PERFORMANCE ESTIMATION OF A LOUDSPEAKER-DRIVEN STANDING-WAVE THERMOACOUSTIC REFRIGERATOR

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Loudspeaker-driven thermoacoustic refrigerators utilize the rich interactions between thermodynamics, acoustics and heat transfer to create a refrigeration effect by using the imposed acoustic wave, which is one form of mechanical work, thus employing the ability of a sound wave to play the role of a compressor and an expander. This work presents experimental data acquired on a standing-wave thermoacoustic refrigerator equipped with a set of structured stacks (200, 400 and 600 cells per square inch) and driven by a commercial loudspeaker. The dimensions of the acoustic resonator was adjusted to enforce equality between the acoustic resonance frequency and the mechanical resonance frequency of the loudspeaker. Experiments were carried-out up to a drive ratio of 11.2%. The results quantify the temporal development of the hot and cold stack temperatures for a range of vital parameters that significantly affect the performance of these refrigerators. The parameters considered were the operating frequency, the input drive ratio, the Lautrec number and the normalized stack position inside the acoustic resonator. When other factors were held constants, the coldest temperature on the stack took place when the operating frequency coincided with the mechanical resonance frequency, and when the stack porosity corresponded to a Lautrec number of 2.8, and when the normalized stack center position was equal to 0.193.

Keywords: standing-wave, thermoacoustic refrigerator, structured stack, loudspeaker-driven refrigeration.

1. Introduction

Thermoacoustic devices have noticeable advantages over conventional energy conversion devices, including the nature of the working fluids which are usually inert gases or mixture of inert gases and thus are safe and environment-friendly with no global warming or ozone depletion hazards; the low need of maintenance due to the lack of moving parts (a single oscillating diaphragm if the refrigerator is driven by a loud-speaker and no moving parts at all if it is driven by a thermoacoustic engine); the inherent simplicity and low cost of operation since the device consists mainly of carefully positioned stack inside a resonator tube of a proper length and geometry; the possibility of using continuous cooling capacity control, as opposed to the on/off control typical in conventional systems; and quiet operation, since the tube holding the system is thick enough not to radiate audible sound levels outside the tube.
The main technical challenges that face these devices are the low power density and the low conversion efficiency. Limitations arise from streaming, acoustic, viscous, and thermal losses [1], as well as from the fact that the system performance is very sensitive to the choice of the design parameters and should be carefully optimized to optimize the conversion efficiency or power density. This particular source is the motivation for this work.

The main components of a TAR are a loudspeaker that converts the electric input power into acoustic power in the form of a pressure wave, and thus plays the role of a compressor/expander in a conventional refrigerator, a stack that is composed of many narrow channels, along which the thermoacoustic interaction occurs, and two heat exchangers (hot and cold) positioned around the stack, and an acoustic resonator housing the stack and the heat exchangers.

The main objective of this study is to quantify how operation at different conditions, namely the operating frequency, the input drive ratio, the stack porosity and the stack position inside the acoustic resonator, affect the hot and cold stack temperatures.

2. Experimental Setup

Figure 1 presents a schematic of the experimental setup used. The system was driven with a sine-wave input generated at the required frequency using a function generator (Tektronix AFG3021B) and amplified using a power amplifier (Bruel & Kjaer type 2734). The amplified wave then was fed to a loudspeaker (Massive Audio TORO 104 subwoofer). This acoustic power then was fed to the resonator, which was made modular with a controlled length in order to enforce that the acoustic resonance frequency is equal to the mechanical resonance of the loudspeaker in order to optimize the electric-to-acoustic conversion efficiency of the speaker and hence performance of the system.

The drive ratio is defined as the ratio between the dynamic pressure at the blind flange at the far end from the loudspeaker and the mean gas pressure. The dynamic pressure was measured using an Endeveco pressure sensor (Meggitt Model 8510B-2) and with a signal conditioner (Meggitt Amplifier type 136). The mean gas temperatures at the hot and cold stack ends were measured using Type-K Omega thermocouples (diameter of 380 µm). The pressure and temperature data were acquired using a data-acquisition card (National Instruments NI 6343, 32 analog inputs, 500 kS/s, 16-bit resolution and ±10 V range) and using Lab View software.

A vacuum pump was used to remove the atmospheric air from the system first before filling the system with argon at atmospheric pressure. Then, the system was purged three times to ensure that the remaining air fraction prior to the experiment was insignificant.

Figure 1: A schematic view of the thermoacoustic refrigerator and the instrumentation used. The wavelength \( \lambda \) is equal to 5.65 m.
The water flow rate in the cold and hot-side heat exchangers were set to 70 mL/hr and 583 L/hr, respectively and. These values resulted in a change in the cold-side water temperature in the range of 4-5 °C, and almost constant hot-side water temperature.

Figure 2 shows the system fundamental acoustical frequency and the harmonics of the complete system. The mechanical resonance frequency of the loud speaker was identified as 57 Hz, indicating that the thermoacoustic refrigerator is operated in a nearly quarter-wave length mode (exactly L/3.22 mode).

3. Results and Discussion

The temporal development of the hot and cold stack temperatures is presented in Fig. 3-6 at different operating frequencies, input drive ratios, Lautrec cumbers (stack porosities) and normalized stack positions, respectively.

3.1. Effects of frequency

In general, the power density in thermoacoustic refrigerators is a linear function of the acoustic resonance frequency [2]. This calls for the use of high resonance frequencies, while considering that the electro-acoustic conversion efficiency of the acoustic speaker is maximized if the speaker is operated at its mechanical resonance frequency. However, some limits apply: (1) for a given gas type and mean gas pressure, a high operating frequency makes the thermal penetration depth smaller which implies the use of small stack spacing, making the manufacture of the stack more difficult and increasing the pressure drop and viscous losses inside the stack; (2) in a constant diameter resonator without stack, the operating frequency is inversely proportional to the resonator length which calls for long resonators to match the acoustic resonance frequency to the mechanical resonance frequency of the acoustic driver; and (3) different acoustic drivers produce acoustic power in a different specific range of frequencies. Therefore, the frequency is a critical factor in the optimization of the loudspeaker-driven TARs [3].
In this work, a range of operating frequencies was studied and the results are presented in Fig. 3, in the range of 80% to 120% the resonance frequency (57 Hz). The results show how operation at the resonance frequency yields the coldest stack temperature, but operation at slightly higher or lower frequencies generate slightly hotter temperatures. For example, operation at a frequency 120% larger than the resonance frequency yields a steady-state cold-side temperature of 18 °C, as opposed to 16.5 °C at the resonance frequency.

3.2. Effects of input drive ratios (β):

Increasing the input drive ratio, up to a certain limit, tends to increase the refrigeration effect and the power density, which must take place if these devices are to compete with conventional technologies. However, some of the limits include: (1) The input drive ratio is limited by the maximum force the acoustic driver can supply (which depends on its power rating and mechanical structure); (2) nonlinear effects will degrade the performance beyond a certain input drive ratio. For this reason, all comparisons with linear theories or DeltaEC simulations typically are carried-out at low input drive ratios.

At large drive ratios, there are different sources of loss in thermoacoustic systems: inherent, viscous, conduction, auxiliary and transduction losses. The inherent loss is generated by the irreversibility associated with the heat transfer between each gas parcel and the stack wall inside the stack channels; the viscous loss is due to the work required to overcome the shear forces in the stack viscous penetration depth as the gas parcels oscillate; the conduction loss is caused by the heat transfer from the hot heat exchanger side to the cold heat exchanger through the stack material as well as through the gas; the auxiliary loss is due to the losses from the cold and hot heat exchangers, and the resonator stack wall to the surrounding; and the transduction loss arises in the acoustic driver due to the Joule’s heating in its copper wires [4]. To keep the flow from turbulent status, the flow Reynolds number, \( Re \), must be kept smaller than 500 [4]. This Reynolds number is defined as:

\[
Re = \left( \frac{u_c \delta_v}{\nu} \right), \tag{1}
\]

where \( u_c \) is the gas particle velocity at the center of the stack, defined as:

\[
u_c = \frac{\left( \beta/BR \right) (a/\gamma) (\sin \zeta_c)}{\left( \beta/BR \right) (a/\gamma) (\sin \zeta_c)} \tag{2}
\]

where \( \beta \) is the drive ratio, \( BR \) is the stack blockage ratio, \( a \) is the speed of sound, \( \gamma \) is the isobaric to isochoric specific heats ratio; \( \nu \) is the kinematic viscosity and \( \delta_v \) is the viscous penetration depth, defined as:

\[
\delta_v = \frac{2 \mu}{\sqrt{\rho m \omega}}, \tag{3}
\]
where $\mu$ is the dynamic viscosity of the gas used, $\rho_m$ is the mean gas density, and $\omega$ is the angular frequency.

![Figure 4: The effects of different drive ratios. This data is for 1-bar argon with a range of drive ratios from 1.5% to 11.2%, a stack of 200 CPSI with a normalized stack position of 0.193 at an operating frequency of 57 Hz.](image)

In this work, the effects of employing five drive ratios, in the range from 1.5% to 11.2%, on the hot and cold stack side temperatures is quantified in Fig. 4. The results reveal that, at the low mean gas pressure employed, increasing the drive ratio increases the temperature difference between the hot and cold stack sides monotonically as well as the cold-side temperature was minimum at the largest drive ratio tested (11.2%).

### 3.3. Effects of Lautrec number, $N_L$:

Thermoacoustic devices two important characteristic length scales, namely the thermal penetration depth $\delta_k$ and the viscous penetration depth $\delta_v$. The thermal penetration depth $\delta_k$ defines the boundary layer of the stack wall where the thermoacoustic effect takes place. Its thickness corresponds to the distance in which heat can transfer through the medium gas during the time interval corresponding to one cycle of acoustic oscillations. On the other hand, the viscous penetration depth $\delta_v$ defines the boundary layer of the stack wall where the gas movement is restrained because of the effects of viscous forces. Within this layer, the loss of kinetic energy is mainly due to the viscous dissipation and one of the design objectives is to minimize the viscous penetration depth $\delta_v$ while maximizing the thermal penetration depth, $\delta_k$. The thermal penetration depth $\delta_k$ can be expressed as:

$$\delta_k = \sqrt{\frac{2K}{\rho_m c_p \omega}}$$  \hspace{1cm} (4)

where $K$ is thermal conductivity of the working fluid, $\rho_m$ is mean gas density and $C_p$ is the isobaric heat capacity of the working fluid.

The Lautrec number, $N_L$, is used as a measure to quantify the ratio between the stack spacing to the thermal penetration depth and is defined as:

$$N_L = \frac{2y_o}{\delta_k}$$  \hspace{1cm} (5)

where $y_o$ is half the stack spacing.

In this work, three stacks of 200, 400 and 600 CPSI, with stack spacing widths $2y_o$ of 1.542, 1.168 and 0.935 mm, respectively, were selected to investigate the system performance in three experiments. The $\delta_k$ and $\delta_v$ values were kept constant in the three experiments at 0.42 and 0.28 mm, respectively, yielding Lautrec numbers of 3.7, 2.8 and 2.2, respectively.
Fig. 5 compares the results for the three stack porosities considered, where the Lautrec number of 2.8 was found to lead to the minimum cold-side gas temperature. Larger and smaller Lautrec numbers suffer from less optimized ratios of thermal and viscous penetration depths to the stack spacing.

![Graph showing temperature over time for different Lautrec numbers.]

Figure 5: The effects of different Lautrec numbers. This data is for 1-bar argon at a drive ratio of 11.2%, different sets of stacks with 200, 400 and 600 CPSI and a normalized stack position of 0.193 at an operating frequency of 57 Hz.

3.4. Effects of normalized stack position, \( \zeta_c \):

The normalized stack position, \( \zeta_c \), quantifies the stack-center position with respect to the acoustic wave and is defined as:

\[
\zeta_c = (2\pi f/a)(X_c),
\]

where \( f \) is the operating frequency and \( X_c \) is the stack center position.

Since the acoustic power output is proportional to the dot product of the dynamic pressure and the gas parcel velocity, it follows that the stack has to be positioned away from pressure and velocity nodes. If the stack is positioned at a pressure antinode which is a velocity node, the dynamic pressure amplitude will be maximum but the gas parcel is stationary with no heat transfer and a zero acoustic power. On the other hand, if the stack is positioned at a velocity antinode, which is a pressure node, the parcel displacement is maximized but because the dynamic pressure is zero, the gas parcel cannot heat up.

Some other factors that control the stack center position is that the closest position the stack can have to the speaker, which is set by the length of the reducing cone plus half the length of the stack, as shown in Figure 1. The reducing cone allows using large-power commercial loud speakers, which typically have large diameters. A long reducing cone may necessitate positioning the stack farther away from the point set by the balance of optimum acoustic power and viscous and thermal relaxation losses. The length of the reducing cone, however, is inversely proportional to the cone angle, which in turn should be small enough to avoid boundary layer separation in this low-Reynolds number oscillating flow to avoid transformation of the kinetic energy of the flow into heat. The trade-off between all these parameters needs to be studied in details. Additionally, in real systems, the existence of the stack alters the standing wave but if the stack is short enough, then this perturbation is not significant, as per the short stack approximation [2].

The present work studies five different stack positions inside the acoustic resonator and quantifies the hot and cold stack temperatures at these positions, in order to investigate the optimum stack position inside the acoustic wave at the conditions tested.
In this work, five stack locations were tested at stack center positions of 6.85, 12.25, 16.75, 21.75 and 26.25 cm, measured as shown in Fig. 1, yielding a range of the normalized stack center position $\zeta$ from 0.072 to 0.276.

For example, at a $\zeta$ value that is too low, 0.072, the stack hot side was only 1 cm away from the resonator end (pressure anti-node), yielding the largest hot-side stack temperatures. At this location, the cold-side temperature experienced a rapid but un-sustained cooling, that was lost because of the conduction heat transfer from the hot to the cold stack side.

Results indicate that starting from $\zeta$ from 0.193, the cooling of the cold-side was sustainable and the minimum cold-side temperature was observed at this normalized stack location. Larger normalized stack positions yield larger cold-side stack temperatures.

4. Summary and Conclusions

The current work investigated the performance of a standing-wave thermoacoustic refrigerator experimentally with the purpose of understanding and quantifying the performance at different operating conditions. The results quantify how deviations from the resonance frequency degrade the cold-side stack temperature, how the increases in the drive ratio improve the cold-side stack temperature, and the values of the Lautrec number and normalized stack position that led to the minimum cold-side stack temperature.

REFERENCES


