SOUND ABSORPTION OF MICRO-PERFORATED PANEL STRUCTURES IN PARALLEL ARRANGEMENT

Hyun-Sil Kim, Pyung-Sik Ma, Yun-Ho Seo, Sang-Ryul Kim, and Bong-Ki Kim
Korea Institute of Machinery and Materials, Acoustics and Noise Research Team, Daejeon, Korea
email: hskim@kimm.re.kr

In this study, wide-band sound absorption using parallel arrangement of different MPPs (micro-perforated plates) is discussed. Each MPP cell is single or multi-layered structures composed of thin elastic plates with or without micro-perforations and rigid MPPs. Firstly, sound absorption of an elastic MPP is investigated. It is found that the combined effect of the elastic behavior and micro-perforation results in a significant increase of the sound absorption coefficient compared to that of a rigid MPP for a very small micro-perforation ratio. Secondly, when different MPP cells are arranged in parallel arrangement, sound absorption is studied using the finite element method (FEM), which yields a significant increase of the sound-absorbing bandwidth compared to that of each MPP cell.

Keywords: micro-perforated plate, sound absorption, cavity

1. Introduction

Micro-perforated panels or plates (MPPs) are free of fibrous or porous materials and resistant to humidity, which makes MPPs as new type of environment-friendly sound absorbers. In general, an MPP is backed by an air cavity and a rigid backing material, and the diameter and perforation ratio are less than 1 mm and 1%, respectively. The sound-absorbing mechanism of an MPP is essentially the Helmholtz resonance, in which sound is absorbed due to friction through narrow apertures. Maa [1, 2] proposed an approximate formula for the impedance of an MPP. Maa’s model is very useful in investigating sound absorption of MPP structures. However, in many cases, it is known that the sound-absorbing bandwidth achieved by a single MPP is limited. Li et al. [3] studied enhancement of the low frequency sound absorption of an MPP by parallel-arranged extended tubes. They developed a theoretical model, and compared the prediction to the experimental results. In this study, sound absorption using parallel arrangement of different MPPs is discussed. Each MPP cell is single or multi-layered structures composed of thin elastic plates with or without micro-perforations and rigid MPPs.

2. Formulation

We consider a thin elastic MPP installed in a rigid duct of which the cross-section is rectangular, $L_1 \times L_2$ as shown in Fig. 1. When a plane wave $Ae^{i(\omega t-kz)}$ is incident onto the MPP, the reflected wave is given by
The wave inside the cavity is given by

\[ p_r = A e^{i(\alpha x - kz)} + \sum_{r=0}^{N_r} \sum_{s=0}^{N_s} D_{rs} \cos\left(\frac{\pi r x}{L_1}\right) \cos\left(\frac{\pi s y}{L_2}\right) e^{i(\alpha x + k_z z)}, \]  

(1)

where \( i = \sqrt{-1} \), \( \omega \) is the angular frequency, \( k = \frac{\omega}{c} \), and \( c \) is the speed of sound in air. The wavenumber \( k_z \) in the \( z \) direction satisfies the following relationship:

\[ \pi^2 \left\{ \left(\frac{r}{L_1}\right)^2 + \left(\frac{s}{L_2}\right)^2 \right\} + k_z^2 = k^2. \]  

(2)

The summation limits \( N_r \) and \( N_s \) in Eq. (1) are determined such that \( k_z \) becomes a real number in Eq. (2). The wave inside the cavity is given by

\[ p_m = \sum_{r=0}^{N_r} \sum_{s=0}^{N_s} \left[ B_{rs} e^{i(\alpha x - k_z z)} + C_{rs} e^{i(\alpha x + k_z z)} \right] \cos\left(\frac{\pi r x}{L_1}\right) \cos\left(\frac{\pi s y}{L_2}\right). \]  

(3)

The boundary condition at the surface of the MPP is

\[ -\frac{1}{i \omega \rho} \frac{\partial p}{\partial z} = \bar{v} \quad \text{at} \quad z = 0, \]  

(4)

while at the rigid backing block

\[ -\frac{1}{i \omega \rho} \frac{\partial p}{\partial z} = 0 \quad \text{at} \quad z = q, \]  

(5)

where \( \rho \) is the density of air and \( \bar{v} \) is the average velocity of the MPP as shown in Fig. 2.
Average velocity $\bar{v}$ is related to the plate velocity $v_p$ and the fluid velocity $v_f$ through the hole as

$$\bar{v} = v_p (1 - \sigma) + v_f \sigma.$$  (6)

In Eq. (6), $\sigma$ is the perforation ratio defined as $\sigma = \pi d^2 / 4l^2$, where $d$ is the diameter of the hole and $l$ is the distance between the holes.

The pressure difference across the plate is related to the impedance of the plate as

$$Z_{\text{resist}} (v_f - v_p) + Z_{\text{react}} v_f = \Delta p = p_r - p_m.$$  (7)

The impedance of the hole, $Z = Z_{\text{resist}} + Z_{\text{react}}$, is given by [1,2]

$$Z_{\text{resist}} = \frac{8\eta_0 h}{(d/2)^2} \left( \frac{1}{32} + \frac{\sqrt{dX}}{32h} \right),$$  (8)

$$Z_{\text{react}} = i \rho \omega h \left( \frac{1}{9 + \frac{X^2}{2}} + \frac{8d}{3\pi h} \right),$$  (9)

where $\eta_0$ denotes the viscosity coefficient of air, $h$ is the thickness of the plate and $X = (d/2)\sqrt{\rho \omega / \eta_0}$. Average velocity of the MPP is related to the plate velocity and the impedance by

$$\bar{v} = \gamma v_p + \frac{\sigma \Delta p}{Z},$$  (10)

in which $\gamma = 1 - \sigma (Z_{\text{react}} / Z)$.

The coefficients $B_{rs}$, $C_{rs}$ and $D_{rs}$ satisfy the following relationship:

$$D_{rs} = -B_{rs} + C_{rs} \text{ at } z = 0,$$  (11)

$$-B_{rs} e^{-ik_a r} + C_{rs} e^{ik_a r} = 0 \text{ at } z = q.$$  (12)

If the plate is simply supported, the displacement of the plate can be expressed as

$$\hat{\xi}(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \phi_m(x)\phi_n(y),$$  (14)

where $\phi_m(x) = \sin(m\pi x / L_1)$, and $\phi_n(x) = \sin(n\pi y / L_2)$. Using Eq. (14), the left-hand side of Eq. (13) can be rewritten as
Multiplying both sides of Eq. (15) by \(\phi_m^*(x)\phi_n^*(y)\) and integrating the expression over the surface of the plates with the use of the orthogonal property of the modes results in

\[
MY^x_n^y(\omega^2_m - \omega^2) a_{mn} = 2AJ^x_0J^y_0 - 2 \sum_{r=0}^{N_r} \sum_{s=0}^{N_s} B^r_{rs} J^x_r J^y_s, \tag{16}
\]

where

\[
Y^x_m = \frac{1}{L_1} \int_0^{L_1} \phi_m^2(x) dx, \tag{17}
\]

\[
J^y_m = \frac{1}{L_1} \int_0^{L_1} \cos \left( \frac{mx}{L_1} \right) \phi_m(x) dx, \tag{18}
\]

and \(Y^x_m\) and \(J^y_m\) are defined in the same manner with respect to \(L_2\) and \(\phi_i(y)\).

Carrying out similar manipulation to the boundary condition results in

\[
\left[ B^r_{rs} \left( \frac{k_z}{\rho \omega} + \frac{2 \sigma}{Z_0} \right) - \frac{k_z}{\rho \omega} \right] I_s = i \omega \gamma \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} J^x_m J^y_n + 2 \frac{\sigma}{Z_0} A \delta_{r0} \delta_{s0}, \tag{19}
\]

where \(\delta_{r0}\) and \(\delta_{s0}\) are 1 when \(r = s = 0\), and zero, otherwise. \(I_r\) is defined as

\[
I_r = \frac{1}{L_1} \int_0^{L_1} \cos^2 \left( \frac{mx}{L_1} \right) dx = \begin{cases} 1/2 & \text{when } r \geq 1 \\ 1 & \text{when } r = 0 \end{cases}, \tag{20}
\]

and \(I_s\) is defined in the same manner with respect to \(L_2\) and \(s\).

When waves propagate through a duct, there exists a cut-off frequency above which waves cannot propagate. It is assumed that the cut-off frequency is low enough such that only \(r = s = 0\) (or \(N_r = N_s = 0\)) is allowed in Eq. (2). Accordingly, Eqs. (2) and (20) become

\[
k_z = k, \quad I_0 = 1. \tag{21}
\]

Substitution of \(N_r = N_s = 0\) into Eqs. (16) and (19), and rearrangement yields

\[
B_{00} (1 + 2 \rho c \sigma / Z) - C_{00} = 2 \gamma \beta (A - B_{00}) + 2 A (\rho c \sigma / Z), \tag{22}
\]

where

\[
\beta = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{i \rho c \omega (J^x_{0m} J^y_{0n})^2}{MY^x_n^y(\omega^2_m - \omega^2)}. \tag{23}
\]

From Eqs. (11), (12) and (22), the ratio of the reflected wave to the incident wave is determined as

\[
D_{00} / A = (Z_{eq} - 1) / (Z_{eq} + 1), \tag{24}
\]

where

\[
Z_{eq} = 1 / (\gamma \beta + \rho c \sigma / Z) - i \cot(kq). \tag{25}
\]

\(Z_{eq}\) can be regarded as the equivalent surface impedance as if the MPP and the cavity were replaced by a single surface. The absorption coefficient \(\alpha\) is defined as
\[ \alpha = 1 - \frac{Z_{eq} - 1^2}{Z_{eq} + 1^2}. \]  

(26)

Note that \( Z_{eq} \) in Eq. (25) can consider a flexible plate without micro-perforation as well as a rigid plate with micro-perforation. When the plate has no micro-perforation, \( \sigma \) is zero. On the other hand, when the plate is rigid, \( \beta \) becomes zero. In Fig. 3, the sound absorption coefficient of a single MPP is compared for elastic and rigid cases. The elastic cases include zero perforation as well as 0.1 % perforation ratio.

![Graph showing absorption coefficient comparison for elastic and rigid cases](image)

Figure 3: Comparison of the absorption coefficients of a single plate for elastic and rigid cases: \( \sigma = 0.0 \% \), and 0.1 %. The plate is simply supported, and the duct has a square cross-section with \( L_1 = L_2 = 280 \) mm. The thickness of the plate and cavity depth are \( h_1 = 0.2 \) mm and \( q_1 = 60 \) mm.

The proposed method can be extended to double elastic MPPs [4], where absorption coefficient is given by

\[ Z_{eq} = \frac{1}{K_j} - i \cot(kq_j) + \frac{1+ \cot^2(kq_j)}{1/K_2 - i \cot(kq_2) - i \cot(kq_1)}, \]  

(27)

in which \( K_j = \gamma_j \beta_j + \rho c \sigma_j / Z_j \), and \( q_j \) is the cavity depth of the \( j \)th MPP (\( j =1, 2 \)).

In Fig. 4, we compared the prediction of the double elastic MPPs to the FEM results, which confirms the accuracy of the proposed method. The plates are simply supported, and the parameters of the double MPPs are \( h_1 = d_1 = 0.2 \) mm, \( h_2 = d_2 = 0.3 \) mm, \( q_1 = q_2 = 30 \) mm, and \( \sigma_1 = \sigma_2 = 0.08 \% \). Although Eq. (14) is valid only for the simply supported condition, displacement of the clamped plate can also be approximated by Eq. (14).
Figure 4: Comparison of the sound absorption coefficients of double-elastic MPPs: prediction vs. the FEM. The plates are simply supported, and the parameters of the double MPPs are $L_1 = 0.26 \text{ m}$, $L_2 = 0.21 \text{ m}$, $h_1 = d_1 = 0.2 \text{ mm}$, $h_2 = d_2 = 0.3 \text{ mm}$, $q_1 = q_2 = 30 \text{ mm}$, and $\sigma_1 = \sigma_2 = 0.08 \%$.

3. MPPs in parallel arrangement

As shown in Fig. 5, two MPPs are arranged in parallel with different perforation ratio $\sigma_1$ and $\sigma_2$ with the same cavity depth (Fig. 5(a)), or different cavity depth $q_1$ and $q_2$ with the same perforation ratio (Fig. 5(b)).

![Parallel arrangement of two MPPs](image)

Figure 5: Parallel arrangement of two MPPs: (a) different perforation ratios with the same cavity depth, (b) different cavity depths with the same perforation ratio

When two surfaces with different surface impedance are arranged in parallel, the total impedance is given by [3]
\[
\frac{1}{Z} = S_1 + S_2,
\]  

(28)

where \(S_1\) and \(S_2\) are area ratio to the total area.

Figure 6: Sound absorption coefficients of two single MPPs in parallel arrangement. Half of the area has perforation ratio \(\sigma = 0.0\%\), while \(\sigma = 0.08\%\) for other half.

Figure 7: Sound absorption coefficients of two single MPPs in parallel arrangement. Half of the area has perforation ratio \(\sigma = 0.4\%\), while \(\sigma = 0.08\%\) for other half.
In Fig. 6, absorption coefficient of the parallel-arranged MPP with the same cavity depth is compared to the results where whole surface is covered by the MPP with $\sigma = 0.0 \%$ or $\sigma = 0.08 \%$. In Fig. 7, the case with the same cavity depth is compared to the results of the single MPP when $\sigma = 0.4 \%$ or $\sigma = 0.08 \%$. In Fig. 8, absorption coefficient of the parallel-arranged MPP with different cavity depth is compared to the results where cavity depth is constant across the whole surface with $q = 30 \text{ mm}$ or $q = 60 \text{ mm}$, and $\sigma = 0.08 \%$. In Figs. 6 ~ 8, it was assumed that $S_1 = S_2$. It is observed that combined MPPs in parallel arrangement show more wide-band sound absorption than that which is obtained by a single MPP covering the whole area.

![Graph](image)

Figure 8: Sound absorption coefficients of two single MPPs in parallel arrangement. Half of the area has cavity depth $q = 60 \text{ mm}$, while $q = 30 \text{ mm}$ for other half.

REFERENCES