This study deals with development of a high-frequency measurement method of normal-incidence sound absorption coefficient in impedance tube. The authors have previously proposed a practical method using multiple microphones. In our proposed method, to extract the normal-propagating wave factor through a cross section of a cylindrical tube in the frequency range in which obliquely propagating waves can exist, four microphones are located with one in each quarter of the circumference and their signals are summed. The normal-incidence absorption coefficient is calculated from the frequency response function between the normal-propagating factors of two cross sections. We have confirmed so far theoretically and experimentally that the proposed method is useful for measuring the normal-incidence absorption coefficient for the commonly used sound-absorbing materials when scattering on the specimen surface is small. However, if a large amount of scattering on the specimen surface exists, the normal-incidence absorption coefficient cannot be measured accurately because the normally reflected sound wave might decrease due to the scattering. Hence, in this study, a monitoring method of the scattering is proposed. As an indicator of the scattering effect, which can show the reliability of measurement results, a normal-incidence (1, 0)-mode-reflection coefficient and a total power reflection coefficient in a tube are defined. They are derived from the difference between the signals of the two microphones placed on opposite sides of the tube. Finally, the experimental results are shown, confirming the validity of the proposed method.

Keywords: normal-incidence, sound absorption coefficient, acoustic impedance tube

1. Introduction

The two-microphone transfer-function method using an acoustic impedance tube [1] is commonly used to measure the normal-incidence sound absorption coefficient of materials. The frequency measurable by the method is limited by the diameter of the tube because the measurement must satisfy the condition that only a normal-propagating wave exists in the tube. Therefore, to measure a high-frequency normal-incidence absorption coefficient, it is necessary to use a small diameter tube as it requires only a small sample. However, when the small size sample is used, the ununiformity of the sample material and the constraint of the sample at the tube wall can affect the measured result. On the other hand, if a reflection matrix, which expresses the relation between the incident acoustic modes and the reflected acoustic modes in the tube, can be identified, the normal-incidence absorption coefficient can determined above the cut-on frequency of the higher-order modes. Åbom, M. proposed the method, by which sound waves in a cylindrical tube are identified using multiple
microphones [2]. Schultz, T. et al. developed a method of measuring the reflection coefficient matrix for multiple acoustic modes in a rectangular duct using eight microphones [3]. They calculated all the elements of the reflection coefficient among the acoustic modes, including the reflection coefficient of the reflected (0, 0) mode to the incident (0, 0) mode, which corresponds to the normal-incidence reflection coefficient. However, these methods are complicated, even when measuring just the normal-incidence absorption coefficient, because they require multiple measurements for multiple sound sources and inverse matrix calculation. Therefore, the author proposed a simple practical method to measure normal-incidence sound absorption coefficient at frequencies beyond the cut-on frequency of higher-order modes using four or eight microphones under an assumption that a large amount of scattering on the specimen surface does not exist [4].

In order to extract the normal propagating wave factor through a cross section of a cylindrical tube, four microphones are located with one in each quarter of the circumference and their signals are summed. The normal-incidence absorption coefficient is calculated from the frequency response function between the normal-propagating factors of two cross sections, which are placed at a prescribed distance. However, when scattering on the specimen surface is large, the normal-incidence absorption coefficient cannot be measured accurately because the normal-incidence wave is converted to obliquely reflected waves and the amount of normal-reflected waves might decrease. Hence, in this study, a monitoring method of the scattering is proposed. As an indicator of the scattering effect, a normal-incidence (1, 0)-mode-reflection coefficient and a total power reflection coefficient in a tube are defined, considering the sound pressure difference between the signals of the two microphones placed on opposite sides of the tube. By monitoring these reflection coefficients, measurement can be performed as confirming the reliability of the measured results. The experimental results are shown, confirming the validity of the proposed method.

2. Theory

2.1 Eight-microphone method

First, the measurement method that the author proposed [4] are reviewed. In this study, a cylindrical tube with radius \(R\) (Fig. 1) is considered. The sound pressure in the tube is expressed as

\[
p(r_T, \theta, z, t) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} C_{mn} J_m(k_{r(m,n)} r_T) \times \left[ e^{j\omega t} \left( A_{mn} e^{-j\beta_{r,m,n} z} + B_{mn} e^{j\beta_{r,m,n} z} \right) + e^{j\omega t} \left( A_{mn} e^{-j\beta_{r,m,n} z} + B_{mn} e^{j\beta_{r,m,n} z} \right) \right] e^{j\omega r},
\]

where \(r_T\) denotes the distance from the center of the circular cross section and \(\theta\) is the angle from the \(x\)-axis. \(J_m\) is the Bessel function of the first kind. \(m\) and \(n\) denote the acoustic modal order in the circumferential direction and in the radial direction, respectively. \(C_{mn}\) is a normalization factor and \(A_{mn}, B_{mn}, A'_{mn}, B'_{mn}\) are the amplitudes of waves. \(k_{z(m,n)}\) is the wave number in \(z\)-direction as \(k_{z(m,n)} = \left( k_0^2 - k_{r(m,n)}^2 \right)^{1/2}\) where \(k_0 = \omega/c\) (\(c\): sound speed) and \(k_{r(m,n)}\) is the wave number in the cross section of the tube which satisfies the following equation derived from the boundary condition at the inner surface of the tube:

\[
\frac{dJ_m(k_{r(m,n)} r)}{dr} \bigg|_{r=R} = 0.
\]

Here, \(k_{r(m,n)} R\) that satisfies Eq. (2) is expressed as \(\lambda_{m,n}\). Figure 2 shows each acoustic mode and the corresponding value of \(\lambda_{m,n}\). The (0, 0) mode is equivalent to a plane sound wave propagating in the direction normal to the cross section, and the other higher-order modes are equivalent to obliquely propagating waves. The obliquely propagating waves for which \(k_0 < k_{r(m,n)}\) cannot propagate in the tube because the wave number becomes purely imaginary and the wave becomes evanescent. Therefore, the minimum frequency at which the \((m, n)\) mode can propagate is
which is the so-called cut-on frequency of the mode. In the two-microphone transfer-function method, measurement is conducted under conditions where only the (0, 0) mode can propagate in the tube. Thus, the measurable frequency in the conventional method is below $f_{c,0,1}$.

In the proposed method, four measurement points, indicated as $r_1$, $r_2$, $r_3$, and $r_4$ in Fig. 1, are considered. These points are located on the inner surface of the tube at every quarter of the perimeter. The proposed method considers the frequency range below $f_{c,0,1}$ in which the waves of the (0, 0), (1, 0), and (2, 0) modes can propagate. In this frequency range, the pressures at these measurement points are expressed as follows:

$$p(r_q) = \sum_{m=0}^{n} C_{m} f_{m} \left( \lambda_{m,0} \right) 
\times \left[ e^{-j m \theta_q} \left( A_{m0} e^{-j k_{l}(a+z_A)} + B_{m0} e^{j k_{l}(a+z_A)} \right) + e^{j m \theta_q} \left( A_{m0} e^{j k_{l}(a+z_A)} + B_{m0} e^{-j k_{l}(a+z_A)} \right) \right] e^{j \omega t},$$

where $q$ ($q = 1, 2, 3, \text{ and } 4$) denotes the measurement point number, $\theta_q$ is the position angle of each point and $z_A$ is the distance of the cross section A from the specimen surface.

Now, the sum of the pressure signals at these measurement points is considered. This can be derived from Eqs. (4) as follows:

$$p_{\sum A} = p(r_1) + p(r_2) + p(r_3) + p(r_4)
= 4 \left( A_{0,0} e^{-j k_{l} z_A} + B_{0,0} e^{j k_{l} z_A} \right) e^{j \omega t}. $$

This equation implies that summing the signals causes a filtering effect that cancels the signals due to the (1, 0) and (2, 0) modes and yields the normal-propagating factor. As Fig. 2 shows, the sound pressures at two opposing points, such as $r_1$ and $r_3$, have the same amplitude and are antiphase for the (1, 0) mode, although the node of the mode is not deterministic. Moreover, the pressures at two neighboring points, such as $r_1$ and $r_2$, also have the same amplitude and are antiphase for the (2, 0) mode. This is the reason why summing the signals can cancel the effect of the higher-order modes.

The sums of the measurement signals at the four points are obtained for two cross sections, A and B in Fig. 1, which are placed at a prescribed distance $s$, and the normal-propagating factor on each cross section is extracted. Then, the frequency response function for the normal-propagating factors between the two cross sections can be calculated as $H_{AB} = P_{sB} / P_{sA}$, where $P_{sA}$ and $P_{sB}$ are the frequency spectra of the sum of the measurement signals at cross section A and B, respectively. Then, the normal-incidence sound absorption coefficient can be obtained using
\[ \alpha = 1 - |r|^2, \]  
where \( r \) denotes normal-incidence pressure reflection coefficient:

\[ r = \frac{H_{AB} - e^{-jkz}}{e^{jkz} - H_{AB}^*}. \]

Using this eight-microphone method, measurement can be performed in a wider frequency range than that in the conventional two-microphone method with the same measurement procedure.

### 2.2 Monitoring method of the scattering effects

The proposed method is based on the assumption that the amount of scattering on a specimen surface is not large. If a large amount of scattering exists, the normal-incident sound wave may be reflected obliquely in the frequency region, in which the higher-order acoustic modes can propagate in the tube. In other words, the (0, 0) mode can be converted to the other higher-order modes due to the scattering on the specimen surface. In this case, it is considered that the normal-incidence absorption coefficient cannot be measured accurately even using the proposed eight-microphone method. Hence, in order to confirm the reliability of measured results, a monitoring method of the scattering effects is investigated in this study.

Here, the sound pressure difference between the signals of the two microphones placed on opposite sides of the tube are considered. \( \Delta p_{hA} \) is the difference between \( p(r_1) \) and \( p(r_3) \) and \( \Delta p_{vA} \) is the one between \( p(r_2) \) and \( p(r_4) \). These can be expressed from Eq. (4) as follows:

\[
\begin{align*}
\Delta p_{hA} &= p(r_1) - p(r_3) \\
&= 2e^{j\alpha_1/2} \left[ C_{10} J_1(\lambda_{10}) \left( A_0^\sigma e^{-jkz_{10}}A_0^T + B_0^\sigma e^{j(kz_{10})}A_0^* \right) + C_{10} J_1(\lambda_{10}) \left( A_0^T e^{-jkz_{10}}A_0^\sigma + B_0^T e^{j(kz_{10})}A_0^* \right) \right], \\
\Delta p_{vA} &= p(r_2) - p(r_4) \\
&= 2e^{j\alpha_1/2} \left[ -C_{10} J_1(\lambda_{10}) \left( A_0^\sigma e^{-jkz_{10}}A_0^T + B_0^\sigma e^{j(kz_{10})}A_0^* \right) + C_{10} J_1(\lambda_{10}) \left( A_0^T e^{-jkz_{10}}A_0^\sigma + B_0^T e^{j(kz_{10})}A_0^* \right) \right].
\end{align*}
\]

Then, the frequency response function of \( \Delta p_{hA} \) to \( p_{sA} \) and that of \( \Delta p_{vA} \) to \( p_{sA} \) are expressed as

\[
\begin{align*}
H_{hA} &= \frac{\Delta p_{hA}}{p_{sA}} = C_{10} J_1(\alpha_{10}) \frac{C_0^\sigma e^{-jkz_{10}}A_0^\sigma + B_0^\sigma e^{j(kz_{10})}A_0^* + r_{0^\sigma a}^\sigma e^{-jkz_{10}} + B_{0^\sigma a}^\sigma e^{j(kz_{10})}}{2C_{00}}, \\
H_{vA} &= \frac{\Delta p_{vA}}{p_{sA}} = C_{10} J_1(\alpha_{10}) \frac{-r_{0^\sigma a}^\sigma e^{-jkz_{10}} - \beta_{0^\sigma a}^\sigma e^{j(kz_{10})}A_0^\sigma + r_{0^\sigma a}^T e^{-jkz_{10}} + B_{0^\sigma a}^T e^{j(kz_{10})}}{2C_{00}},
\end{align*}
\]

where \( r_{0^\sigma a} = A_0^\sigma /B_0 \) denotes a normal-incidence reflection coefficient, \( r_{0^\tau a} = A_0^\tau /B_0 \) denote reflection coefficients of the reflected (1, 0) mode to the incident (0, 0) mode in the \( \sigma \) direction and that in the \( \tau \) direction, respectively. \( \beta_{0^\sigma a}^\sigma = B_{0^\sigma a}^\sigma /B_0 \) and \( \beta_{0^\tau a}^\tau = B_{0^\tau a}^\tau /B_0 \) denote ratios of the incidence (1, 0) mode to the incident (0, 0) mode in the \( \sigma \) direction and that in the \( \tau \) direction, respectively.

In the same way, \( H_{hB} \) and \( H_{vB} \) can be derived on the cross section B. Then, the pressure reflection coefficient of the reflected (1, 0) mode to the incident (0, 0) mode can be derived as follows:

\[
\begin{align*}
&\left( r_{0^\sigma a}^\sigma e^{-jkz_{0^\sigma a}} + e^{-jkz_{0^\sigma a}} \right) e^{jkz_{10}} \\
&\times \left( H_{hA} + jH_{vA} \right) - \left( r_{0^\sigma a}^\sigma e^{-jkz_{0^\sigma a}} + e^{-jkz_{0^\sigma a}} \right) e^{jkz_{10}} e^{j\alpha_a} \left( H_{hB} + jH_{vB} \right),
\end{align*}
\]
Similarly, the ratios of the incidence (1, 0) mode to the incident (0, 0) mode are derived as
\[
\beta_{0\rightarrow1}^\sigma = \frac{C_{00}}{C_{10}J_1(\alpha_{10})} \left( r_{0\rightarrow1} e^{-j\theta_{0\rightarrow1}} + e^{j\theta_{0\rightarrow1}} \right) \left( H_{hA} + jH_{vA} \right) - \left( r_{0\rightarrow0} e^{-j\theta_{0\rightarrow0}} + e^{j\theta_{0\rightarrow0}} \right) e^{j\theta_{0\rightarrow1}} e^{j\theta_{0\rightarrow0}} \left( H_{hB} + jH_{vB} \right),
\]
\[
\beta_{0\rightarrow1}^\tau = \frac{C_{00}}{C_{10}J_1(\alpha_{10})} \left( r_{0\rightarrow1} e^{-j\theta_{0\rightarrow1}} + e^{j\theta_{0\rightarrow1}} \right) e^{j\theta_{0\rightarrow1}} e^{j\theta_{0\rightarrow0}} \left( H_{hB} + jH_{vB} \right) - \left( r_{0\rightarrow0} e^{-j\theta_{0\rightarrow0}} + e^{j\theta_{0\rightarrow0}} \right) e^{-j\theta_{0\rightarrow1}} e^{-j\theta_{0\rightarrow0}} \left( H_{hB} + jH_{vB} \right).
\]

From Eqs. (12) to (15), a total power reflection coefficient in a tube can be calculated as follows:
\[
R_{tube} = \frac{R_{0\rightarrow0} + R_{0\rightarrow1}}{1 + B_{0\rightarrow1}},
\]
where
\[
R_{0\rightarrow0} = r_{0\rightarrow0}^* r_{0\rightarrow0}^*,
\]
\[
R_{0\rightarrow1} = \frac{k_{z(1,0)}}{k_0} r_{0\rightarrow1}^\sigma r_{0\rightarrow1}^\tau + \frac{k_{z(1,0)}}{k_0} r_{0\rightarrow1}^\tau r_{0\rightarrow1}^\sigma \text{ and}
\]
\[
B_{0\rightarrow1} = \frac{k_{z(1,0)}}{k_0} \beta_{0\rightarrow1}^\sigma \beta_{0\rightarrow1}^\tau + \frac{k_{z(1,0)}}{k_0} \beta_{0\rightarrow1}^\tau \beta_{0\rightarrow1}^\sigma.
\]

3. Experiments

3.1 Experimental setup

In order to verify the validity of the proposed method, experiments were conducted. Figure 3 shows the schematic view of the experimental setup. The impedance tube is made of acrylic and the inner diameter \(D\) is 100 mm. The cut-on frequencies are 2029, 3364 and 4213 Hz for the (1, 0), (2, 0) and (0, 1) modes, respectively. The upper limit measurement frequency which is determined by the microphone distance \(s=25\) mm is 6237 Hz. Eight 1/4 inch microphones were used and were positioned in accordance with the theory. White noise was used as the noise source. In this study, the sum of the microphone signals was obtained in the frequency domain in order to correct for the amplitude and phase mismatches among the microphones.
3.2 Experimental results

Figure 4 shows the measured normal-incidence sound absorption coefficient of 25-mm-thick melamine foam as an example. In the figure, the faint solid line indicates the result obtained by two-microphone measurement (Mics. 1 and 5). The bold solid line indicates the result obtained by eight-microphone measurement. For comparison with the results of the conventional method, result measured using a small diameter tube (B&K 4206, $D = 29$ mm) is also shown. The figure shows that the dependence of the absorption coefficient on the frequency is reasonably smooth below about 4 kHz when using eight microphones, whereas the absorption coefficient fluctuates above the cut-on frequency of the $(1, 0)$ mode in the case of using only two microphones. In the frequency range in which a smooth curve for the absorption coefficient is obtained by the eight-microphone measurement, the absorption coefficient almost matches the value measured using a small-diameter tube.

In order to consider the scattering effects, normal-incidence absorption coefficient and the power reflection coefficients of the specimens, on which surface some scattering occurs, were measured. As Fig. 5 shows, the target specimen is 25-mm-thick melamine foam on which surface a polypropylene circular patch ($D=50$ mm, $t=0.4$ mm) is attached. The patch may disturb the acoustic field in the tube, scattering the normal-incidence wave in the frequency region in which the higher-order modes can propagate. Figure 5(a) is in the case that the patch was attached at the centre of the specimen. Figure 5(b) is in the case that the patch was attached at a position shifted from the centre.
Figure 5: Specimens under test. 25-mm-thick melamine foam on which surface a polypropylene circular patch ($D=50$ mm, $t=0.4$ mm) is attached.

(a) The patch is located at the centre.  
(b) The patch is located at a position shifted from the centre.

Figure 6: Measured results of 25-mm-thick melamine foam on which surface the polypropylene circular patch ($D=50$ mm, $t=0.4$ mm) is attached at the centre.

(a) Normal-incidence absorption coefficient.  
(b) Power reflection coefficients.

Figure 7: Measured results of 25-mm-thick melamine foam on which surface the polypropylene circular patch ($D=50$ mm, $t=0.4$ mm) is attached at a position shifted from the centre.

(a) Normal-incidence absorption coefficient.  
(b) Power reflection coefficients.
Figure 6 shows the measured absorption coefficient and the power reflection coefficients in the case that the polypropylene circular patch is attached at the centre of the specimen. As Fig. 6(b) shows, $R_{0\rightarrow 1}$ is small and $R_{0\rightarrow 0}$ is almost identical to $R_{tube}$ through the frequency range below $f_{c \ 0,1}$. This implies that the effect of the scattering is small in $R_{0\rightarrow 0}$. It is considered that in this condition, the scattering on the surface might not occur because the (1, 0) and (2, 0) modes, which nodal lines are located through the centre of the cross section of the tube, cannot be exited. Figure 7 shows the experimental results in the case that the patch is attached at a position shifted from the centre. In this case, the scattering occurs and the (1, 0) and (2, 0) mode can be excited. The measured normal-incidence absorption coefficient is larger than that in the case in Fig. 6, although they should ideally be the same. The curve slightly fluctuates above $f_{c \ 1,0}$, unlike that in Fig. 6. As Fig. 7(b) shows, the value of $R_{0\rightarrow 1}$ is larger than that in Fig. 6 through the frequency range and the maximum value is 0.054. Moreover, $R_{tube}$ is larger up to 0.053 than $R_{0\rightarrow 0}$. This difference implies the amount of the effect of the scattering. These results show that during the absorption measurement, the reliability of measured data can be checked by monitoring $R_{0\rightarrow 1}$ or $R_{tube}$.

4. Conclusions

Using the proposed eight-microphone method, the normal-incidence sound absorption coefficient can be measured at about twice the frequency of the conventional two-microphone transfer-function method under the assumption that the amount of scattering on the specimen surface is sufficiently small. If the scattering is large, however, the normal-incidence absorption coefficient cannot be measured accurately even using the proposed method. In practical measurement, the problem of the proposed eight-microphone method is that it is hard to tell how reliable the measured result is. Hence, the monitoring method of the scattering effects on a specimen surface has been proposed. The normal-incidence (1, 0)-mode-reflection coefficient and the total power reflection coefficient in a tube were derived by considering the difference between the signals of the two microphones placed on opposite sides of the tube. The experiments were conducted to demonstrate the validity of the proposed method. The theoretical consideration and the experimental results showed that by monitoring the scattering effects, measurement can be performed as confirming the reliability of the measured result.

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