In order to solve the problem of multi-dimensional vibration isolation, the multi-dimensional vibration isolation platform based on the improved 3-RR^U parallel mechanism is obtained by replacing cylindrical pair with parallelogram structure in the traditional 3-RRC parallel mechanism. The kinematics and the dynamics of multi-dimensional vibration isolation platform is analyzed. The parameter sensitivity analysis of the natural frequency is performed based on the dynamic model which is more precise than previous research because both the mass of links and the stiffness of joints are taken into account in the derivation of the dynamic model. The results indicate that the mass of links and the stiffness of joints have important influences on the natural frequencies of the vibration isolation platform. The researches in this paper can provide the reference for the design and optimization for the multi-dimensional vibration isolation applications based on the parallel mechanism.

Keywords: parallel mechanism; sensitivity; natural frequency

1. Introduction

The parallel mechanisms have large stiffness, high precision and many degrees of freedom. They can be used to isolate the multi-dimensional vibration problems, which can be extensively observed in many mechanical systems. Many researches have been conducted to investigate the relevant applications. Xu used the parallel mechanism damping device as the stretcher of ambulance [1]. Wu made use of the parallel mechanism to design the shock-absorbing seat [2]. Fu regarded the parallel mechanism as the basis of the spacecraft vibration platform [3]. Liu designed a frequency modulated vibration damping platform which is based on the 3-PRC parallel mechanism [4]. Among many kinds of parallel mechanisms, 3-RRC parallel mechanism is an important candidate which have been used for vibration isolations with three translational degrees of freedom. Yin analyzed the workspace of 3-RRC parallel mechanism [5]. Zhou designed a novel algorithm to determine the workspace 3-RRC parallel mechanism [6]. Guo analyzed the mechanism accuracy of 3-RRC parallel robot [7].

Although 3-RRC parallel mechanism is widely used, there exist some drawbacks on the 3-RRC parallel mechanism. Therefore, the 3-RR^U parallel mechanism is put forward by improving the traditional 3-RRC parallel mechanism. Compared with 3-RRC parallel mechanism, 3-RR^U parallel mechanism has better mechanical properties and is more suited to be used as the base for the vibration isolation platform. In the previous research on the 3-RR^U parallel mechanism, the effects of the links and other factors on the dynamic characteristics are ignored for the sake of simplification. However, the mass of links can be important when small or miniature vibration isolation systems are to be analyzed. In addition, in practical engineering, the torsion springs can be installed at the different locations to
improve the stiffness of the mechanism. The installation of the torsion springs at the different position have not been theoretically studied yet.

This paper makes use of the sensitivity to study the influences of the link mass and position of the torsion on the natural frequencies. The results provide theoretical reference and basis for the design of parallel vibration isolation mechanism and multi-dimensional vibration reduction application.

2. Improved 3-RR^U parallel mechanism

As shown in Fig. 1(a), the configuration of 3-RRC parallel mechanism is composed of the upper (moving) platform, the lower (static) platform and three branch chains connecting to the upper platform and the lower platform. There are two connecting rods on each chain. The kinematic pairs are R pair, R pair and C pair from bottom to top for every chain. The 3-RRC parallel mechanism has three translational degrees of freedom.

As the spatial distribution of three branches of the mechanism is asymmetrical, it is easy to cause unevenness when the platform applied with the loads. Moreover, the C pair is easy to produce self-locking. In order to improve the mechanical performance of the mechanism, the C pairs of the branches of the mechanism can be replaced by the parallelogram structures, and the three branched chains are arranged symmetrically between the upper and lower platforms. An improved 3-RR^U parallel mechanism is obtained, as shown in Fig. 1(b). The mechanism is symmetrical in space. Since all kinematic pairs are rotational pairs, the mechanism’s positive and negative load transfer performance is improved. Also, the mechanism is not easy to produce self-locking phenomenon. 3-RR^U parallel mechanism can be used to build multi-dimensional vibration isolation system.

![Schmetica diagram of 3-RRC](image1)

![Schmetica diagram of 3-RR^U](image2)

Figure 1: Configurations of 3-RRC and 3-RR^U parallel mechanism

3. Analysis of kinematics and dynamics

As shown in Fig. 2, the coordinate system $O - XYZ$ is set up at the center of parallel mechanism static platform (bottom platform). The $X$ axis points to one of the rotating subcenter of the static platform. The $Z$ axis is vertical up and the $Y$ axis can be determined by satisfying the right-hand rule. The moving coordinate system $P - xyz$ is established at the center of the moving platform (upper platform). The $x$ axis is parallel to the $X$ axis, and the $z$ axis is parallel to the $Z$ axis. In the coordinate projection of the mechanism, $l_1$, $l_2$ and $d_i$ are the length of rod, $A_iB_i$, $B_iC_i$ and parallel structure $C_iD_i$, respectively. $\alpha_i$, $\beta_i$ and $\gamma_i$ are the included angles between each rod and the $Z$ axis. $\theta_i$ is the rotation angle of the local coordinate system $E_i - uvw$ relative to the moving coordinate system $P - xyz$, $\theta_i = (0^\circ, 120^\circ, -120^\circ)$, $\phi_i$ presents the included angle between $PE_i$ and the $x$ axis. $r$ is the distance from $P$ to $E_i$. $R$ is the distance between point $O$ on the static platform to the vertical plane of
the axis of the active pair. \( h \) is the distance from the axis of active pair to the static platform. \( g \) is the distance from the cross bar of the parallelogram structure to the moving platform.

The homogeneous coordinate of the point \( O \) in the moving coordinate system on the static platform can be written as

\[
T_{OP} = -T_{PO} = (-x_p, -y_p, -z_p, 1)^T \tag{1}
\]

The local coordinate system \( E_i -uvw \) is established at the branched chain \( E_i \). The homogeneous coordinate of point \( O \) in the local coordinate system can be written as

\[
T_{OE_i} = \begin{bmatrix}
-R + l_1 \sin \alpha_i + l_2 \sin \gamma_i \\
\frac{d_i \sin \beta_i}{d_i - g - l_1 \cos \alpha_i - l_2 \cos \gamma_i} \\
1
\end{bmatrix} \tag{2}
\]

The transformation matrix between the local coordinate system \( E_i -uvw \) to the moving coordinate system \( P -xyz \) is

\[
E_i = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 & r \cos \varphi_i \\
\sin \theta_i & \cos \theta_i & 0 & r \sin \varphi_i \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{3}
\]

Combining the Eqs. (1) and (3) gives

\[
T_{OE_i} = E_i^{-1} \cdot T_{OP} = \begin{bmatrix}
-x_p \cos \theta_i - y_p \sin \theta_i - r \cos (\varphi_i - \theta_i) \\
x_p \sin \theta_i - y_p \cos \theta_i - r \sin (\varphi_i - \theta_i) \\
-z_p \\
1
\end{bmatrix} \tag{4}
\]

Eight inverse solutions of the mechanism will be obtained through Eqs. (2) and (4). The initial position of the mechanism is shown in Fig. 2. The analytical solution is obtained as follows

\[
\beta_i = \sin^{-1}\left(\frac{x_p \sin \theta_i - y_p \cos \theta_i - r \sin (\varphi_i - \theta_i)}{d_i}\right) \tag{5}
\]

\[
\gamma_i = \pi - \sin^{-1}\left(\frac{C^2 + D^2 + B^2 - A^2}{2B \sqrt{C^2 + D^2}}\right) - \tan^{-1}\left(\frac{D}{C}\right) \tag{6}
\]

\[
\alpha_i = \sin^{-1}\left(\frac{C^2 + D^2 + A^2 - B^2}{2A \sqrt{C^2 + D^2}}\right) - \tan^{-1}\left(\frac{D}{C}\right) \tag{7}
\]
where \( A = l_1, B = l_2, C = R - x_p \cos \theta_i - y_p \sin \theta_i - r \cos (\phi_i - \theta_i), \) \( D = z_p - h - g - \sqrt{d_i^2 - [x_p \sin \theta_i - y_p \cos \theta_i - r \sin (\varphi_i - \theta_i)]^2}. \)

Combining Eqs. (2) and (4), one can obtain

\[
\dot{\alpha} = J^{-1} v_p
\]  

where \( \dot{\alpha} = (\dot{\alpha}_1, \dot{\alpha}_2, \dot{\alpha}_3)^T, v_p = (\dot{x}_p, \dot{y}_p, \dot{z}_p)^T \) are the angular velocity of the active pair and the velocity of point \( P \) on the moving platform. \( J \) is the Jacobian matrix of them. The elements of the matrix in its inverse matrix are

\[
J_{11} = \frac{- \cos \beta_i \cos \theta_i \sin \gamma_i + \sin \beta_i \sin \theta_i \cos \gamma_i}{l_1 \cos \beta_i (\cos \alpha_i \sin \gamma_i - \sin \alpha_i \cos \gamma_i)} \\
J_{12} = \frac{- \cos \beta_i \cos \theta_i \sin \gamma_i + \sin \beta_i \sin \theta_i \cos \gamma_i}{l_1 \cos \beta_i (\cos \alpha_i \sin \gamma_i - \sin \alpha_i \cos \gamma_i)} \\
J_{13} = \frac{\cos \gamma_i}{l_1 \cos \alpha_i \sin \gamma_i - \sin \alpha_i \cos \gamma_i}
\]

In the same way, one can obtain

\[
\dot{\gamma} = G^{-1} v_p
\]  

where \( \dot{\gamma} = (\dot{\gamma}_1, \dot{\gamma}_2, \dot{\gamma}_3)^T \) is the angular velocity for the rod \( B_i C_i \), and \( G \) is the Jacobian matrix. The elements of the matrix in its inverse matrix are

\[
G_{i1} = \frac{\cos \beta_i \cos \theta_i \sin \alpha_i - \sin \beta_i \sin \theta_i \cos \alpha_i}{l_2 \cos \beta_i (\cos \alpha_i \sin \gamma_i - \sin \alpha_i \cos \gamma_i)} \\
G_{i2} = \frac{\cos \beta_i \sin \theta_i \sin \alpha_i + \sin \beta_i \cos \theta_i \cos \alpha_i}{l_2 \cos \beta_i (\cos \alpha_i \sin \gamma_i - \sin \alpha_i \cos \gamma_i)} \\
G_{i3} = \frac{- \cos \alpha_i}{l_2 \cos \alpha_i \sin \gamma_i - \sin \alpha_i \cos \gamma_i}
\]

In the same way, one can obtain

\[
\dot{\beta} = N^{-1} v_p
\]  

where \( \dot{\beta} = (\dot{\beta}_1, \dot{\beta}_2, \dot{\beta}_3)^T \) is the angular velocity for the side rod \( C_i D_i \), \( N \) is the Jacobian matrix of them. The elements of the matrix in its inverse matrix are

\[
N_{i1} = \frac{\sin \theta_i}{d_i \cos \beta_i}, \quad N_{i2} = \frac{- \cos \theta_i}{d_i \cos \beta_i}, \quad N_{i3} = 0
\]

The dynamic model of the 3-RR^U parallel mechanism is established based on Lagrange equation, which is presented as

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{Q}_i} \right) - \frac{\partial T}{\partial Q_i} + \frac{\partial V}{\partial \dot{Q}_i} + \frac{\partial D}{\partial \dot{S}_i} = \Gamma_i
\]  

where \( T \) is the kinetic energy, \( V \) is potential energy, For the 3-RR^U parallel mechanism, they are respectively given by

\[
T = \frac{1}{2} v_p^T M v_p \\
V = \frac{1}{2} S^T K S
\]
Table 1: Initial parameters of the system

<table>
<thead>
<tr>
<th>Initial parameters of the systems</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of upper platform (kg)</td>
<td>200</td>
</tr>
<tr>
<td>Stiffness of torsion spring (N m/m°)</td>
<td>10000</td>
</tr>
<tr>
<td>Length of every rod (mm)</td>
<td>80</td>
</tr>
<tr>
<td>The radius of the upper and lower platforms (mm)</td>
<td>100</td>
</tr>
<tr>
<td>The angle between the rod $A_i B_i$ and the vertical direction (°)</td>
<td>-60</td>
</tr>
<tr>
<td>The angle between the rod $B_i C_i$ and the vertical direction (°)</td>
<td>60</td>
</tr>
<tr>
<td>The angle between the rod $C_i D_i$ and the vertical direction (°)</td>
<td>0</td>
</tr>
<tr>
<td>Angle of the parallelogram mechanism (°)</td>
<td>90</td>
</tr>
</tbody>
</table>

where $\mathbf{M}$, $\mathbf{K}$ present the mass matrix and the stiffness matrix, respectively. The undamped vibration isolation has no damping term $\mathbf{D}$, neither the generalized force term $\Gamma_i$. The Eq. (18) can be rewritten as

$$\mathbf{M} \ddot{\mathbf{s}} + \mathbf{Ks} = 0 \quad (21)$$

The eigenvalue equation of the natural frequency is

$$|\mathbf{K} - \omega^2 \mathbf{M}| = 0 \quad (22)$$

The sensitivity analysis of the natural frequency to any design variable $x_m$, can be obtained by solving the partial derivative of Eq. (21)

$$\left( \frac{\partial \mathbf{K}}{\partial x_m} - 2\omega_i \frac{\partial \omega_i}{\partial x_m} \cdot \mathbf{M} - \omega_i^2 \frac{\partial \mathbf{M}}{\partial x_m} \right) \cdot \Phi_i + \left( \mathbf{K} - \omega_i^2 \mathbf{M} \right) \frac{\partial \Phi_i}{\partial x_m} = 0 \quad (23)$$

where $\omega_i$ is order circle frequency, and $\Phi_i$ is regular mode of vibration. Then the following equation will be obtained

$$\frac{\partial f_i}{\partial x_m} = \frac{1}{4\pi \omega_i} \Phi_i^T \left( \frac{\partial \mathbf{K}}{\partial x_m} - \omega_i^2 \frac{\partial \mathbf{M}}{\partial x_m} \right) \Phi_i \quad (24)$$

### 4. Analysis of the natural frequency

The initial parameters of the system are shown in Table 1. In Fig. 3, $f_1$ and $f_2$ present two natural frequencies of horizontal direction, respectively. In this example, these two natural frequencies are equal. $f_3$ presents the natural frequency of vertical direction.

The essence of sensitivity is the rate of change, which can directly measure the variation trend of the variable. In each of the subplots of Fig. 3, the sensitivity of each natural frequency decreases with the increase of the mass of rod. This shows that the relationship between the sensitivity and the mass of the rod is negative correlation. As the rod gradually moves away from the static platform, the upper limit of the sensitivity of the natural frequency of each horizontal direction increases. The reason is that the higher the position of the rod is, the more complex the form of motion will be. The form of motion turns from a simple fixed axis motion to a plane complex motion, and eventually change to a space complex motion. As a result, the greater the kinetic energy of the rod, the greater the sensitivity of the natural frequency of the mass of the rod. In Fig. 3(d) and (e), the influence of mass of the rod on the natural frequency of vertical direction is the same as the situation in Fig. 3(c), indicating that the mass of the side link and the upper link mainly influence the natural frequency in the horizontal direction. It shows that the mass of the rod may not have the same influence on the natural frequencies of each order. This influence is directional. The upper limit of sensitivity for every natural frequency in Fig. 3(e) is about six times of Fig. 3(a). Therefore, in engineering practice, changing the mass of links in parallelogram structure should be considered first to meet the requirement of the natural
Figure 3: The influence of the mass of the rod on the sensitivity of the natural frequency

frequency. According to the above conclusion, the more the position of each rod in the branch chain of the parallel mechanism is far away from the static platform, the greater the influence of the mass on the natural frequency. Fig. 4 shows the variation of the sensitivity of the natural frequency of the vertical direction when the torsion spring is added to the different joint positions. The two curves show that the sensitivity of the natural frequency decreases with the increase of the stiffness of the torsion spring, but the torsional spring stiffness between the two links has a greater influence on the sensitivity. Mainly because the potential energy of the torsion spring of the active joints is only related to $\alpha$, and the elastic potential energy of the torsion spring between two links is related to $\alpha$ and $\gamma$. In the light vibration of the system, these two angles vary in the same direction. In the engineering practice, the torsion spring element should be rationally selected according to the demand of vibration isolation.

5. Conclusions

In this paper, a multi-dimensional vibration isolation was obtained by installing the spring at the joints of parallel mechanism. To obtain the base for the multi-dimensional vibration isolation plat-form, the 3-RR^U parallel mechanism was obtained by replacing C pair in 3-RRC parallel mechanism with
the parallelogram structures. The mechanical performance of the 3-RR^U parallel mechanism is improved and some limitation of the 3-RRC is eliminated. The dynamic model of the multi-dimensional vibration isolation platform was established by using the Lagrange’s principle, in which the influences of the links mass are taken into account. The sensitivity analysis was performed to investigate the influence of the link mass and the position of the torsion spring on the natural frequencies of the multi-dimensional vibration isolation platform. The results that that both the link mass and the position of the torsion spring affect the natural frequencies of the system; the influence of the link is increasingly important as the distance between the link and the static platform increase, the influence of the torsion springs on the natural frequency is different if the torsion installed at different position.

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