The acoustic performance of coatings comprising both a periodic layer of voids and a periodic layer of steel cylinders in an elastic medium with a steel backing is analytically and numerically investigated. The analytical model is based on effective medium approximation theory whereby each layer of scatterers is modelled as a homogeneous medium. The numerical model is based on the finite element method. Peaks of high sound absorption are attributed to Fabry-Perot resonance associated with the presence of the voids as well as dipole resonance of the steel scatterers in the elastic medium. The performance of acoustic coatings comprising a layer of voids and a layer of steel inclusions is compared with those comprising two layers of voids or two layers of steel inclusions. The coating comprising a layer of steel inclusions followed by a layer of voids in the direction of sound propagation is shown to have the best broadband sound absorption performance.

Keywords: acoustic coating, phononic crystals, soft elastic material, effective medium approximation, finite element method

1. Introduction

Phononic crystals are composite structures comprising sound scatterers arranged periodically in elastic media, and can be designed to achieve tailored acoustic performance [1]. Two main mechanisms governing the acoustic performance of phononic crystals are destructive wave interference attributed to Bragg scattering due to periodicity of the scatterers and local resonance of the scatterers [2, 3]. An important application of phononic crystals is as an acoustic coating that can be externally applied to marine vessels to minimise underwater noise pollution as well as to absorb external acoustic waves for stealth purposes [4]. Acoustic coatings were traditionally designed using a soft rubber medium comprising periodic cavities [5]. Later, heavy metallic inclusions in a soft rubber were employed as acoustic coatings [6]. When subject to acoustic waves, the presence of voids causes monopole resonance whereas heavy inclusions result in dipole resonance. These resonances may reduce the transmission or reflection of sound from the coatings depending on the type of backing and the nature of the material [7-9]. Scattering of sound waves by voids or heavy inclusions leads to conversion of longitudinal waves into shear waves which are subsequently dissipated in the rubber host medium provided it has high shear damping, resulting in high sound absorption.

Recently, the authors investigated the acoustic performance of a layer of periodic voids in a soft elastic medium attached to a steel plate and reported high sound absorption due to Fabry-Perot resonance [8]. The acoustic performance of a layer of periodic steel inclusions in a soft rubber with a steel backing was further shown to result in high sound absorption due to dipole resonance of the steel
For both the voided and steel inclusions, high sound absorption occurred in a narrow frequency band. In an attempt to broaden the sound absorption performance of an acoustic coating, in this work the presence of both voids and steel inclusions embedded in a soft elastic medium is investigated.

2. Phononic crystal model

A phononic crystal comprising two layers of scatterers in a polydimethylsiloxane (PDMS) medium with a steel backing is examined. The scatterers are steel cylinders and/or cylindrical voids. Four combinations of scatterers in the PDMS are considered, corresponding to

- two layers of cylindrical voids,
- two layers of steel cylinders,
- a layer of steel cylinders followed by a layer of cylindrical voids in the direction of sound propagation,
- a layer of cylindrical voids followed by a layer of steel cylinders in the direction of sound propagation.

Figure 1 schematically shows a phononic crystal comprising a layer of steel cylinders followed by a layer of cylindrical voids in the x-direction. The scatterers are embedded in a PDMS medium of thickness $t$ which is attached to a steel plate of thickness $s$. The number of scatterers in the y-direction is infinite. The radius of the steel scatterers and voids are denoted by $a_s$ and $a_v$. The distance between the scatterers in the x- and y-directions are respectively $g$ and $h$. The layer of scatterers in the PDMS closest to the steel backing plate is at a distance of $d$ from the steel plate. The fluid on the incidence side is water and the fluid on the transmission side is air. The phononic crystal is subject to acoustic plane wave excitation in the x-direction from the left hand side. The geometric and material parameters used in this work are listed in Section 3.
The sound absorption performance of the phononic crystal with the four combinations of scatterers in PDMS is analytically and numerically studied. The analytical model is based on effective medium approximation theory in which each layer of scatterers is modeled as a homogeneous medium with effective geometric and material properties, as described in Refs. [8] and [9]. The absorption coefficient of the layered composite is then calculated from the reflected and transmitted pressures which in turn were obtained using the transfer matrix method [10]. The numerical model is based on the finite element method developed using COMSOL Multiphysics (v5.2a), also described in Refs. [8] and [9].

3. Results and discussion

The material properties of the solid and fluid media are presented in Table 1. The radius of the steel cylinders is \( a_s = 0.5 \) cm and the radius of the cylindrical voids is \( a_v = 0.5 \) cm. The distance between the scatterers in the x- and y-directions are \( g = 1.4 \) cm and \( h = 2 \) cm. The thickness of the steel plate is \( s = 2 \) cm and the thickness of the PDMS medium is \( t = 3 \) cm. The distance between the steel plate and the closest layer of scatterers is \( d = 0.8 \) cm. Figure 2 shows the sound absorption coefficient of the phononic crystal for the four configurations of voids and steel cylinders. The presence of a single or two layers of voids leads to very high sound absorption at a low frequency due to Fabry-Perot resonance [8, 11]. This first peak has also been described as a mass-spring resonance of the system, whereby the mass is mainly contributed by the steel backing plate and the stiffness is contributed by the soft rubber medium. For the phononic crystal comprising a layer of steel cylinders followed by a layer of voids, two peaks of sound absorption occur. The first absorption peak is due to Fabry-Perot resonance associated with the presence of the voids. The second absorption peak at a higher frequency is due to dipole resonance of the steel scatterers. Sound absorption using the phononic crystal comprising two layers of steel cylinders is very small, even at the dipole resonance frequency. For the phononic crystal comprising a layer of voids followed by a layer of steel scatterers, interestingly, the second absorption peak due to dipole resonance of the steel scatterers does not appear. This is because the layer of voids reflects most of the incident acoustic energy for frequencies beyond the first absorption peak, leaving little acoustic energy to initiate dipole resonance of the steel scatterers. The physical mechanism leading to high sound absorption at both the peaks is the conversion of longitudinal waves into shear waves which are subsequently dissipated in the soft rubber medium. Figure 3 shows the deformation of the phononic crystal at the two peak frequencies for the phononic crystal comprising a layer of steel cylinders followed by a layer of cylindrical voids in the direction of sound propagation. At the first absorption peak, the steel plate oscillates with the highest amplitude while the phononic crystal is stationary as shown in Fig. 3(a), confirming the mass-spring resonance of the system. At the second absorption peak corresponding to dipole resonance frequency of the steel scatterers in PDMS, the steel cylinders oscillate in the elastic medium while the steel backing plate is stationary, as shown in Fig. 3(b).

<table>
<thead>
<tr>
<th>Solids</th>
<th>Density (kg/m³)</th>
<th>Bulk modulus (GPa)</th>
<th>Shear modulus (GPa)</th>
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</thead>
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<tr>
<td>Rubber</td>
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<td>1+0.01i</td>
<td>0.006+0.0018i</td>
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<td>175</td>
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<td>1500</td>
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<td>340</td>
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</table>

The phononic crystal comprising a layer of steel cylinders followed by a layer of cylindrical voids in the direction of sound propagation is shown to have the best broadband sound absorption
Figure 2: Sound absorption coefficient of phononic crystals comprising two layers of voids (blue lines), two layers of steel cylinders (green lines), a layer of cylindrical voids followed by a layer of steel cylinders (red lines), and a layer of steel cylinders followed by a layer of cylindrical voids (black lines), obtained analytically (solid lines) and numerically (dashed lines).

Figure 3: Deformation of the phononic crystal comprising a layer of steel cylinders followed by a layer of voids at the peak sound absorption frequencies of (a) 510 Hz and (b) 2910 Hz.

performance. The effects of the radius of the voids and steel cylinders on the acoustic performance of the phononic crystal are investigated by separately varying these parameters. Figure 4 presents the sound absorption coefficient of the phononic crystal in which the radius of the voids is varied. A reduction in the radius of the voids moves the first peak to a higher frequency. The amplitude of sound absorption at the first peak reduces slightly; however, the absorption amplitude in the region between the two peaks improves significantly as the void radius is reduced. The frequency as well as the amplitude of sound absorption at the second absorption peak does not significantly change due to a change in the void radius. Figure 5 presents the absorption coefficient of the phononic crystal in which the radius of the steel scatterers is varied. The first peak in the sound absorption coefficient does not vary in frequency or magnitude as the steel cylinder radius is varied. The absorption amplitude at the second peak reduces as the radius of the steel cylinders is reduced.
4. Summary

The sound absorption performance of a phononic crystal comprising a layer of cylindrical voids and a parallel layer of steel cylinders in a soft elastic medium attached to a steel plate has been presented. Results are compared with the same phononic crystal comprising two layers of voids or two layers of steel cylinders. Peaks of high sound absorption are shown to occur attributed to dipole resonance of the steel cylinders and Fabry-Perot resonance due to the presence of the voids. However, one of these peaks does not appear when the phononic crystal consists of two layers of voids, two layers of steel cylinders or when the layer of steel cylinders is placed between the layer of voids and the steel backing plate. The phononic crystal comprising a layer of steel cylinders followed by a layer of voids in the direction of sound propagation is shown to have the best broadband sound absorption performance. For this combination of scatterers, the effect of void radius and steel cylinder radius on the acoustic performance is also investigated.
REFERENCES


