [sec]. Therefore, the number of data is 3000. Furthermore, music sounds were reproduced by three kinds of CDs. The music sounds used in the experiment are shown as follows:

Music sound 1: Artist: Eagles, Song title: New Kid in Town  
Label: Asylum Records

Music sound 2: Artist: Eagles, Song title: Hotel California  
Label: Asylum Records

Music sound 3: Artist: HY, Song title: Letter  
Label: Independent Music

Let us define the music sound 1 with road traffic noise as Case A. And, let us define the music sound 2 with road traffic noise as Case B. Furthermore, let us define the music sound 3 with road traffic noise as Case C. Based on the actual measurement data of environmental noise, the boundary value $C$ obtained by the discriminant analysis is estimated for each data and the experimental values of the probability density function are also obtained. Figure 3 shows the estimated value of boundary value obtained by the discriminant analysis and the experimental value of the probability density function for Case A. In this case, the estimated boundary value $\hat{C}$ is equal to 83.1 [dB] and the existence probability $\alpha$ is equal to 0.48 ($\beta = 0.52$). The estimated boundary values $\hat{C}$ were obtained by the discriminant analysis for Case B and Case C, respectively. In these cases, the average value of the background noise and the average value of the objective noise are fairly near, and

![Figure 3: Experimental value of probability density function $P(x)$ and the estimated $\hat{C}$ for Case A.](image-url)

\[ \hat{C} = 83.1 \text{ [dB]} \]
the measured value of the probability density function is unimodal. It can be seen that the estimated values of the estimated boundary values \( \hat{C} \) are near the center of the experimental probability density functions. Table 3 shows the estimated results of \( L_{eq} \) by using the propose method. As shown in this table, the estimated values of \( L_{eq} \) agree well with the experimental ones.

Table 3 Estimated results of \( L_{eq} \) using the proposed method

<table>
<thead>
<tr>
<th>Case</th>
<th>( L_{eq} ) of background noise [dB]</th>
<th>( L_{eq} ) of objective noise [dB]</th>
<th>( L_{eq} ) of mixed noise [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Estimate</td>
<td>Experiment</td>
</tr>
<tr>
<td>Case A</td>
<td>78.9</td>
<td>78.3.</td>
<td>81.5</td>
</tr>
<tr>
<td>Case B</td>
<td>78.9</td>
<td>79.4</td>
<td>81.7</td>
</tr>
<tr>
<td>Case C</td>
<td>82.5</td>
<td>82.9</td>
<td>85.5</td>
</tr>
</tbody>
</table>

6. Conclusion

In this paper, a practical method was proposed for the purpose of separating the \( L_{eq} \) evaluation indices of the objective and background noises. Here, the proposed method do not need any prior information, by separating the background and objective noises as a probability phenomenon. That is, this method can statistically determine the adequate boundary value between the background noise and the objective noise by using the discriminant analysis method. In the experimental consideration, the validity of the proposed method was confirmed by applying it to the simulation experiment. In this case, Gaussian random numbers with the known means and variances of the background and objective noise fluctuations were used. The effectiveness of the proposed method was experimentally confirmed by applying the actual data measured in a room. The music sounds and road traffic noise were employed in this experiment. The estimated values agreed well with the experimental values.

This research is still in the early stage and the work reported here has focused principally on its methodological aspects. Accordingly, several future problems remain. This method must be applied to many other actual problems to broaden and confirm its practical effectiveness. For example, this method must be applied to a separation problem of factory and road traffic noises.

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REFERENCES


