BANDWIDTH ANALYSES OF A MEMBRANE TYPE SOUND ABSORBER WITH MAGNETIC NEGATIVE STIFFNESS

J. Zhao, X. Li, Y. Wang, W. Wang, L. Zhu and B. Zhang

Beijing Key Lab of Environmental Noise and Vibration, Beijing Municipal Institute of Labor Protection, Beijing, China
email: junjuanzhao@sina.com

A magnetic membrane type sound absorber (MMSA) with negative stiffness is proposed in this paper to implement a compact design for low frequency sound absorption. The negative stiffness produced by the magnet can offset the stiffness of the cavity and shifts the absorption peak towards lower frequencies. The performance of this sound absorber has been investigated experimentally before. In this paper, the MMSA’s bandwidth and resonant frequency properties are calculated, simulated and analyzed. The simulate result verify that, the magnetic membrane type sound absorber with negative stiffness has a broader bandwidth, compared with the membrane absorber which increasing surface density mass to realize the low frequency sound absorption. The result also demonstrates that the MMSA can realize a low frequency sound absorption conveniently and adjustably due to the nonlinear magnetic negative stiffness in compact cavity. Therefore, an appropriate choice of the design parameters can implement a compact low frequency and broadband magnetic membrane type sound absorber with negative stiffness.

Keywords: membrane sound absorber, magnetic negative stiffness, low frequency

1. Introduction

A typical membrane absorber is composed of a single-leaf impervious membrane placed in front of a rigid wall, with an air cavity in-between [1, 2]. The sound-absorption properties of such an absorber are mainly influenced by the surface density, membrane tension, and back-cavity depth. It can extend the absorption bandwidth to lower frequencies usually at the cost of a large cavity depth. For resonance-type sound absorbers such as membrane absorbers and Microperforated panels (MPPs), it is a challenging task to realize the low-frequency sound absorption with a compact design.

Special designs have been proposed to fulfil these seemingly contradictory requirements. For instance, Tao et al. presented a composite absorber in which the back wall of the cavity is replaced by a close-box loudspeaker with a shunted circuit [3]. In this device, the loudspeaker absorbs low-frequency sounds when the electrical parameters of the shunted circuit are properly adjusted. Additionally, the development of acoustic metamaterials with unusual constitutive effective parameters [4-12] which are not found in nature has significantly enhanced our ability in sound wave manipulation. Mei et al. studied a thin-film acoustic metamaterial comprising an elastic membrane decorated with asymmetric rigid platelets [13]. The metamaterial aims to totally absorb low-frequency airborne sounds at selective resonance frequencies ranging from 100 Hz to 1000 Hz. Ma et al. reported on a novel acoustic metasurface with hybrid resonances that can achieve robust impedance matching and perfect absorption [14]. This type of metasurface has a very narrow but tunable total absorption bandwidth. Chiu et al. introduced negative stiffness, which was realized using magnetic force, to enhance the low-frequency performance of a duct noise control device [15]. Furthermore,
Zhang et al. studied a thin, broadband sound absorber, which was implemented using electro-mechanical coupling [16].

The performance of the magnetic membrane absorber (MMSA) with negative stiffness has been investigated experimentally and theoretically [17]. It can realize a low frequency sound absorption conveniently and adjustably due to the nonlinear magnetic negative stiffness in cavity. In this paper, the bandwidth and resonant frequency of the MMSA is calculated and analyzed in detail. The analyses verify that the MMSA has a broader absorption bandwidth compared with the membrane absorber which increasing surface density mass to realize the low frequency sound absorption,

2. The MMSA absorber structure

The MMSA considered herein is illustrated in Fig. 1(a). It consists of a container and a polyethylene terephthalate (PET) membrane onto which a small iron plate with negligible mass is glued. The membrane is fixed to the edges of a container, forming a closed cavity. Directly underneath the iron plate, a magnet is installed through a screw which can tune the magnet top height. The membrane radius, surface density, cavity depth and magnet top height are denoted as \( a, m, d \) and \( h \), respectively.

![Diagram of the MMSA with negative stiffness.](image)

3. Introduction

The acoustic impedance of such a system can be obtained using the impedance type of electro-acoustic analogy. Basically, the resonant system contains the mass-resistance in series with the mechanical compliance of the membrane, magnetic compliance of the magnet, and cavity reactance of the air space. The acoustic impedance \( W \) of the absorber can be calculated using

\[
W = r + W_m + W_s + W_n
\]

where

\[
W_m = j\omega m
\]

\[
W_s = -j\rho c_0 \cot \frac{\omega d}{c_0} = -j \frac{s}{\omega}; s = \omega \rho c_0 \cot \frac{\omega d}{c_0}
\]

\[
W_n = -j \frac{k_n}{\omega}
\]

and \( \rho \) is the density of air, \( c_0 \) is the velocity of sound in air, \( \omega = 2\pi f \), \( f \) is the frequency, \( m \) (kg/m\(^2\)) is the surface density of the membrane, \( r \) is the acoustic resistance of the membrane, normally, which depends mainly on the mounting conditions, and \( s \) is the stiffness of the air in the cavity. Then, \( W \) for the whole structure can be expressed as
\[ W = r + W_m + W_s + W_n = r + j(\omega m - \frac{s}{\omega} - \frac{k_n}{\omega}) \] (8)

For a sound absorber with magnetic negative stiffness construction, the resonance frequency \( f_0 \) is given by

\[ f_0 = \frac{1}{2\pi} \sqrt{s + \frac{k_n}{m}} = \frac{1}{2\pi} \sqrt{\frac{s'}{m}} \] (9)

so that the specific acoustic impedance \( W' \) can be calculated by

\[ W' = \frac{W}{\rho_c_0} = \frac{r + W_m + W_s + W_n}{\rho_c_0} = r' + f \left( \frac{2\pi f \cdot m - \frac{s'}{2\pi f}}{\rho_c_0} \right) \] (10)

Then, the absorption coefficient \( \alpha \) of the MMSA absorber can be calculated by

\[ \alpha = 4r' \sqrt{(r' + 1)^2 + \left( \frac{2\pi f \cdot m - \frac{s'}{2\pi f}}{\rho_c_0} \right)^2} \] (11)

and

\[ M(\omega) = \frac{\omega m}{\rho_c_0} \]
\[ K(\omega) = \cot \left( \frac{\omega d}{c_0} + \frac{k_n}{\omega \rho_c_0} \right) \]

When, \( M(\omega) = K(\omega) \), the resonance absorption coefficient is

\[ \alpha_{max} = \frac{4r}{(r + 1)} \] (12)

Then, half-absorption coefficient of the MMSA can be calculated by

\[ \alpha_h = \frac{1}{2} \alpha_{max} = \frac{4r}{(r + 1)^2 + \left( \frac{1}{\rho_c_0} - \cot \left( \frac{\omega d}{c_0} + \frac{k_n}{\omega \rho_c_0} \right) \right)^2} \] (13)

From equal (12) and (13)

\[ \cot \frac{\omega d}{c_0} + \frac{k_n}{\omega \rho_c_0} = \frac{\omega d}{\rho_c_0} (r + 1) \] (14)

\[ k(\omega) = \cot \frac{\omega d}{c_0} + \frac{k_n}{\omega \rho_c_0} \] (15)

\[ m(\omega) = \frac{\omega m}{\rho_c_0} (r + 1) \]
The resonance and half-absorption points of the MMSA are shown graphically in Fig. 2 of the \( \frac{om}{\rho c_0} \) and \( \cot\left(\frac{\omega d}{c_0}\right) + k_n/\omega pc_0 \) curves, referring to the \( \frac{om}{\rho c_0} \) and \( \cot\left(\frac{\omega d}{c_0}\right) \) curves of the magnetic stiffness absence, the resonance point shifts from \( f_0' \) to \( f_0 \). According to Eq.(9), in order to achieve low resonant frequency sound absorption, \( m \) can be increased or \( s' \) can be decreased. Whatever \( r \) equals, the absorption bandwidth shown in Fig. 2 becomes narrower with increasing \( m' \). These results demonstrate that simply introducing negative stiffness can enable the dual requirements of low-frequency and broad-bandwidth sound absorption to be achieved. The analyses results demonstrate that simply introducing negative stiffness can enable the dual requirements of low-frequency and broad-bandwidth sound absorption to be achieved.

### Figure 2: Absorption band of the MMSA.

**4. Conclusions**

In summary, the bandwidth properties of MMSA are investigated, it is revealed that the MMSA can realize a low and adjustably frequency sound absorption conveniently due to the nonlinear magnetic negative stiffness in cavity. Especially, it has a broader bandwidth compared with the membrane absorber that realizes a low frequency sound absorption by increasing surface density mass. The analyses results demonstrate that simply introducing negative stiffness can enable the dual requirements of low-frequency and broad-bandwidth sound absorption to be achieved.

**5. Acknowledgements**

This work is supported by National Natural Science Foundation of China No.11704035 and No.11604015; Beijing Natural Science Foundation No.1172007, No.1182011 and No.1164013; Beijing Nova Program No. Z181100006218018; Xicheng District excellent talent project No.20160060; Beijing Academy of Science and Technology Youth Core Plan No. YC201707.

**REFERENCES**


