AN ANALYTICAL RESEARCH ON THE VIBRATIONAL CHARACTERISTICS OF A PROPULSION SHAFT COUPLED WITH A CONICAL-CYLINDRICAL SHELL SYSTEM

Dongchen Xie, Cong Zhang and Xincong Zhou

School of Energy and Power Engineering, Key Laboratory of Marine Power Engineering & Technology(Ministry of Communications), National Engineering Research Center for Water Transport Safety, Wuhan University of Technology, Wuhan 430063, China

Email: zhangcong@whut.edu.cn

Li Qin

Key Laboratory of Metallurgical Equipment and Control Technology (Ministry of Education), Hubei Key Laboratory of Mechanical Transmission and Manufacturing Engineering, Wuhan University of Science and Technology, Wuhan 430081, China

Abstract: The ship is excited by the excitations on the propeller and the hull, which leads to coupling vibration between the propulsion shaft and the shell. This paper chooses the propulsion shaft with the conical-cylindrical shell as the object. Analytical method is used to describe the vibration of the conical-cylindrical shell and the multi-span shaft. Transversal continuity conditions between shell and shaft are built at the junction and the supporting bearings. Then the equations are solved. Excitations at the shaft end and excitations on the shell are considered. The characteristics of the vibration of the shaft with conical-cylindrical shell are discussed. The effects of different excitations and coupling performances between different sub-structures vibration of the system are analyzed.

Keywords: shaft vibration, conical-cylindrical shell, coupling effects, analytical method

1. Introduction

When the ship is navigating, the hull and propulsion shaft are excited by the different unbalance loads from waves and propulsion system. The research on the vibration of hull and propulsion shaft is very important for promoting the ship characteristics. Much of the earlier researches are about the characteristics of the cylindrical shell and the conical shell. The equations of motion for cylindrical shell and conical shell has been summarized by Leissa [1]. The free vibrational characteristics of isotropic coupled cylindrical-conical shell are solved using a wave solution to describe the displacements of the cylindrical shell and a power series solution to solve the displacements of the conical shell [2]. A modified variational approach for dynamic analysis of stiffened conical-
cylindrical shells is introduced by Qu and Chen, et al. [3-4]. The structural and acoustic response of a coupled conical-cylindrical-conical shell is solved by Caresta and Kessissoglou [5].

As the propulsion shaft is connected with shell via bearings, in recently years, many researchers focus on the vibration of propulsion shaft with shell, especially in longitudinal direction. S.Merz [6] built the model of submerged shaft-hull by FEM and BEM and analyzed the longitudinal transmission of vibration and sound. Caresta [7] used the equations of cylindrical shell, conical shell, shaft and actuators at thrust bearing and continuity equations to analyze the characteristics of vibration and the active control on longitudinal vibration.

Based on the existed references, in this paper, the model of a propulsion shaft coupled with a conical-cylindrical shell system is built. Boundary equations of shaft and shell and continuity equations at junctions are presented. Radial bearings are also considered in the model, as the connection between shaft and shell. Characteristics of vibration under loadings on the shaft and shell are discussed.

2. Dynamic Modelling

The model of a propulsion shaft coupled with a conical-cylindrical shell system is shown in Fig. 1. The propulsion shaft is connected to the coupled shell via intermediate bearing and stern bearings. In this model, bearings are simplified to the springs.

![Figure 1: The model of the propulsion shaft coupled with conical-cylindrical shell](image)

2.1 Dynamic response of a cylindrical shell

The cylinder is modeled using Flügge equations of motion which are given by [8]:

\[
\frac{\partial^2 u_c}{\partial x^2} + \frac{(1 - \mu_c)}{2a^2} (1 + \beta^2) \frac{\partial^3 u_c}{\partial \theta^2} + \frac{1}{2a} \frac{\partial^3 v_c}{\partial x \partial \theta} + \frac{\mu_c}{a} \frac{\partial w_c}{\partial x} - \beta^2 a \frac{\partial^3 w_c}{\partial x^3} + \beta \frac{(1 - \mu_c)}{2a} \frac{\partial^3 w_c}{\partial x \partial \theta^2} = 0
\]

\[
-\frac{\gamma}{c_{lc}^2} \frac{\partial^2 u_c}{\partial t^2} = 0
\]

\[
\frac{1 + \mu_c}{2a} \frac{\partial^3 u_c}{\partial x \partial \theta} + \frac{(1 - \mu_c)}{2} \frac{\partial^3 v_c}{\partial x^2} + \frac{\partial^3 v_c}{\partial \theta^2} + \frac{\partial w_c}{\partial \theta} + \beta^2 \left( \frac{3(1 - \mu_c)}{2} \frac{\partial^3 v_c}{\partial x^2} - \frac{(3 - \mu_c)}{2} \frac{\partial^3 w_c}{\partial x^2 \partial \theta} \right)
\]

\[
-\frac{\gamma}{c_{lc}^2} \frac{\partial^2 v_c}{\partial t^2} = 0
\]
\[-\beta^2 \left( a^2 \frac{\partial^4 w_c}{\partial x^4} + 2 \frac{\partial^4 w_c}{\partial x^2 \partial \theta^2} + \frac{1}{a^2} \frac{\partial^4 w_c}{\partial \theta^4} - \frac{1}{a^2} \frac{\partial^3 u_c}{\partial x^3} + \frac{(1-\mu_c)}{2a} \frac{\partial^3 u_c}{\partial x \partial \theta^2} - \frac{(3-\mu_c)}{2a^2} \frac{\partial^3 v_c}{\partial x^2 \partial \theta} + \frac{2}{a^2} \frac{\partial^2 w_c}{\partial \theta^2} \right) \]

\[-\frac{\mu_c}{a} \frac{\partial u_c}{\partial x} - \frac{1}{a^2} \left( \frac{\partial v_c}{\partial \theta} + w_c (1+\beta^2) \right) - \frac{\gamma c_{Le}}{c_{Le}^2} \frac{\partial^2 w_c}{\partial \theta^2} = 0\]

where \( a \) is the mean radius of the shell, and Poisson’s ratio of the hull. \( c_{Le} = \sqrt{E_c / \left[ \rho_c \left(1-\mu_c^2\right) \right]} \) is the longitudinal wave speed. \( \beta = h_c / \left(\sqrt{12a}\right) \) is a non-dimensional thickness parameter. \( h_c \) is \( E_c \), \( \rho_c \) and \( \mu_c \) are respectively the thickness, Young’s modulus, density and the Possion’s ratio.

The displacement in axial, circumferential and radial can be expressed as:

\[ u_c(x, \theta, t) = U_c e^{jk_c x} \cos(n\theta)e^{-j\omega t} \]  

\[ v_c(x, \theta, t) = V_c e^{jk_c x} \sin(n\theta)e^{-j\omega t} \]  

\[ w_c(x, \theta, t) = W_c e^{jk_c x} \cos(n\theta)e^{-j\omega t} \]  

where \( U_c \), \( V_c \), \( W_c \) are the amplitude of displacement in axial, circumferential and radial directions.

For each \( k_{nj} (i=1:8) \), they are function of \( W_{nj} (i=1:8) \).

### 2.2 Dynamic response of a conical shell

The equations of motion are given by Leissa [1] and Caresta [9]:

\[ L_{11} u_{co} + L_{12} v_{co} + L_{13} w_{co} - \rho_{co} h_{co} \frac{\partial^2 u_{co}}{\partial t^2} = 0 \]  

\[ L_{21} u_{co} + L_{22} v_{co} + L_{23} w_{co} - \rho_{co} h_{co} \frac{\partial^2 v_{co}}{\partial t^2} = 0 \]  

\[ L_{31} u_{co} + L_{32} v_{co} + L_{33} w_{co} - \rho_{co} h_{co} \frac{\partial^2 w_{co}}{\partial t^2} = 0 \]  

where \( \rho_{co} \) and \( h_{co} \) are the density and the thickness of the conical shell. The differential operators \( L_{ij} \) are given by Caresta [9]. The displacement in parallel to the shell surface, normal to the shell surface and radial are expressed as:

\[ u_{co}(x_c, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{m, n, co, n} \cos(m \theta) \cos(n \theta)e^{-j\omega t} = \sum_{n=0}^{\infty} u_{co, n} \cdot x_n \cos(n \theta)e^{-j\omega t} \]  

\[ v_{co}(x_c, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} b_{m, n, co, n} \cos(m \theta) \sin(n \theta)e^{-j\omega t} = \sum_{n=0}^{\infty} v_{co, n} \cdot x_n \cos(n \theta)e^{-j\omega t} \]
where $u_{co,n}, v_{co,n}, w_{co,n}$ are function of $u_{co,i} (i=1:8)$.

### 2.3 Dynamic response of a propeller shaft

For each segment of shaft beam, the bending dynamic equation of motion is as follows, according to the Euler-Bernoulli theory:

$$EI \frac{d^4 U_i(x,t)}{dx^4} + \rho \frac{d^2 U_i(x,t)}{dt^2} = 0, i = 1,2,\ldots,n+1$$  \hspace{1cm} (13)

where $t$ is time. $\rho$ is mass density per unit length. $E$ is the Young’s modulus and density of the shaft. $I_i$ is the moment of inertia of each segment of shaft.

According to Clough [10], the displacement of shaft can be solved from Eq. (13) as

$$U_i(x,t) = \Phi_i(x)Z_i(t)$$  \hspace{1cm} (14)

where $Z_i(t)$ is generalized coordinate and

$$\Phi_i(x) = A_i \cos a_{si}x + B_i \sin a_{si}x + C_i \cosh a_{si}x + D_i \sinh a_{si}x$$  \hspace{1cm} (15)

where $A_i, B_i, C_i, D_i$ are the coefficients of each segment. $\Phi_i(x)$ is the mode function. $a_{si}$ is given by

$$a_{si}^4 = \frac{\omega_s^2 \rho_{si}}{E_i I_{si}}$$  \hspace{1cm} (16)

where $\omega_s$ is the natural frequency.

### 2.4 Boundary and continuity equations

The responses for the shaft with coupled shell can be solved by boundary condition and continuity conditions. For the shell with the free conical end and cylindrical end, the boundary conditions are:

Conical end:

$$F_{co,x} = 0 \quad F_{co,\theta} = 0 \quad F_{co,r} = 0 \quad M_x = 0$$  \hspace{1cm} (17-20)

Cylindrical end:

$$F_{c,x} = 0 \quad F_{c,\theta} = 0 \quad F_{c,z} = 0 \quad M_{c,x} = 0$$  \hspace{1cm} (21-24)
The continuity conditions of the junction of conical-cylindrical shell are:

\[ u_c = \bar{u}_{co}, \quad v_c = \bar{v}_{co}, \quad w_c = \bar{w}_{co}, \quad \frac{\partial w_c}{\partial x} = \frac{\partial w_{co}}{\partial x} \]

\[ N_{c,x} = \tilde{N}_{co,x}, \quad N_{c,\theta} + \frac{M_{c,\theta}}{R} = \tilde{N}_{co,\theta} + \frac{M_{co,\theta}}{R}, \quad V_{c,x} = \tilde{V}_{co,x}, \quad M_{c,x} = M_{co,x} \]  

(25-32)

where

\[ \tilde{F}_{co,x} = F_{co,x} \cos \alpha - F_{co,\theta} \sin \alpha \]

(33)

\[ \tilde{w}_{co} = u_{co} \sin \alpha + w_{co} \cos \alpha \]

(34)

For the shaft, the boundary conditions of the free ends are:

\[ EI_1 \frac{\partial^3 w(x,t)}{\partial x^3} \bigg|_{x=0} = 0, \quad EI_1 \frac{\partial^2 w(x,t)}{\partial x^2} \bigg|_{x=0} = 0 \]

(35-36)

\[ EI_1 \frac{\partial^3 w(x,t)}{\partial x^3} \bigg|_{x=L} = 0, \quad EI \frac{\partial^2 w(x,t)}{\partial x^2} \bigg|_{x=L} = 0 \]

(37-38)

At the bearing where connects shaft and the shell, the continuity conditions of the shaft parts are:

\[ w_n(x_n') = w_{n+1}(x_n') \]

\[ \frac{\partial w_n(x_n')}{\partial x} = \frac{\partial w_{n+1}(x_n')}{\partial x} \]

\[ EI_n \frac{\partial^3 w_n(x_n')}{\partial x^3} = EI_{n+1} \frac{\partial^3 w_{n+1}(x_n')}{\partial x^3} + k_n \left( w_n(x_n') - \tilde{w}_{\partial,ij} \right) \]

(39-42)

\[ EI_n \frac{\partial^2 w_n(x_n')}{\partial x^2} = EI_{n+1} \frac{\partial^2 w_{n+1}(x_n')}{\partial x^2} \]

where \( x'_n, x'_n \) are the left side and right side of the shaft part, respectively. \( k_n \) is the stiffness of the bearing. Meanwhile, the continuity conditions of the shell parts are:

\[ u_{co,i} = u_{co,i+1}, \quad v_{co,i} = v_{co,i+1}, \quad w_{co,i} = w_{co,i+1}, \quad \frac{\partial w_{co,i}}{\partial x} = \frac{\partial w_{co,i+1}}{\partial x} \]

\[ F_{co,x,i} = F_{co,x,i+1} + k_n \left( w_n(x_n') - \tilde{w}_{\partial,co,i} \right) \sin \alpha, \quad F_{co,\theta,i} = F_{co,\theta,i+1} \]

(43-50)

\[ F_{co,x,i} = F_{co,x,i+1} + k_n \left( w_n(x_n') - \tilde{w}_{\partial,co,i} \right) \sin \alpha, \quad F_{co,\theta,i} = F_{co,\theta,i+1} \]
where
\[
\tilde{w}_{i,o,d} = w_{i,o} \cos \alpha + u_{o,d} \sin \alpha
\]  

(51)

3. Results and discussion

The model of the propulsion shaft coupled with conical-cylindrical shell is built using the following parameters as Fig. 2 shown: \( L_{co} = 5.08\) m, \( R_1 = 0.3\) m, \( R_2 = 0.9\) m, \( L_{cy} = 2.25\) m, \( R_s = 0.035\) m, \( L_s = 5.88\) m. The thickness of conical-cylindrical shell is 0.006 m. The material properties for steel are: density \( \rho = 7800\) kg\(m^{-3}\), Young’s modulus \( E = 2.1 \times 10^{11}\) N\(m^{-2}\) and Poisson’s ratio \( \mu = 0.3\). The aft stern bearing is located at 1.24 m from the propeller end of shaft and 0.84 m from the conical end of shell. The forward bearing and intermediate bearing are 1.35 m and 1.89 m from the propeller end of shaft, respectively. The stiffness of each bearing is given by \( K_b = 1 \times 10^9\) N\(m^{-2}\).

The excitations at the end of conical shell and the end of the shaft are considered respectively to discuss the responses of shaft and coupled shell under loading at different places. The observed points are all at the junction of bearings. \( C_1, C_2, C_3 \) are on the shell and \( S_1, S_2, S_3 \), while \( S_1, S_2, S_3 \) are on the shaft. The results are shown in Fig. 3-4.

Figure 2: The excitations on the model
It can be seen from the figures that the displacement of shell is larger in Fig. 3 than in Fig. 4, while the displacement of shaft is larger in Fig. 3 than in Fig. 4, due to the different locations of the excitations. However, in both cases, transversal displacement of shell is larger than the displacement of shaft, which means that the shell is easier to vibrate than the shaft, no matter when the excitation is on the shell or on the shaft, due to their characteristics. Moreover, the shaft vibration is less influenced by the excitation on the shell but the vibration of shell is obviously sensitive to the excitation on the shell. As the transversal vibration of the shell are related to the sound transmission, the vibration transmitted from shaft via bearings are needed to be noticed.

4. Conclusions

An analytical method to solve the vibration of the model of a propulsion shaft coupled with a conical-cylindrical shell system is introduced in this paper. Radial bearings are also considered in the model, as the connection between shaft and shell and they are simplified to springs. Boundary equations of shaft and shell and continuity equations at junction are presented. Characteristics of vibration under loadings on the shaft and shell are discussed. The conclusions are in the model of a shaft coupled with a conical-cylindrical shell, shell is easier to vibrate than the shaft, no matter when the excitation is on the shell or on the shaft. Moreover, the shell is obviously sensitive to the excitation on the shell. Thus, the vibration transmitted from shaft via bearings are needed to be noticed to decrease to sound transmission due to vibration of the shell in the direction of normal to the shell.
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REFERENCES