ACTIVE ACOUSTIC METASURFACE WITH AN ARRAY OF PIEZOELECTRIC MEMBRANES AND LOCAL CONTROL OF ACOUSTIC IMPEDANCE

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It is known that acoustic metasurfaces (AMS) represent powerful tools which can be used for the construction of impedance or phase gratings to control the reflected sound field. The main drawback of AMSs resides in the narrow operational frequency range, which can be avoided using the active control of acoustic impedance of the particular unit cell of the AMS. This can be achieved by introducing the curved piezoelectric membrane shunted by an active electronic circuit as a unit cell of the AMS according to the technique known as the active elasticity control. It is natural that large planar AMSs require the construction of arrays with a large number of actively tuned electroacoustic resonators. This puts considerable requirements on the complexity of the control electronics. In this paper, we introduce a novel concept of the electronic control of the acoustic impedance in the large array of actively tuned electroacoustic resonators by means of the methods of Active Matrix Addressing and Pulse Amplitude Modulation. We demonstrate the applicability of these two methods to control the acoustic properties of AMS by means of numerical simulations.

Keywords: active acoustic metasurface, piezoelectric membrane, elasticity control, acoustic impedance, discrete control

1. Introduction

Acoustic metamaterials and metasurfaces (AMS) represent perspective candidates for an efficient suppression of noise transmission [1,2,3]. The resonant nature of the AMSs as well as their enhanced coupling to incident sound waves give rise to their narrow-band functionality [4]. The limitation of the narrow-band operation can be avoided by the replacement of the passive acoustic structures in the acoustic resonators by tunable acoustic elements, which yields the construction of active acoustic metasurfaces (AAMS). The tunable acoustic elements can be fabricated as electroacoustic transducers, e.g., piezoelectric membranes [5,6,2] or electrodynamic loudspeakers [7]. These can be connected to passive [3] or active [2,7] electrical networks.

The effect of the active electrical network, which is connected to the piezoelectric membrane (PM), can be explained according to the method of active elasticity control (AEC) by Date et al. [8]. The elastic properties of piezoelectric actuators can be controlled to a large extent by means of the active shunt circuit, which has a negative capacitance. Since the elastic properties of the PM have an
influence on the specific acoustic impedance of the PM, the AEC serves as an efficient tool for the control of the acoustic impedance of the AAMS. If one manages to precisely tune the acoustic impedance independently over the whole area of the AAMS, it is possible to construct acoustic impedance gratings (or acoustic phase gratings), which can be used for the control of the sound field [9].

This approach has, however, some restrictions. The adjustment of the elastic properties requires a precise control of the negative capacitance of the shunt circuit. Any discrepancy from the required value of the connected external capacitance results in the failure of the operation of the device. In addition, each piezoelectric transducer in the array of AAMS cells requires its own independent active shunt circuit, which results in an enormous complexity of the electronic components of the system.

The aforementioned issues have motivated the work presented in this paper. We propose an efficient approach to control the elastic properties of piezoelectric membranes in the array of cells in the AAMS. Our approach is based on the method of Active Matrix Addressing (AMA) and the Pulse Amplitude Modulation (PAM) of the driving signal. The main objective of this work is to give an evidence by means of numerical simulations that PAM offers a method for the control of the required charge on PM electrodes, which is sufficiently accurate for the application within the AEC.

2. Methods

In this section, the application of AMA and PAM to the AEC of the piezoelectric membrane is introduced.

2.1 Active matrix addressing in the acoustic metasurface

Figure 1 shows the top and bottom views of the AAMS, which consists of an array of the tunable acoustic resonator (TAR) cells, which employ the piezoelectric membrane (PM) as the electroacoustic transducer. The array of the TAR cells consists of $m$ rows and $n$ columns. The number of connectors needed to address every PM in the particular cell is equal to $m + n$. In order to drive each PM in the AAMS independently, each PM is connected to a simple switch-device, which delivers free charges to the PM electrodes actively at particular times.

The connectors of the switching devices in the array of TAR cells are connected with the active shunt circuit through a multiplexer. The active shunt circuit measures the voltage on the electrode of the PM in the particular TAR cell and delivers charges, which correspond to the required value according to the AEC method. Depending on the number of the TAR cells in the AAMS, the charges must be delivered to the particular electrode with a duty cycle, which is inversely proportional to the number of the TAR cells in the AAMS. Therefore, the PAM is used for driving the PM in the TAR
2.2 Lumped model of the metasurface cell

Figure 2 shows the lumped model of a single PM in the TAR cell of the AAMS. The dynamics of the PM in the considered frequency range is identical with the simple spring-mass system, where the spring is replaced by the piezoelectric actuator. The equation of motion of the system is given by

$$m \frac{d^2}{dt^2} [\Delta l(t) + u_{vib}(t)] = -F(t) - b \frac{d}{dt} \Delta l(t),$$

(1)

where $\Delta l(t)$, $u_{vib}(t)$, $-F(t)$, $m$, and $b$ are the elongation of the piezoelectric membrane, displacement of the source of vibration, force applied from the piezoelectric actuator, mass of the object, and the damping coefficient of the piezoelectric actuator due to internal mechanical losses, respectively. The coupling between the force $F(t)$ and the elongation of the piezoelectric membrane $\Delta l(t)$ is given by the piezoelectric constitutive equations

$$Q = d_{31} \frac{l}{h} F + \varepsilon_{33}^T \frac{A}{h} U,$$

(2)

$$\Delta l = s_{11}^E \frac{l^2}{A h} F + d_{31} \frac{l}{h} U,$$

(3)

where $\varepsilon_{33}^T$, $s_{11}^E$, and $d_{31}$ are the particular components of permittivity, elastic stiffness, and piezoelectric coefficients tensors, respectively. The symbols $l$, $h$, and $A$ stand for the length of the PM along the direction of its deformation, thickness of the PM, and the area of the electrodes, respectively. The driving force for the PM vibration is given by the difference of acoustic pressures at the both sides of the AAMS and it is schematically indicated by the black zig-zag line. The PM is connected to the active external electronic circuit, which has a negative capacitance equal to $-C$.
By combining Eqs. (1)-(4), we obtain the following set of equations of motion

\[ \Delta l''(t) + \delta \Delta l'(t) + \omega_r^2 \Delta l(t) = -\xi Q(t) - u''_{\text{vib}}(t), \]  
\[ Q'(t) = i(t), \]

where

\[ \omega_r = \frac{Ah}{l\sqrt{m s_{11}\varepsilon_{33} A (1 - k^2)}}, \]
\[ \delta = \frac{b}{m}, \]
\[ \xi = \frac{h}{ml (1 - k^2)} \sqrt{\frac{k^2}{s_{11}\varepsilon_{33}}}, \]
\[ k^2 = \frac{d^2}{s_{11}\varepsilon_{33}}. \]

The voltage on the PM electrodes can be expressed in the form

\[ U(t) = m\xi \Delta l(t) + Q(t) \left( \frac{h}{\varepsilon_{33} A} + \frac{m \omega_r^2}{\xi} \right). \]

When the PM is connected to the external circuit of the capacitance \( C \), there is a coupling between the charge and voltage on the PM electrodes given by

\[ Q(t) = -CU(t). \]

### 3. Results

In order to achieve the key objective of this paper, i.e., to demonstrate that it is possible to construct the acoustic phase grating using an array of PM which are controlled with a single active shunt circuit, we proceed in two steps. In the first step, we calculate analytically the optimal time dependence of the charge \( Q(t) \), which yields the situation, when the vibration of the PM \( \Delta l(t) \) is virtually equal to zero. This situation corresponds to the perfect sound shield, which is characterized by the infinite acoustic impedance of the particular PM. Further analysis presented in Ref. [2] reveals that it is possible to change the phase of the transmitted sound wave around this operational point from negative to positive values. In the second step, we perform the exact numerical simulation of the response of the PM under the simultaneous excitation by the source of vibration \( u_{\text{vib}}(t) \) and the compensating electrical driving by means of the charge \( Q(t) \).
3.1 Required time dependence of charge

Assume that the state quantities $\Delta l(t), F(t), U(t), Q(t), F(t)$, and $u_{vib}(t)$ have the harmonic time dependence with the complex amplitudes $\Delta l_0, F_0, U_0, Q_0, F_0$, and $u_{vib,0}$, respectively. By the substitution of the harmonic time dependencies into the equations of state (5), one immediately gets the solution in the form

$$\Delta l_0 = -\frac{l^2 m \omega^2}{A \varepsilon_{33}^T + A \varepsilon_{33}} \left(\frac{d_{31}^2 - s_{11}^E \varepsilon_{33}^T}{d_{31}^2 - s_{11}^E \varepsilon_{33}} + Ch s_{11}^E\right) u_{vib,0},$$  \hspace{1cm} (11)

$$Q_0 = -\frac{h l m \omega^2 A c d_{31}}{A \varepsilon_{33}^T + A \varepsilon_{33}} \left(\frac{d_{31}^2 - s_{11}^E \varepsilon_{33}^T}{d_{31}^2 - s_{11}^E \varepsilon_{33}} + Ch l^2 s_{11} \varepsilon_{33} \omega (m \omega - ib)\right) u_{vib,0},$$  \hspace{1cm} (12)

It follows from Eq. (11) that, when the external capacitance $C$ is equal to

$$C = -\varepsilon_{33}^T (1 - k^2) \frac{A}{h},$$  \hspace{1cm} (13)

the amplitude of the PM vibration $\Delta l_0$ is equal to zero. By the substitution of Eq. (13) into Eq. (12), we obtain the optimal amplitude of charge on the PM electrodes, which yield the perfect sound shield

$$Q_{0,\text{opt}} = \frac{lm \omega^2 (s_{11}^E \varepsilon_{33}^T - d_{31}^2)}{d_{31} h} u_{vib,0}.$$  \hspace{1cm} (14)

3.2 Numerical simulation

The optimal time dependence of charge $Q(t)$ for the considered harmonic time dependence of the vibration/acoustic source, which was calculated in the above section, was approximated by means of the PAM driving current $i(t)$. Figure 3 shows the results of the numerical solution of the system of differential equations Eqs. (5) and (6). Figure 3 shows the normalized values of the charge on the PM electrodes for four different driving currents $i(t)$: disconnected PM electrodes (black), continuous drive (red), pulsed amplitude modulation driving current with the duty cycle 1/35 and with 20 and 80 rectangular pulses per period (purple) and (turquoise), respectively. The green curve indicates...
the pulsed amplitude modulation driving current with duty cycle 1/16 and with 80 rectangular pulses per period. Figure 4 shows normalized displacement of the PM in the corresponding situations of 4 driving currents.

Our simulations indicate that the key parameter, which is responsible for the efficiency of the vibration control, is the number of samples per period of the driving signal. Figure 4 clearly indicates that the duty cycle of the driving current has no effect on the vibration control efficiency. On the other hand, the extremely small values of duty cycle may produce implementation difficulties due to the increase in the peak current values of the driving signal.

4. Conclusion

We have proposed an efficient approach to control the elastic properties of piezoelectric membranes in the array of cells in the AAMS. Our approach has been based on the method of Active Matrix Addressing (AMA) and the Pulse Amplitude Modulation (PAM) of the driving signal. We have successfully demonstrated that it is possible to achieve the control of the acoustic impedance at the arbitrary cell of the AAMS by means of the AMA and the pulse amplitude modulation.

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