DEPENDENCE OF CHANNEL RADIUS AND ACOUSTIC IMPEDANCE ON ENTROPY FLUX DENSITY PRODUCTION OF TRAVELING WAVE THERMOACOUSTIC ENGINES

Mariko Senga
Tokai University, Graduate School of Science and Technology, Kanagawa, Japan
email: senga.mariko@gmail.com

Shinya Hasegawa
Tokai University, School of Engineering, Kanagawa, Japan

The acoustic intensity can be amplified when a sudden temperature gradient exceeding the threshold value in the axial direction of the bundle of the narrow tube (the regenerator) and inputting traveling wave in the direction in which the temperature gradient increases. This phenomenon is called a traveling wave thermoacoustic engine, and it is classified as a heat engine which is possible to perform the isothermal reversible thermodynamic cycle. However, in an actual traveling wave thermoacoustic engine, there are irreversibilities such as dissipation and heat transport from cold to hot by oscillating flow. These irreversibilities causes the decrease in thermal efficiency.

In this study, the entropy flux density production per unit length in the regenerator was numerically analyzed by changing absolute value of normalized acoustic impedance and ratio of the flow channel radius to the thermal penetration depth for the traveling wave thermoacoustic engine. Under analysis conditions of this study, the entropy flux density production per unit length does not depend on the absolute value of the normalized acoustic impedance, and became the minimum value when the ratio of the flow channel radius to the thermal penetration depth was about 0.127.

Keywords: thermoacoustic engine, traveling wave, entropy production

1. Introduction

When a sudden temperature gradient exceeding the threshold value is given in the axial direction of the bundle of the narrow tube (hereinafter, the regenerator), the acoustic wave is excited by the interaction between the tube wall and the gas in the tube, and the energy conversion of the heat flux density $Q$ and the acoustic intensity $I$ is performed [1, 2]. This phenomenon can be regarded as a heat engine using acoustic waves and this is called a thermoacoustic engine. The interconversion of $Q$ and $I$ in the regenerator of the thermoacoustic engine can be expressed as follows according to the first law of thermodynamics.

$$\frac{d}{dx}(Q + I) = 0.$$  \hspace{1cm} (1)

Here, $x$ is defined as the propagation direction of acoustic waves as positive along the axial direction of the tube. The energy conversion of the thermoacoustic engine can be classified into two types by focusing on the phase difference $\phi$ between the complex pressure amplitude $P$ and the cross sectional mean complex velocity amplitude $U$ in the tube [3]. One is the standing wave type, and the other is the traveling wave type. The energy conversion of the standing wave type is performed by the irreversible thermodynamic cycle, and the thermal efficiency is generally low. On the other hand,
the traveling wave type can be classified as a heat engine which is possible to perform the isothermal reversible thermodynamic cycle “ideally” [3]. However, in an actual traveling-wave thermoacoustic engine, the irreversibilities occur due to the viscous dissipation [4] and the heat transport from cold to hot by the oscillating flow [5,6], thus it is difficult to execute the isothermal reversible thermodynamic cycle. Since the irreversibilities often cause decrease in the thermal efficiency, it is important to realize the isothermal reversible energy conversion even when using the thermoacoustic phenomenon as an engine.

In general, the reversible and irreversible characteristics of a heat engine can be evaluated by a measure such as the entropy production. If the entropy production is large, it indicates that the irreversibility is large. As mentioned above, since the thermoacoustic engine is also a heat engine that performs interconversion between $Q$ and $I$, it is possible to use the entropy production as the measure for evaluating the reversible or the irreversible. Here, assuming that the entropy flux density $S$ in the regenerator is $Q/T_m$ ($T_m$: the cross sectional mean temperature), the entropy flux density production per unit length $dS/dx$ can be expressed as follows [7].

$$\frac{dS}{dx} = \frac{d}{dx} \left( \frac{Q}{T_m} \right) = \frac{1}{T_m} \frac{dQ}{dx} - \frac{Q}{T_m^2} \frac{dT_m}{dx}. \quad (2)$$

As shown in Eq. (1), since $dQ/dx = -dI/dx$ from the first law of thermodynamics, Eq. (2) can be rewritten.

$$\frac{dS}{dx} = -\frac{1}{T_m} \frac{dI}{dx} - \frac{Q}{T_m^2} \frac{dT_m}{dx}. \quad (3)$$

It is important to decrease $dS/dx$ in order to make the energy conversion in the regenerator reversible. The first term on the right side and the second term on the right side of Eq. (3) represent the entropy flux density production per unit length due to $I$ and $Q$ respectively [7]. When $dI/dx$ is positive value, the thermoacoustic phenomenon can operate as an engine. In this case, $Q$ become necessarily negative value from the second law of thermodynamics. For this reason, there exists a condition that minimizes the difference between the first term on the right side and the second term on the right side of Eq. (3) under a constraint condition. In this study, the objective is to search for the condition that minimize the irreversibility in the regenerator of the traveling wave thermoacoustic engine. We use $dS/dx$ as the index of the evaluation, and $dS/dx$ in the regenerator under a certain constraint condition is analyzed.

2. Analysis Method

In this study, $dS/dx$ in Eq. (3) is used to search for conditions that decrease the irreversibilities of energy conversion. First, $dI/dx$ and $Q$ in Eq. (3) are found. Here, $I$ is expressed as follows.

$$I = \frac{1}{2} \text{Re}[\bar{P}U]. \quad (4)$$

Therefore, $dI/dx$ is

$$\frac{dI}{dx} = W = \frac{1}{2} \text{Re} \left[ \frac{d\bar{P}}{dx} U \right] + \frac{1}{2} \text{Re} \left[ \bar{P} \frac{dU}{dx} \right]. \quad (5)$$

Here, $\bar{P}$ represents the complex conjugate. Also, $Q$ can be written as [2],

$$Q = \frac{1}{2} c_p \rho_m \text{Re}[T\bar{U}] - I . \quad (6)$$
\[ C_p, \rho_m, \text{ and } T \text{ are the isobaric specific heat, the mean density, and the cross-sectional mean complex temperature amplitude, respectively. The axial change of } P \ (dP/dx), \text{ the axial change } U \ (dU/dx) \text{ and } T \text{ are expressed from thermoacoustic theory.} \ [7, 2] \]

\[
\frac{dP}{dx} = -j\omega \rho_m (1 - \chi_v) U. \tag{7}
\]

\[
\frac{dU}{dx} = -j\omega P_m \left[1 - (\gamma - 1)\chi_\alpha\right] P + \frac{\chi_\alpha - \chi_v}{(1 - \chi_v)(1 - \sigma)} T_m \frac{dT_m}{dx} U. \tag{8}
\]

\[
T = \frac{1}{C_p \rho_m} \left(1 - \chi_\alpha\right) P - \left(\frac{(1 - \chi_\alpha) - \sigma(1 - \chi_v)}{(1 - \chi_v)(1 - \sigma)}\right) \frac{1}{j\omega} \frac{dT_m}{dx} U. \tag{9}
\]

Here, \( j \): the imaginary unit, \( \omega \): the angular frequency, \( \gamma \): the specific heat ratio, \( P_m \): the mean pressure, \( \sigma \): Prandtl number. Furthermore, \( \chi_\alpha, \chi_v \) are thermoacoustic functions related to thermal and viscosity depending on \( r/\delta_a \) \[1\]. In the case of a circular flow path, \( \chi_\alpha, \chi_v \) are written as follows.

\[
\chi_{\alpha,v} = \frac{2J_1 ((j - 1)r/\delta_{\alpha,v})}{(j - 1)r/\delta_{\alpha,v}}J_0 ((j - 1)r/\delta_{\alpha,v}). \tag{10}
\]

\( J_1 \) and \( J_0 \) are the Bessel functions of the 1st and 0th order, \( r \) is the flow channel radius, \( \delta_a, \delta_v \) are the thermal penetration depth and the viscous penetration depth, respectively. The value of \( r/\delta_a \) can be used as an indicator of the thermodynamic cycle through which the gas in the flow channel of the regenerator passes. In the case of \( r/\delta_a << 1 \), \( r \) is sufficiently smaller than \( \delta_a \), so that heat from the tube wall is easily transmitted, and the thermodynamic cycle is performed isothermally reversible. In \( r/\delta_a \sim 1 \), the gas exchange the heat with the tube wall with delay and undergoes the irreversible cycle. At \( r/\delta_a >> 1 \), the gas cannot perform heat exchange with the tube wall, so the thermodynamic cycle becomes the adiabatic reversible cycle. Substituting Eqs. (7) ~ (9) into Eqs. (5), (6) and expanding them respectively, \( W \) and \( Q \) can be expressed as following equation.

\[
W = W_{\text{prog}} + W_{\text{stand}} + W_v + W_p, \tag{11}
\]

\[
W'_v = \frac{\omega P_m}{2} \text{Im} \left[\frac{1}{1 - \chi_v}\right] |U|^2,
\]

\[
W_p = \frac{1}{2} \frac{(\gamma - 1)\omega}{\gamma P_m} \text{Im}[\chi_\alpha] |P|^2,
\]

\[
W_{\text{prog}} = \frac{1}{2} \text{Re}[b] \frac{1}{T_m} \frac{dT_m}{dx} |P||U| \cos \varphi,
\]

\[
W_{\text{stand}} = \frac{1}{2} \text{Im}[b] \frac{1}{T_m} \frac{dT_m}{dx} |P||U| \sin \varphi,
\]

\[
b = \frac{\chi_\alpha - \chi_v}{(1 - \sigma)(1 - \chi_v)}.
\]

\[
Q = Q_{\text{prog}} + Q_{\text{stand}} + Q_D, \tag{12}
\]

\[
Q_{\text{prog}} = -\frac{1}{2} \text{Re}[g] |P||U|,
\]

\[
Q_{\text{stand}} = -\frac{1}{2} \text{Im}[g] |P||U| \sin \varphi,
\]

\[
Q_D = \frac{C_p \rho_m}{2\omega} \text{Re} \left[\frac{1}{1 - \chi_v}\right] \text{Im}[g_D] |U|^2 \frac{dT_m}{dx},
\]
\[ g = \frac{\chi_a - \bar{x}_v}{(1 + \sigma)(1 - \bar{x}_v)}, \]
\[ g_D = \frac{(\chi_a - \bar{x}_v) - (1 - \sigma)\chi_v}{(1 - \text{Re}[\chi_v])(1 - \sigma^2)}. \]

Here, \( W_r \): the acoustic intensity loss due to viscosity, \( W_p \): the acoustic intensity loss due to thermal diffusion by \( |P|^2 \), \( W_{\text{prog}} \): the energy conversion by traveling wave phase, and \( W_{\text{stand}} \): the energy conversion by standing wave phase. Furthermore, \( Q_{\text{prog}} \): the heat flux density due to traveling wave component, \( Q_{\text{stand}} \): the heat flux density due to standing wave component, \( Q_D \): the heat flux density due to heat transport from cold to hot by the oscillating flow [5,6].

In this study, \( \varphi = 0 \) is assumed in order to investigate the energy conversion of the traveling wave thermoacoustic engine. In the case of \( \varphi = 0 \), \( W_{\text{stand}} \) and \( Q_{\text{stand}} \) are 0. Thus, \( W_r, W_p, W_{\text{prog}}, Q_{\text{prog}}, Q_D \) will be discussed. In addition, following relations,
\[ \theta = \frac{\lambda}{T_m} \frac{dT_m}{dx}, \quad \lambda = \frac{c}{\omega}, \quad |z_n| = \frac{1}{\rho_m c} |P| |U|, \quad \frac{1}{P_m} = \frac{\gamma}{\rho_m c^2}, \quad \rho_mC_p = \frac{\rho_m c^2}{T_m(y - 1)}, \]

are used to the equations (11) and (12), and rewrite them as follows. Here, \( c \) is the adiabatic acoustic velocity.

\[ W_v = \frac{|P||U|}{2\lambda} \text{Im}\left[\frac{1}{1 - \chi_v} \frac{1}{|z_n|}\right]. \tag{13} \]
\[ W_p = \frac{|P||U|}{2\lambda} (\gamma - 1) \text{Im}[\chi_a]|z_n|. \tag{14} \]
\[ W_{\text{prog}} = \frac{|P||U|}{2\lambda} \text{Re}[b] \theta. \tag{15} \]
\[ Q_{\text{prog}} = -\frac{|P||U|}{2\lambda} \lambda \text{Re}[g]. \tag{16} \]
\[ Q_D = \frac{|P||U|}{2\lambda} \frac{\lambda}{(y - 1)} \text{Re}\left[\frac{1}{1 - \chi_v}\right] \text{Im}[g_D] \theta \frac{1}{|z_n|}. \tag{17} \]

\(|z_n|\) is the absolute value of the normalized acoustic impedance obtained by ratio between absolute value of the specific acoustic impedance (=\(|P| / |U|\)) and the characteristic acoustic impedance of the traveling wave in free space \( \rho_m c \). Therefore, substituting Eq. (13) - (17) for Eq. (3), \( dS/dx \) can be expressed as

\[ \frac{dS}{dx} = \frac{-1}{T_m} \frac{|P||U|}{2\lambda} \left( \frac{1}{\gamma - 1} \text{Re}\left[\frac{1}{1 - \chi_v}\right] \text{Im}[g_D] \theta^2 \frac{1}{|z_n|} - \text{Re}[g] \theta + \text{Im}\left[\frac{1}{1 - \chi_v}\right] \frac{1}{|z_n|}\right) \]
\[ + (\gamma - 1) \text{Im}[\chi_a]|z_n| + \text{Re}[b] \theta. \tag{18} \]

In this study, \( dS/dx \) is analyzed using Eq. (18) and search conditions that can decrease irreversibility. In Eq. (18), parameters such as \( \gamma \) and \( \lambda \) which change with physical property of gas and parameters such as \( T_m \) included in \( \theta \) are can be set by the experimenter. Therefore, it can be seen that \( dS/dx \) depends on \( |z_n| \) and \( \chi_a, \chi_v \). Furthermore, \( \chi_a, \chi_v \) depends on \( \rho_\delta a_n \), as described above. Hence, in this report, the dependence of \( dS/dx \) on \( \rho_\delta a \) and \( |z_n| \) is investigated by using Eq. (18). The parameters are set as shown in Table 1. \( |U| \) is fixed in this analysis and \( |P| \) is obtained from Eq. (19).

\[ |P| = |z_n| \rho_m c |U|. \tag{19} \]
Table 1 Parameters used in analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_m$</td>
<td>450 [K]</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>125.7</td>
</tr>
<tr>
<td>$</td>
<td>U</td>
</tr>
<tr>
<td>$</td>
<td>P</td>
</tr>
<tr>
<td>$</td>
<td>\rho_m c</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.701</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>5.657 [m]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.655</td>
</tr>
<tr>
<td>$r/\delta_\alpha$</td>
<td>0.1 ~ 10</td>
</tr>
</tbody>
</table>

$\delta_\alpha$ : 0.560 [mm] $r/\delta_\alpha$ was set to 0.1 to 10 by changing $r$ from 0.006 to 5.596 mm.

3. Analysis results

Figure 1 shows results of $dS / dx$ when varying $r/\delta_\alpha$ and $|z_n|$. In Fig. 1, the horizontal axis shows $r/\delta_\alpha$, and the solid, dotted, dashed-dotted lines refer to $|z_n| = 1, 10, 30$, respectively. From this result, even changing $|z_n|$, there was no significant difference in $dS / dx$ in the range of all $r/\delta_\alpha$ under the analytical condition of this study. For the range of $r/\delta_\alpha < 1$, the irreversibility due to viscous dissipation $W_\nu$ is large because $r$ is sufficiently smaller than $\delta_\alpha$. Therefore, the entropy flux density production per unit length due to viscous dissipation increases. On the other hand, when $1 < r/\delta_\alpha < 10$, the heat transport from cold to hot by oscillating flow becomes large, and the entropy flux density production per unit length has a peak in the vicinity of $r/\delta_\alpha \sim 1.7$. The reason why $dS / dx$ decrease for $1 < r/\delta_\alpha$ is that it approaches adiabatic reversible process since $r$ becomes sufficiently larger than $\delta_\alpha$. As a result, the minimum point of $dS / dx$ exists at $r/\delta_\alpha = 0.127$ without dependence on $|z_n|$ was obtained.

![Graph](image)

Fig. 1 $dS / dx$ vs. $r/\delta_\alpha$.

4. Conclusion

In this study, the entropy flux density production per unit length of the regenerator was analyzed while changing absolute value of normalized acoustic impedance and ratio of the flow channel radius to the thermal penetration depth for the traveling wave thermoacoustic engine. Under the analytical condition in this study, the entropy flux density production per unit length was independent of the absolute value of the normalized acoustic impedance. Furthermore, it depended on the ratio of the
flow channel radius to the thermal penetration depth, and entropy flux density production per unit length became a local minimum value when this value was about 0.127.

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