In this paper, we propose a method capable of detecting the convergence of the coefficients of the pre-estimating filters applied to active noise control systems. In general, the secondary and the feedback paths are manually estimated before starting the active noise control. In practical use, the paths is desired to be automatically estimated with a designed error even where the power of the primary noise disturbing the estimation is unknown and fluctuates. The impulse responses of the paths should be moreover assumed to be unknown. Then, it is impossible to monitor the transition of the estimation error directly. In this paper, we solve the first issue which the power of the primary noise is unknown and fluctuates, by applying a step size control method to updating the coefficients. Moreover, we propose the method of using two adaptive filters, whose step sizes are different, for detecting the convergence of the coefficients. The second issue, which the impulse response is unknown, can be thereby solved. Concretely, the first order recursive filter expression of the normalized least mean square algorithm is used for the detection. According to the expression, the square sum of the difference between the coefficients of the adaptive filters converges on the magnitude of the designed estimation error multiplied by the square of the difference between the step sizes. The completion of the estimation of the path can be judged by detecting that the square sum of the difference decreased to the magnitude. We finally verify using computer simulations that the proposed method can successfully detect the completion of the estimation.

Keywords: pre-estimating filter, convergence detection

1. Introduction

Feedforward type active noise control systems [1] detecting a primary noise with a microphone, inevitably form a feedback path and a secondary path. In general, the paths are manually estimated by individually applying an adaptive filter before starting the control [2]. However, the paths are practically desired to be automatically estimated.

Three methods are accordingly proposed on the subject of the automatic estimation. One is the method of detecting the convergence of the gradient vector used for updating the coefficients of the adaptive filter [3]. Another is the method of using the cross-correlation between the output of the adaptive filter and the remainder subtracted that from the output of the acoustic path [4]. The other is the method of updating the coefficients for a required time [5]. These methods, however, cannot be applied to practical systems which the power of the primary noise arriving at the microphone is...
supposed to be unknown and to fluctuate. Moreover, these methods cannot guarantee the coefficients to have converged with a designed estimation error.

As for the simultaneous equations method \[6, 7, 8\], the previous estimation of the secondary path is unnecessary, and a little differently, the estimation of the feedback path is not required unless howling occurs. Nevertheless, the feedback path is desired to be sufficiently cancelled, because the cancellation error of the feedback path increases the non-minimum phase components in the optimum noise control filter decreasing the output of the error microphone to zero \[9\]. The increase of the non-minimum components degrades the noise reduction effect.

The upper limit of the cancellation error not degrading the noise reduction effect depends on the shape of the impulse response of the secondary path. Two upper limits are shown based on simulations. One is –10 dB derived by using the practical impulse responses whose direct sound component is considerably large in comparison to the reflection those \[10\]. The other is –20 dB shown by using artificial impulse responses whose components consist of exponentially decade random numbers \[11\]. In practical use, the coefficients of the adaptive filter needs to be estimated with the designed error.

For guaranteeing the coefficients of the adaptive filter to converge with the designed error, the power of the primary noise needs to be evaluated. In this study, we substitute the short time square sum of the difference between the outputs of the adaptive filter and the microphone for the power of the primary noise. The square sum is then overestimated by the estimation error. The step size applied to the adaptive algorithm is thereby estimated less than the desired one, which operates so as to reduce the estimation error to smaller than the designed level. Moreover, the degree of the overestimation decreases with the convergence of the coefficients and finally to negligible level.

A certain time measurement is necessary for evaluating the power of the primary noise. In this study, we apply the block implementation type adaptive algorithm to updating the coefficients. The short time power of the primary noise can be thereby evaluated. Moreover, we use white noise, whose power is constant, as the reference signal for estimating the acoustic path. The block can be consequently fixed at a length. We next apply the step size control method proposed in \[12, 13\] to the adaptive algorithm. The coefficients are thereby guaranteed to converge with the designed error even where the power of the primary noise fluctuates.

The remained issue is the derivation of the method of detecting the convergence of the coefficients with the designed error. In this study, we propose to use the square sum of the difference between the coefficient vectors of two adaptive filters, to which two different step sizes are applied, for the detection. The convergence of the coefficient vector can be judged from the decrease of the square sum to less than the prescribed level. We finally verify using computer simulation that the proposed method successfully works.

2. Necessity of feedback path cancellation

Figure 1 shows the configuration of the feedforward type system applying the simultaneous equations method to active noise control, where \(N(z)\) is the primary noise detected by the noise detection microphone \(M_d\), \(X(z)\) is the input signal of the noise control filter \(H(z)\), \(E(z)\) is the output signal of the error microphone \(M_e\), \(P(z)\) is the primary path from the noise detection microphone \(M_d\) to the error microphone \(M_e\), \(B(z)\) is the feedback path from the loudspeaker \(S_p\) to the noise detection microphone \(M_d\), \(C(z)\) is the secondary path from the loudspeaker \(S_p\) to the error microphone \(M_e\), and \(\hat{B}(z)\) is the feedback control filter used for canceling the feedback path, \(S(z)\) is the auxiliary filter identifying the overall path from the input of the noise control filter to the output of the error microphone. The signals, the acoustic paths and the filters are thus denoted with \(z\)-transform.

In practical use, the cancellation error of the feedback path,

\[
\Delta B(z) = B(z) - \hat{B}(z),
\]

should be supposed to be not zero. The reference signal used for updating the coefficients of the noise
control filter $H(z)$ then changes from $N(z)$ to

$$X(z) = \frac{N(z)}{1 - \Delta B(z)H(z)}. \quad (2)$$

$X(z)$ is thus used as the input signal, and $E(z)$ can be regarded as the output signal in the system shown in Fig. 1.

Figure 1: Configuration of feedforward type system applying simultaneous equations method to active noise control.

In this study, we call $\tilde{C}(z)$ “modified secondary path”. Accordingly, the optimum noise control filter reducing the output of the error microphone to zero changes to

$$H_{opt}(z) = \frac{P(z)}{\tilde{C}(z)} = \frac{P(z)}{C(z) - \Delta B(z)P(z)}. \quad (4)$$

Equation (4) states that the estimation of not $C(z)$ but $\tilde{C}(z)$ is necessary for reducing the primary noise. The simultaneous equations method [6, 7, 8] directly derives the optimum noise control filter without estimating $\tilde{C}(z)$. The simultaneous equations method is characterized by the auxiliary filter used for estimating the overall path shown in Fig. 2. After the estimation, the auxiliary filter satisfies the following relation,

$$S(z) = P(z) + \tilde{C}(z)H(z) \quad (5)$$

Figure 2: Configuration of overall path from input of noise control filter to output of error microphone.

Figure 2 shows the configuration simplified so that the mechanism of the active noise control can be understood at a glance. As seen from the simplified configuration, the cancellation error $\Delta B(z)$ changes the secondary path from $C(z)$ to

$$\hat{C}(z) = C(z) - \Delta B(z)P(z). \quad (3)$$

In this study, we call $\hat{C}(z)$ “modified secondary path”. Accordingly, the optimum noise control filter reducing the output of the error microphone to zero changes to

$$H_{opt}(z) = \frac{P(z)}{\hat{C}(z)} = \frac{P(z)}{C(z) - \Delta B(z)P(z)}. \quad (4)$$

Equation (4) states that the estimation of not $C(z)$ but $\hat{C}(z)$ is necessary for reducing the primary noise. The simultaneous equations method [6, 7, 8] directly derives the optimum noise control filter without estimating $\hat{C}(z)$. The simultaneous equations method is characterized by the auxiliary filter used for estimating the overall path shown in Fig. 2. After the estimation, the auxiliary filter satisfies the following relation,

$$S(z) = P(z) + \hat{C}(z)H(z) \quad (5)$$
including two unknowns, \( P(z) \) and \( \tilde{C}(z) \).

The simultaneous equations method solves Eq. (5) by giving two different coefficients to the noise control filter. The auxiliary filter then forms two independent equations,

\[
S_1(z) \approx P(z) + \tilde{C}(z)H_1(z) \tag{6}
\]

\[
S_2(z) \approx P(z) + \tilde{C}(z)H_2(z), \tag{7}
\]

where \( H_1(z) \) and \( H_2(z) \) are the transfer functions of the noise control filters whose coefficients are different. The optimum noise control filter is derived to

\[
H_{opt}(z) = -\frac{P(z)}{\tilde{C}(z)} \approx \frac{S_1(z)H_2(z) - S_2(z)H_1(z)}{S_2(z) - S_1(z)} \tag{8}
\]

by substituting the solutions of Eqs. (6) and (7) into Eq. (4).

The simultaneous equations method can thus reduce the primary noise without previously estimating the secondary and the feedback paths unless the cancellation error causes howling. However, the denominator of Eq. (4), especially \( \Delta \text{B}(z)P(z) \), increases the non-minimum phase components degrading the noise reduction effect. This means that the sufficient cancellation of the feedback path is requisite for reducing the primary noise.

3. Guarantee of convergence with designed error

In practical systems, the power of the primary noise disturbing the estimation of the coefficients of the adaptive filter is supposed to be unknown and to fluctuate. In this study, we apply the block implementation normalized least mean square (NLMS) algorithm formulated by

\[
H_{n+1} = H_n + \mu_n \sum_{j=nJ_n}^{(n+1)J_n} e_j X_j
\]

\[
+ \frac{\sum_{j=nJ_n}^{(n+1)J_n} e_j^2 X_j^T X_j}{J_n}
\]

(9)

to the estimation of the coefficients of the adaptive filter [12, 13], where \( H_n \) is the coefficient vector given to the adaptive filter at the \( n \)th block, \( j \) is the sample time index, \( e_j \) is the remainder subtracted the output of the adaptive filter from that of the microphone, \( J_n \) is the variable block length, \( X_j \) is the input signal vector of the adaptive filter. We moreover evaluate the short time power of the primary noise at

\[
Q_n = \sum_{j=nJ_n}^{(n+1)J_n} e_j^2
\]

(10)

The short time power is then overestimated by the difference between the outputs of the microphone and the adaptive filter. However, the difference decreases with the convergence of the coefficients, and finally to a negligible extent [12].

Moreover, the power of the reference signal used for estimating the coefficients of the adaptive filter can be stabilized [12] by extending the block length until satisfying the following relation,

\[
P_n = \sum_{j=nJ_n}^{(n+1)J_n} X_j^T X_j \geq \sigma_x^2 J_n I_0,
\]

(11)
where $\sigma^2 x$ is the average power of the reference signal, $J_0$ is the lower limit of the block length necessary for estimating the power of the primary noise. Controlling the step size by using

$$\mu_n = \frac{2C_0P_n}{Q_nJ_n} J_n$$

(12)
can guarantee that the estimation error decreases to the designed level $C_0$ even where the power of the primary noises fluctuate[12, 13].

4. Method for detecting convergence of coefficients

In practical systems, the impulse response of the acoustic path should be supposed to be unknown. Then, we cannot calculate the estimation error between the impulse response samples of the acoustic path and the coefficients of the adaptive filter. A novel method is required for detecting the convergence of the coefficients with the designed error. In this study, we propose the method of using two adaptive filters for the detection.

The first order recursive filter expression of the NLMS algorithm[14] can be used for verifying that the convergence can be detected by using the proposed method. Actually, the NLMS algorithm updating the $m$th coefficient of the adaptive filter, $H_j(m)$, at $j$ sample time can be expressed as

$$H_{j+1}(m) = H_j(m) + \mu \frac{e_j x_j(m)}{P_j},$$

(13)

where $\mu$ is the step size, $x_j(m)$ is the $m$th element of the reference signal vector $X_j$ and

$$P_j = X_j^T X_j.$$  

(14)

According to [14], Eq.(13) can be rearranged as

$$H_{j+1}(m) = H_j(m)\alpha_j(m) + h(m)(1 - \alpha_j(m)) + \frac{\mu}{P_j} \sum_{i=0, i\neq m}^{I=1} \{\Delta_j(i)x_j(i) + n_j\} x_j(m)$$

(15)

where $h(m)$ is the $m$th impulse response sample of the acoustic path,

$$\alpha_j(m) = 1 - \frac{x_j^2(m)}{P_j},$$

(16)

$$\Delta_j(i) = h(i) - H_j(i),$$

(17)

and $n_j$ is the primary noise. The expression can be developed to the block implementation type:

$$H_{n+1}(m) = H_n(m)\alpha_n(m) + h(m)\{1 - \alpha_n(m)\} + \frac{\mu_n}{P_n} \sum_{j=n,0}^{(n+1)J_0} \sum_{i=0, i\neq m}^{I=1} \{\Delta_n(i)x_j(i) + n_j\} x_j(m),$$

(18)

where

$$\Delta_n(i) = h(i) - H_n(i),$$

(19)

$$\alpha_n(m) = 1 - \frac{\mu_n}{P_n} \sum_{j=n,0}^{(n+1)J_0} x_j^2(m).$$

(20)

Equation (18) indicates that the adaptive filter coefficient $H_{n+1}(m)$ is estimated as the step response generated by applying the $m$th element of the impulse response of the acoustic path to the
recursive filter. As seen from the expression, the coefficient \( H_{n+1}(m) \) fluctuates around \( h(m) \) after the convergence, as

\[
\lim_{n \to \infty} H_{n+1}(m) = h(m) + \frac{\mu_n}{P_n} \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0 \atop i \neq m}^{I-1} \{ \delta_n(i) + n_j \} x_j(m),
\]

where \( \delta_n(i) \) denotes the estimation error involved in the coefficient \( H_n(i) \). The second term of Eq. (21) shows that the coefficient \( H_{n+1}(m) \) fluctuates around to \( h(m) \), which expresses the designed estimation error and can be expressed as

\[
\delta_{n+1}(m) = \frac{\mu_n}{P_n} \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0 \atop i \neq m}^{I-1} \{ \delta_n(i) + n_j \} x_j(m).
\]

In practical use, the power of the reference signal is supposed to be less than that of the primary noise. \( \delta_n(i) \) is then expected to be negligible in comparison with \( n_j \) after the convergence. Equation (21) can be accordingly approximated to

\[
\delta_{n+1}(m) \approx \frac{\mu_n}{P_n} \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0 \atop i \neq m}^{I-1} n_j x_j(m).
\]

We next apply another step size \( a\mu_n \) to the second adaptive filter, where

\[
a < 1.
\]

Then, the fluctuation of the coefficient of the second adaptive filter can be expressed by

\[
\delta'_{n+1}(m) \approx \frac{a\mu_n}{P_n} \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0 \atop i \neq m}^{I-1} n_j x_j(m) = a\delta_{n+1}(m).
\]

In this study, we note that the difference between \( \delta_{n+1}(m) \) and \( \delta'_{n+1}(m) \) is equal to that of the coefficients of the adaptive filters. The square sum of the difference,

\[
d_{n+1}(m) = \frac{(1 - a) \mu_n}{P_n} \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0 \atop i \neq m}^{I-1} n_j x_j(m) = (1 - a)\delta_{n+1}(m),
\]

can be evaluated by

\[
D_n = \frac{(H_1^n - H_2^n)^T(H_1^n - H_2^n)}{H_1^n H_1^n^T},
\]

where \( H_1^n \) and \( H_2^n \) denote the coefficient vectors of the adaptive filters, respectively. Equations (26) and (27) show that the square sum \( D_n \) converges on the \((1 - a)^2\) times of the designed estimation error. The convergence of the coefficients can be judged by monitoring the transition of \( D_n \).
5. Verification by simulation

Figure 3 shows the impulse response of the acoustic path, whose samples are given the series of exponentially decayed regular numbers. The other conditions are shown below.

- Reference signal: white noise.
- Primary noise: white noise.
- Power ratio of reference signal to primary noise: –6 dB and 0 dB.
- Block length: $J_0 = 16$.
- Designed estimation error: –30 dB.
- Step size ratio: $a = 0.9$.

As seen from Eq. (27), $D_n$ diverges when the initial coefficient vectors, $H^1_0 = H^2_0 = 0$, are given to the adaptive filters. In this simulation, we give $H^2_0 = [0.1 \ 0 \ \cdots \ 0]$ to one of the adaptive filters. Under the conditions, the difference $d_{n+1}(m)$ gives the relation,

$$d_{n+1}(m) = (1 - a) \delta_{n+1}(m) = 0.1 \delta_{n+1}(m).$$

Equation (29) states that the square difference $D_n$ decreases to 20 dB less than the designed estimation error. In the conditions shown above, the designed estimation error is –30 dB, which states that the convergence of the coefficients can be judged by detecting the decrease of $D_n$ to –50 dB.

Figure 4 shows the transition of the estimation error and the square difference where the power ratio of the reference signal to the surrounding noise changes from 0 dB to –6 dB.
the power ratio of the reference signal to the primary noise is changed from 0 dB to −6 dB at the indicated point. This result shows that the estimation error nevertheless decreases steadily to the designed estimation error of −30 dB and that the convergence of the coefficients can be judged by detecting the decrease of the square sum $D_n$ to less than −50 dB.

6. Summary

In this paper, we have proposed the method capable of detecting the convergence of the coefficients of the adaptive filter with the designed estimation error, and then have shown that the method successfully works. In the near future, we will apply the proposed method to re-estimating the feedback path under active noise control.

REFERENCES