1. Introduction

During the operation of the ship, the abnormal vibration noise is inevitable. The theoretical core of detecting the abnormal vibration noise of the ship is to find out the main noise source or transmission path acoustic fault [1].

In the past, many methods have been proposed to detect damages in the structure, e.g., modal parameter method, frequency response function method and etc. However, these methods rely on the finite element model to simulate the numerical model accurately, which makes it difficult to detect damage online. Recently, damage detection using transmissibility function has been put forward. This method can directly utilize structural response data, and can realize online detection, which has obvious advantages in large structure testing.

Transmissibility function relationship between frequency response functions was analyzed by Liu and Ewins [2], where transmissibility function was defined as the ratio of frequency response functions of two point for multi-degree-of-freedom system. A more generalized transmissibility concept was proposed as a powerful tool to modal analyze model by [3]. Translational and curvature transmissibility functions were adopted to detect and locate damages on a cantilever beam by [4]. Moreover, the research demonstrated that more accurate damage detection could be achieved by choosing a reasonable frequency range of transmissibility function. In addition, transmissibility function were analytically derived by [5] for detecting and locating damage in...
linear and nonlinear structures. The research by [6] demonstrated that transmissibility function analysis was able to detect a single bolt loosening. Furthermore, Kess and Adams [7] analyzed the influence of operational and environmental variability on the damage indicator, and the results showed that the accuracy and reliability of transmissibility function analysis could be improved by identifying specific frequency ranges. Recently, Yi and Zhu [8,9] developed a mobile sensing system which is capable of maneuvering on the surface of ferromagnetic materials. Transmissibility function analysis was embedded in mobile sensing nodes; using data collected by mobile sensing nodes, on-board computation was successfully conducted to detect damage on a steel frame.

This paper is structured as follows. In Section 2, the theory of transmissibility function and damage detection methods are described. In Section 3, we verify the whole process of damage detection by simulation. Section 4 describes the experimental setup, and proves that transmissibility function can be used for actual damage detection. Finally, the conclusion drawn from this study are summarized in Section 5.

2. Theory

The dynamic equilibrium equation of a linear multiple-degree-of-freedom system is given by the well-known second-order differential equation:

\[ M \ddot{x}(t) + C \dot{x}(t) + K x(t) = f(t). \]  

(1)

Where M, C, and K are the mass, damping, and stiffness matrices of the system, respectively. \( f(t) \) is the input force vector, and \( x(t) \) contains the responses of each degree-of-freedom of the system. Transforming to the frequency domain leads to the following frequency response function:

\[ X(\omega) = H(\omega) F(\omega). \]  

(2)

With

\[ H(\omega) = (K - \omega^2 M + i\omega C)^{-1}. \]  

(3)

The direct transmissibility between point i and a reference point j is defined as

\[ T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} . \]  

(4)

where \( X_i(\omega) \) and \( X_j(\omega) \) are the responses measured at DOF i and DOF j.

From Eq. (2), transmissibility function can be derived as:

\[ T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{H_{ik}(\omega) F_k(\omega)}{H_{jk}(\omega) F_k(\omega)} = \frac{H_{ik}}{H_{jk}} . \]  

(5)

Since the dynamic responses of structures change in the case damage occurs it is natural to expect that also transmissibility function will change if damage occurs.

This paper makes further data analysis on transmissibility function via principal component analysis. Using available transmissibility function, matrix \( [T]_{M \times N} \) be formed which has M rows of transmissibility function, each with N measure point:

\[ T = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & t_{22} & \cdots & t_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \cdots & t_{nm} \end{bmatrix} . \]  

(6)

Each column of transmissibility function matrix \( T(t) \) is centralized and the covariance matrix is obtained.
\[ S = \frac{1}{(n-1)} * T^T T \]  
Where \( T \) is the matrix after centralizing. Then, each of principal component of transmissibility function can be calculated: \( y_1, y_2, \ldots, y_m \)

Their corresponding eigenvectors are given: \( \lambda_1, \lambda_2, \ldots, \lambda_m \)

The loading of each of principal component can be calculated from :

\[ p = \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i} \]

(8)

In general, the first few principal component contains most of the original data information. By utilizing the first few principal component, principal component assurance criterion can be defined to visually and effectively judge the damage of the structure.

\[ PCAC = \frac{\left( \sum_{i=1}^{n} y_i^T y_i^m \right)^2}{\left( \sum_{i=1}^{n} y_i^T y_i \right) \left( \sum_{i=1}^{n} y_i^m y_i^m \right)} \]

(9)

Where \( y_i \) and \( y_i^m \) are principal component of transmissibility function in healthy and measurement state.

If PCAC=1, the structural is in a healthy state, if PCAC<1, damage occurred. Considering the noise signals, the threshold of principal component assurance criterion equal to 0.9 [10], and the structure is in a healthy state when PCAC>0.9. When PCAC<0.9, damage occurred.

3. Simulation

3.1 Numerical model

Fig. 1 shows the finite element model in ANSYS software to simulate 10mm thick steel plate of ship structure, the concrete material properties adopted in the model: elastic modulus is 2.07 \times 10^{11} \text{ N/m}^2, density is 7800 \text{ kg/m}^3, the poisson's ratio is 0.3. For simulating actual deformation of ship structure, applying different force in the center of the disc to produce certain deformation. Four different forces are applied to the numerical model, and the maximum deformation are 1.3mm, 1.9mm, 2.5mm and 3.0mm. Then, the deformation model is analyzed to obtain the frequency response function of relevant points on the plate. As shown in the Fig. 1(a), an excitation is applied at location 0. Acceleration records are collected at location 1,2,3,4,and 5.
3.2 Transmissibility function analysis

In practical engineering, noise influence on measurement results. Considering the effect of noise for measurement results, Random White Gaussian Noise is obtained through MATLAB, and it is added to the frequency response function to simulate the influence of noise in the actual process. In healthy state, namely, under the condition of undeformation, the acceleration transmissibility function $T^i_j$ of the adjacent two points is calculated. Then transmissibility function matrix $T = \begin{bmatrix} T^1_2 & T^2_2 & T^3_2 & T^4_2 \\ T^1_3 & T^2_3 & T^3_3 & T^4_3 \\ T^1_4 & T^2_4 & T^3_4 & T^4_4 \end{bmatrix}$ is composed, and the principal component analysis is carried out. The loading of each principal component is obtained, as shown in Fig. 2.

Fig 1: (a) Numerical model after deformation; (b) The position of the measurement points on the model

Fig 2: The loading of each principal component
The same process to do to obtain the result of principal component analysis under several different deformation state. Through analysis, the loading of the first two principal component has a larger percentage, which means a lot of raw information is contained. Two-dimensional PCA scatter-plot is made by utilized the first two principal component. Considering that the peak value of the transmissibility function is quite different from the trough, the principal component data is converted to dB.

In healthy state, the first principal component is the horizontal axis, the second principal component is the vertical axis, and scatter diagram is made by two principal components. In the same way, the scatter plot of principal components can be made under different deformation state. Then the scatter plots under different deformation are made in the same coordinate with the scatter plots in the healthy state, and the results are shown in Fig. 3.

Fig 3 : Damage detection under different case

The principal component assurance criterion (PCAC) of transmissibility function matrix under different deformation is calculated to make a more intuitive judgment on whether the structure is damaged. The results are shown in Table 1.
Table 1 : The principal component assurance criterion (PCAC)

<table>
<thead>
<tr>
<th>Deformation</th>
<th>1.3mm</th>
<th>1.9mm</th>
<th>2.5mm</th>
<th>3.0mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCAC</td>
<td>0.6671</td>
<td>0.6341</td>
<td>0.6040</td>
<td>0.5902</td>
</tr>
</tbody>
</table>

As we can see, the values in table are less than the threshold (i.e. 0.9). At the same time, with the increase of the deformation, the values are more smaller. Therefore, it is effective to detect damage by using transmissibility function from simulation.

4. Experimental example

Experiments are carried out to validate the proposed method in this subsection. In the experiment, the vibration signal measured is analyzed by the transmissibility function, and the damage indicator is used to judge the damage.

4.1 Experimental setup

Fig. 4 shows a schematic of the experimental setup. It consists of a circular steel plate with a thickness of 1.6mm and a radius of 290mm. The circular steel plate is rigidly clamped with two steel flange plates with a thickness of 32mm, and the two flanges are clamped together using ten 24M steel bolts. The entire assembly was suspended from two steel eyes. A shock-setup was installed at the side of the disc to generate different degree of damage by adjusting the swing angle of shock-setup. In order to cause obvious damage to the disk, four damage cases with a larger interval angle between each other to be done i.e. 20°, 30°, 35°, 40°. As shown in Fig. 4, five acceleration sensors were installed at the other side of the disc. After each shock, the disc was knocked by hammer and the vibration signal of corresponding points was recorded by acceleration sensors.

Fig 4: The whole experimental setup
**4.2 Transmissibility function analysis**

The experimental frequency variation of test point acceleration for the first 1000Hz was used as the analysis signal of transmissibility function. In healthy state, the acceleration transmissibility function of two adjacent points on the disk is calculated as a matrix $T_i$, i.e. $T_2^1$, $T_3^2$, $T_4^3$ $T_5^4$. Next, the detailed analysis process is consistent with Section 3, The scatter plot of different shock angles is shown below:

Fig 5: Damage detection under different case

Table 2 is the principal component assurance criterion (PCAC) value calculated by using principal component analysis in various impact angle. According to the data in the Table 2, value are less than the threshold(i.e.0.9). Moreover, the values are more small with the increase of the deformation. So it is effective to detect damages by using the transmissibility function.

**Table 2: The principal component assurance criterion (PCAC)**

<table>
<thead>
<tr>
<th>Different shock</th>
<th>20°</th>
<th>30°</th>
<th>35°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCAC</td>
<td>0.4209</td>
<td>0.3677</td>
<td>0.3627</td>
<td>0.3594</td>
</tr>
</tbody>
</table>
5. Conclusion

In this paper, a method of path fault detection based on transmissibility function is proposed and the results show that:

1) According to the damage indicator in this paper, the transmissibility function can be used to detect some path faults.

2) As the damage become serious, the damage indicator shows certain regularity. As a result, the damage indicator taken in this paper can reflect the change of damage to a certain extent.

3) The method proposed in this paper is based on the measured vibration response signal. It only needs to obtain the response signal of the structure under random excitation, and the method is simple and suitable for online monitoring.

In this paper, the transmissibility function is used to detect damage, but it is necessary to discuss the damage quantification and the detection of complex structural damage under the certain condition of limited sensors.

References

7 Kess, H., D.Adams, Investigation of operational and environmental variability effects on damage detection algorithms in a woven composite plate, Mechanical Systems and Signal Processing.21(6)(2007)2394-2405.