ACTIVE DOUBLE-PANELLED SOUND INSULATING STRUCTURE BASED ON ACTIVE ACOUSTIC BOUNDARY METHOD

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Based on the active acoustic boundary method, the active double-panelled sound insulating structure, which consists of the double-panelled structure and the panel acoustic actuator arranged on the boundary of the air interlayer sound field is presented. Simply supported panels are used to as the active acoustic boundary and the control forces are exerted on the panels. Using the acoustoelasticity theory, a computational model is established to calculate the optimal control forces with the minimum radiated sound power and the minimum air interlayer sound power as control targets. On the basis of established computational model, the sound transmission loss (STL) and the response of each subsystem of the active double-panelled sound insulating structure are studied and the corresponding results before and after controlling are compared. The simulation results indicate that the active acoustic boundary control strategy can effectively improve the sound insulation performance of double-panelled structure and better control effect can be achieved with the control target of minimum radiated sound power compared with that with the minimum air interlayer sound power. The active acoustic boundary has no influence on the vibration response of the upper panel, while the vibration response of the lower panel and the air interlayer sound power are suppressed effectively.

Keywords: Active acoustic boundary method; Acoustoelasticity theory; Sound Transmission loss; Active double-panelled sound insulating structure; Radiated sound power

1. Introduction

As people require better riding comfort, the problem of vibration and sound radiation of shell structures are brought to the forefront. Based on the law of mass action [1], the only factor which influences the sound insulation of a single compact panel is the mass per unit area, and it is more difficult to insulate low frequency sound than high frequency. For high frequency noise, by using the double-panel can improve the effect of sound insulation. But for the very low-frequency noise, the double-panel is not superior to the single panel. This is due to the resonance of the plate-air-plate causes the performance degradation of double-panelled structure [2].

Technically speaking, there are two kinds of noise control method. [3] The passive control method is effective for high frequency noise, but the control effect to low frequency noise is not very good [4]. To Active noise control of double-panelled structure, Pan and Bao[5] theoretically
studied different ways of controlling sound transmission of double-panelled structure, indicated that there were two kinds of mechanism, modal reconstruction and modes suppression. Carneal and Fuller[6] researched on active noise control of double-panelled structure shows that the high stiffness radiation panel and exerting control force on the radiation panel can improve the effect of sound insulation.

Research above shows that in low frequency band, the active noise control can significantly improve the sound insulation performance of structure. However, directly exerting control force causes structural fatigue. On the basis of previous research, a double-panelled structure active control model is established, the active acoustic boundary is simply-supported panel embedded in the air interlayer boundaries and the control forces exert on it. Based on the acoustoelasticity theory[7], the coupling relationship between sub-systems is established. The optimal control force is calculated with the control targets of minimizing the radiated sound power and minimizing the air interlayer sound power. With the help of simulation tools, the feasibility of the control strategy presented in this article is verified.

2. Establishment of sandwich structure model

As shown in Figure 1, The vibration control equation of the upper and lower elastic panels are

\[ D_i \nabla^4 w_i + m_i \frac{\partial^2 w_i}{\partial t^2} - [P_{in} - P_g] = 0 \]  

\[ D_i \nabla^4 w_i + m_i \frac{\partial^2 w_i}{\partial t^2} - [P_g - P_r] = 0 \]

\[ D_i \] is the bending stiffness of the elastic panel, \( m_i \) is the surface density of the elastic panel, \( w_i \) is the vibration displacement, \( i = 1,2 \). \( P_{in} \) is the sound pressure of the incident sound field. \( P_g \) and \( P_r \) are the sound pressure of interlayer sound field and radiated sound field respectively.

These physical quantities meet the following governing equation [8]:

\[ \nabla^2 p_i = \frac{1}{c_0^2} \frac{\partial^2 p_i}{\partial t^2}, \quad i = g, r \]

\[ \text{Active Acoustic Boundary} \]

\[ \text{Simply supported panels} \]

\[ \text{Air Interlayer} \]

\[ \text{Incident wave} \]

\[ \text{Active Acoustic Boundary} \]

\[ \text{Active Acoustic Boundary} \]

Figure 1: Schematic diagram of active double-panelled sound insulating structure based on active acoustic boundary. (a) three-dimensional model; (b) computational model.

Similarly, the vibration governing equations of active acoustic boundary panel are as follows

\[ D_i \nabla^4 w_i + m_i \frac{\partial^2 w_i}{\partial t^2} - [f_{i,r} \delta(y - y_{i,r}, z - z_{i,r}) - P_g] = 0 \]

\[ D_i \nabla^4 w_i + m_i \frac{\partial^2 w_i}{\partial t^2} - [P_g - f_{r,i} \delta(y - y_{i,r}, z - z_{r,i})] = 0 \]
where $D_1$ and $D_r$, $m_i$ and $m_r$, $w_i$ and $w_r$ respectively is the bending stiffness, surface density, cross-sectional area and vibration displacement of the active acoustic boundary panels. $f_{i,c}$ and $f_{r,c}$, $(v_{i,c}, z_{i,c})$ and $(v_{r,c}, z_{r,c})$ respectively is the magnitude and position of the control forces applied on.

The upper and lower elastic panels are simply supported on the sound barriers, which meet:

$$x = 0, a : w_i = w_2 = 0, \frac{\partial^2 w_i}{\partial x^2} = \frac{\partial^2 w_2}{\partial x^2} = 0 \quad (6a) \quad y = 0, b : w_i = w_2 = 0, \frac{\partial^2 w_i}{\partial y^2} = \frac{\partial^2 w_2}{\partial y^2} = 0 \quad (6b)$$

Similarly, the active acoustic boundary panels on the left and right sides are simply supported on the rigid boundary of the air interlayer sound field, which meet:

$$z = z_c - \frac{s_r}{2}, w_i = w_r = 0, \frac{\partial^2 w_i}{\partial z^2} = \frac{\partial^2 w_r}{\partial z^2} = 0 \quad (7a) \quad y = y_c - \frac{s_r}{2}, w_i = w_r = 0, \frac{\partial^2 w_i}{\partial y^2} = \frac{\partial^2 w_r}{\partial y^2} = 0 \quad (7b)$$

$s_y$ and $s_z$ respectively is the geometric dimension of the active acoustic boundary panels on the left and right sides.

On the boundary of structure and sound field, the boundary conditions are:

$$z = 0 : \frac{\partial p_g}{\partial z} = \rho_0 \omega^2 w_i \quad (8a), \quad z = h : \frac{\partial p_g}{\partial z} = \rho_0 \omega^2 w_2 \quad (8b), \quad z = h : \frac{\partial p_g}{\partial z} = \rho_0 \omega^2 w_r \quad (8c)$$

$$x = 0 : \frac{\partial p_g}{\partial x} = \rho_0 \omega^2 w_i \quad (8d), \quad x = a : \frac{\partial p_g}{\partial x} = \rho_0 \omega^2 w_r \quad (8e)$$

At the same time, other boundaries of sandwich cavity are assumed to be acoustic rigid boundaries.

3. Solution of acoustic vibration coupling characteristics of sandwich structure

The upper and lower panels of the double-panelled structure are simply supported, so their vibration displacements can be expressed as simply supported modal function:

$$w_i(x,y,t) = \phi^T(x,y) \alpha_i e^{j(\omega t-k z)} \quad (9a), \quad w_r(x,y,t) = \phi^T(x,y) \alpha_2 e^{j(\omega t-k z)} \quad (9b)$$

The vibration modal coefficient vector of the two elastic panels are $\alpha_i$ and $\alpha_2$, $\phi(x,y)$ is presented as the corresponding simply supported modal function. Similarly, the vibration displacement of the simply supported active acoustic boundary plate can be expressed as

$$w_i(y,z,t) = \psi^T(y,z) \alpha_i e^{j(\omega t-k z)} \quad (10a), \quad w_r(y,z,t) = \psi^T(y,z) \alpha_2 e^{j(\omega t-k z)} \quad (10b)$$

The vibration modal coefficient vector of active acoustic elastic panels on the left and right sides is $\alpha_i$ and $\alpha_2$ respectively, the corresponding simply supported modal function is $\Psi(y,z)$. The sound field distribution of radiated sound field is expressed as the combined form of structural modal function [9]

$$p_g(x,y,z,t) = \phi^T(x,y) \zeta e^{j(\omega t-k z)-\alpha t)} \quad (11)$$

where $\zeta$ is the sound pressure coefficient vector. The sandwich sound field can be expressed in the form of the superposition of modal function:

$$p_g(x,y,z,t) = \Psi_g^T(x,y,z) \mu e^{j(\omega t-k z)} \quad (12)$$

$\Psi_g(x,y,z)$ is the modal function vector of the sandwich sound field, $\mu$ is the corresponding modal vector. $\Psi_g(x,y,z)$ meets the following equation [7]:

$$\nabla^2 \Psi_g(x,y,z) = -\frac{1}{c_0^2} \Omega_g \Psi_g(x,y,z) \quad (13)$$

$\Omega_g$ is the natural frequency matrix of the rigid cavity.

By boundary conditions (8c), we can obtain
\[ \zeta = j \frac{\rho_0 \omega^2}{k_z} a_z \]  

Applying acoustoelastic theory [14], by Green formula, we can obtain

\[ \left[ \psi_g \frac{\partial^2 p_x}{\partial \psi} - \nabla^2 \psi_g p_x \right] V = \int \left( \psi_g \frac{\partial p_x}{\partial n} - \frac{\partial \psi_g}{\partial n} p_x \right) dS \]  

\( V_g \) is the volume of the cavity and \( V_g \) is surrounded by \( \hat{S} \). \( n \) is the outward normal of rigid cavity and we take outward direction positive. Substitute (3), (8a), (8B), (8D), (8e), (12) and (13) in (15), and introduce the acoustic damping coefficient \( \chi_3 \), we can obtain

\[ L_x \mu = -T_{a_3} a_3 + T_{g_3} a_3 + T_{r_3} a_r \]  

As to upper panel of the double-panelled structure, substitute (9a) in (1), using the orthogonality of vibration modal function, introduce the structural damping coefficient \( \chi_1 \), gives

\[ M_1 \left( \Omega^2 - j \chi_1 \omega \Omega_1 - \omega^2 I \right) a_1 + \Gamma_{1,g} \mu = F \]  

let

\[ Z_{1,g} a_1 + \Gamma_{1,g} \mu = F \]  

Then

\[ Z_{1,g} a_1 + \Gamma_{1,g} \mu = F \]  

Similarly, substitute (9b) in (2), using the orthogonality of vibration modal function, introduce the structural damping coefficient \( \chi_2 \), we can obtain

\[ \left[ M_2 \left( \Omega^2 - j \chi_2 \omega \Omega_2 - \omega^2 I \right) + j \frac{\rho \omega^2}{k_z} \Gamma_{2,v} \right] a_2 - \Gamma_{2,v} \mu = 0 \]  

let

\[ Z_{2,v} a_2 - \Gamma_{2,v} \mu = 0 \]  

Then

\[ Z_{2,v} a_2 - \Gamma_{2,v} \mu = 0 \]  

For the Active acoustic boundary plate, Similarly, substitute (10a) in (4), using the orthogonality of vibration modal function, introduce the structural damping coefficient \( \chi_2 \), we get

\[ M_1 \left( \Omega^2 - j \chi_2 \omega \Omega_2 - \omega^2 I \right) a_1 + \psi(y_1, z_1) f_{1,v} + \Gamma_{1,v} \mu = 0 \]  

let

\[ Z_{1,v} a_1 + \Gamma_{1,v} \mu = \psi(y_1, z_1) f_{1,v} \]  

Then

\[ Z_{1,v} a_1 + \Gamma_{1,v} \mu = \psi(y_1, z_1) f_{1,v} \]  

Similarly, substitute (10b) in (5), using the orthogonality of vibration modal function, introduce the structural damping coefficient \( \chi_r \), gives

\[ M_r \left( \Omega^2 - j \chi_r \omega \Omega_r - \omega^2 I \right) a_r - \Gamma_{r,v} \mu + \psi(y_r, z_r) f_{r,v} = 0 \]  

let

\[ Z_{r,v} a_r - \Gamma_{r,v} \mu = -\psi(y_r, z_r) f_{r,v} \]  

Then

\[ Z_{r,v} a_r - \Gamma_{r,v} \mu = -\psi(y_r, z_r) f_{r,v} \]  

4. **Optimal control force**

For the active noise control of the double-panelled structure, the control purpose is to improve the sound insulation performance and to reduce the radiation noise that structure radiates to the
surroundings. In terms of the transmission path of acoustic wave in the sandwich structure, it can be considered from the following two aspects:

(1) Reducing the acoustic power of the radiated sound field by directly minimizing the radiated sound power.

(2) Cutting off the transmission path of acoustic wave and reduce the sound power in the acoustic transmission path.

### 4.1 Optimal control force with the control target of minimum radiated sound power

From equation (18),(22)and(24),we can obtain

\[ a_t = Z_{t, s}^{-1} F - Z_{t, s}^{-1} F \Gamma_{t, s} \mu \] (25a)

\[ a_r = Z_{r, s}^{-1} \psi(y_{r, r}, z_{r, r}) f_{r, r} - Z_{r, s}^{-1} \Gamma_{r, s} \mu \] (25b)

\[ a_r = -Z_{r, s}^{-1} \psi(y_{r, r}, z_{r, r}) f_{r, r} + Z_{r, s}^{-1} \Gamma_{r, s} \mu \] (25c)

Substitute (25) in (16), gives

\[ \mu = -Z_{t, s}^{-1} \Gamma_{t, s} f + Z_{t, s}^{-1} \Gamma_{t, s} \] (26)

Substitute (26) in (20), we get

\[ a_{\mathbf{z}} = -Z_{t, s}^{-1} \Gamma_{t, s} f + Z_{t, s}^{-1} \Gamma_{t, s} \] (27)

Then formula (27) can be simply expressed as

\[ a_{\mathbf{z}} = \Xi \mu + \Xi \mathbf{f} \] (28)

Therefore, the radiation pressure of the radiated sound field can be obtained by (11) and (28).

\[ p_r(x, y, z) = \int \frac{\rho_0 c_0}{2S} \phi^*(x, y, z) \Xi \mu + \Xi \mathbf{f} \] (29)

The average radiated sound power of the radiated sound field is defined as

\[ W_r = \frac{1}{2S \rho_0 c_0} \left( \int p_r p_r^* dS \right) \] (30)

Substitute (29) in (30), we can get

\[ W_r = c_A + f_c^H \mathbf{b}_A + \mathbf{b}_A^H f_c + f_c^H \mathbf{a}_A \mathbf{f}_c \] (31)

From (31), we can obtain the optimal control force

\[ f_c = -a_A^{-1} \mathbf{b}_A \] (32)

Then, the minimum value of the average radiated sound power of the radiated sound field is equal to

\[ W_{r, o} = c_A - \mathbf{b}_A^H a_A^{-1} \mathbf{b}_A \] (33)

### 4.2 The optimal control force with the control target of minimum sound power of sandwich sound field

From formula (18),(20),(22),(24), we can get

\[ a_t = Z_{t, s}^{-1} F - Z_{t, s}^{-1} F \Gamma_{t, s} \mu \] (34a)

\[ a_r = Z_{r, s}^{-1} \psi(y_{r, r}, z_{r, r}) f_{r, r} - Z_{r, s}^{-1} \Gamma_{r, s} \mu \] (34c)

\[ a_r = -Z_{r, s}^{-1} \psi(y_{r, r}, z_{r, r}) f_{r, r} + Z_{r, s}^{-1} \Gamma_{r, s} \mu \] (34d)

Substitute (34) into (16), then

\[ \mu = -Z_{t, s}^{-1} \Gamma_{t, s} f + Z_{t, s}^{-1} \Gamma_{t, s} \psi(y_{r, r}, z_{r, r}) f_{r, r} - Z_{r, s}^{-1} \Gamma_{r, s} \psi(y_{r, r}, z_{r, r}) f_{r, r} \] (35)

Then formula (35) can be simply expressed as

\[ \mu = \Xi \mu + \Xi \mathbf{f} \] (36)

Therefore, Sound pressure distribution of the interlayer sound field can be obtained by formula (12) and formula (36):

\[ p_s(x, y, z) = \psi^*(x, y, z) \Xi \mu + \Xi \mathbf{f} \] (37)

Similarly, the average sound power of the interlayer sound field is defined as
\[ W_g = \frac{1}{2V_g \rho c_0} \text{Re} \left( \int_{V_g} p_g p_g^* dV \right) \] (38)

Substitute (37) into (38), we can get
\[ W_g = c_A + f_c^H b_A + b_A^H f_c + f_c^H A f_c \] (39)

The optimal control force obtained by formula (39) is
\[ f_c = -a_A^{-1} b_A \] (40)

The minimum value of sound power of the air interlayer sound field is equal to
\[ W_{g,\sigma} = c_A - b_A^H a_A^{-1} b_A \] (41)

Assuming the average sound power of the incident sound field is \( W_i \), The average sound power of radiated sound field is \( W_r \), The average sound power of sandwich sound field is \( W_g \). We define the transmission loss of structure as
\[ \text{STL} = -10 \log \left( \frac{W_r}{W_i} \right) \] (42)

We define the average vibrational kinetic energy of the structure as
\[ E_i = \frac{1}{2S_i} m_i \text{Re} \left( \int_{S_i} v_i v_i^* dS \right) \] (43)

In the formula, \( i \) can be taken as the incident panel (upper panel), i.e. the average kinetic energy \( E_{up} \), or the average kinetic energy \( E_{low} \) of the lower panel or the average kinetic energy of the left \( (E_{c,l}) \) and right \( (E_{c,r}) \) boundary panels.

5. Simulation Analysis

In order to verify the feasibility of the control strategy and explore the control mechanism, a simulation model is set up in Figure 1. The control force is applied \( (v_{c,r}, z_{c,r}) \) and \( (v_{c,l}, z_{c,l}) \) are both \( (0.25s_y, 0.25s_z) \). The computing wave band of the whole system is 0~500Hz.

Fig. 2 shows the sound transmission loss curve of double-panelled system before and after controlling. From the figure we can see, in the range of 0~70Hz and 220~500Hz, Both control strategies improve the sound insulation performance of the double-panelled structure. But taking the sound power as the control target is better than the air interlayer sound power.

![Figure 2: Comparison of STL with different control targets](image)

In order to illustrate the change of energy in each part of the system (upper and lower panel, air interlayer sound field and radiated sound field) before and after controlling, Fig. 3~ Fig. 8 is respectively obtained. It can be seen from Fig. 3 that the change of kinetic energy of the upper panel is very small, which indicates that the active acoustic boundary panel has little influence on the vibration performance of the upper panel. In Fig. 4, the vibration of the radiation panel is effectively suppressed after control. In Fig. 5, before and after control, the distribution of the interlayer sound...
field changes obviously, the control effect of interlayer sound power under these two control targets is similar. The kinetic curve of left and right panels in the same control targets in Fig.6 and Fig.7 are different, it is due to the acoustic field oblique incidence, especially the influence of azimuth, which makes the whole system not completely symmetrical.

From formula (11), the sound pressure of radiated sound field is expressed by the vibration modal of radiation panel. It can be seen from Fig.(4) and Fig.(8) that the sound power of radiated sound field is approximately as same as the kinetic energy of the radiation panel. Compare with taking minimum sandwich sound power as control target, the control effect of minimum radiated sound power is better which is shown from the fact that the kinetic energy and radiation sound power of radiation structure is smaller.

Figure 3: Comparison of vibrational kinetic energy of the upper panel of simply supported double-panelled systems with different control targets.

Figure 6: Comparison of vibrational kinetic energy of the left panel considered as the active acoustic boundary of simply supported double-panelled systems with different control targets.

Figure 4: Comparison of vibrational kinetic energy of the lower panels of simply supported double-panel systems with different control targets.

Figure 7: Comparison of vibrational kinetic energy of the right panel considered as the active acoustic boundary of simply supported double-panelled systems with different control targets.

Figure 5: Comparison of sound power of the air interlayer sound field of simply supported double-panelled systems with different control targets.

Figure 8: Comparison of sound power of the radiated sound field of simply supported double-panelled systems with different control targets.
6. Conclusion

This paper presents active acoustic insulation double-panelled structure based on active acoustic boundary method. Firstly, the Calculation model of double-layer plate active sound insulation structure based on active acoustic boundary is established, that is active acoustic boundary is arranged on the air interlayer boundary of double-panelled structure. In this paper, simply supported plate is used to substitute the active acoustic boundary and the control force is applied on it. The coupling relation among subsystems is derived by using acoustoelasticity theory. The optimal control force is obtained by taking the minimum radiated sound power and the minimum sound power of the interlayer sound field as control targets. The following conclusions are obtained by calculation and analysis: The active acoustic boundary control strategy can effectively improve the sound insulation performance of the double-panelled structure, especially in low frequency band. Taking the minimum radiated sound power as control target is better than the minimum sound power of the interlayer sound field. After control, the active acoustic boundary has little effect on the vibration response of the incident panel. The vibration kinetic energy of the radiation panel and the sound power of the air interlayer sound field are effectively suppressed. In future work the effect of the dimensions of the active acoustical boundary on STL and responses of the subsystems can be taken into account.

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