A PREDICTION MODEL FOR METRO TRAIN INDUCED VIBRATION OF TOPSIDE BUILDINGS ON UNDERGROUND METRO GARAGE

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This paper presents a prediction model for metro train induced vibration of the topside building on the underground metro garage. The model is developed based on the vehicle-track coupled dynamics and is comprised of two subsystems. The first subsystem is the train-track coupled dynamic subsystem and the second subsystem is the track-garage-building coupling dynamic subsystem. The relationship between the two subsystems are wheel/rail forces which can be obtained using the first subsystem. In the first subsystem, the train is marshalled by several vehicles and each vehicle is simplified as multi-rigid-body system, and the rail is modelled as Timoshenko beam. Track geometric irregularities and the interaction between train and railway turnout are the excitations and the wheel/rail interactions are modelled by Hertz nonlinear elastic contact theory. In the second subsystem, the underground metro garage, the topside buildings on the garage and the soil around the garage are modelled using finite elements, and the equivalent elastic boundaries are applied to reduce the impact of artificial boundaries on the calculation results.

Keywords: building vibration, vehicle-track coupled dynamics, topside buildings, metro train

1. Introduction

As an efficient, fast, safe and comfortable mean of transportation, the subway has been used in more and more cities to solve the traffic congestion. In China, there are many cities are in the plan and construction of urban rail transit, and the scale of construction is the biggest in the world[1]. However, the construction of subway is huge, expensive and of occupying a lot of land. Due to the shortage of land source and the high cost of land acquisition, how to use the land efficiently has become a focus in the subway construction and urban planning. The metro garage is the largest intensive land occupation, so developing the topside buildings on the metro garage is an effective mean of improving the efficiency of land use and increasing the profit for subway operation. Unfortunately, the running train could induce the vibration of topside buildings which has quite adverse influences on the structure safety of buildings, the operating of precision instruments and the life quality of residents in the buildings.

The issues on the train induced vibration of topside buildings on garage belongs to the environmental vibration induced by moving train. In general, the train is simplified as rigid multi-body system and linear or nonlinear wheel/rail interaction is used to calculation the wheel/rail forces[2-4], and many methods can be used to establish the dynamic model of the track, soil and buildings, such as analytical method[5-7], the semi-analytical and semi-numerical method[8-12] numerical method[13-17], field measurement[18-20] and the hybrid method of in-situ test and simulation[21-25].
This paper focuses on the prediction of the metro train induced vibrations of topside buildings on underground metro garage. The underground metro garage is of two floors and an operating control centre (OCC) and a residential quarter is on the garage. The garage and the OCC have been completely constructed and comes into service. The residential quarter contains 15 residential buildings and is under planning. The location of buildings relative to the garage is shown as Fig. 1. In order to estimate the train induced vibrations of the residential buildings, a prediction model is developed in this paper. Due to the structure of the garage and the buildings are complex, the pure numerical method is adopted. Firstly, a vertical train-track coupling dynamic model is established, based on the theory of vehicle-track coupling dynamics\cite{3,4}, to obtain the vertical wheel/rail forces. Then, the track-garage-buildings coupling dynamic model is established by the FEM and the moving wheel/rail forces are the input excitations to calculate the vibrations of the garage and the buildings.

\section{Prediction model}

\subsection{Vertical Train-track coupling dynamic model}

In the garage, the rails are discretely supported on the floor by fasteners. The thickness of the floor is large, so it will deform small induced by the moving train, because the velocity of the train entering and exiting the garage is low. So the effect of the floor deformation on the wheel/rail can be ignored. Based on the idea of vehicle-track coupling dynamics, a vertical train-track coupling dynamic model is proposed, as shown in figure 2. The train is composed of many vehicles and each vehicle is a multi-rigid body of 10 DOFs including the floating and nodding of the car body, the floating and nodding of each bogie and the floating of each wheel. The rail is modelled by a long enough simply supported Timoshenko beam and the fasteners are modelled by the discrete linear spring-damping elements. The wheel/rail force is calculated by the nonlinear Hertz contact theory.
The equations of motion of each vehicle can be easily derived according to D’Alembert’s principle, which can be described in form of second order differential equations in the time domain:

\[ \mathbf{M}_v \ddot{\mathbf{z}}_v (t) + \mathbf{C}_v \dot{\mathbf{z}}_v (t) + \mathbf{K}_v \mathbf{z}_v (t) = \mathbf{G}_v + \mathbf{H}_v \mathbf{p}_v (t) \]  

where \( \mathbf{M}_v, \mathbf{C}_v \) and \( \mathbf{K}_v \) are the mass, damping and stiffness matrix of the vehicle, \( \mathbf{z}_v (t), \dot{\mathbf{z}}_v (t) \) and \( \ddot{\mathbf{z}}_v (t) \) are the displacement, velocity and acceleration vector of the vehicle, \( \mathbf{G}_v \) is the gravity vector of the vehicle, \( \mathbf{p}_v \) is the vector comprised of the wheel/rail forces applied at the 4 wheelsets, and \( \mathbf{H}_v \) is the transition matrix which transforms the wheel/rail forces into the load vector applied on the vehicle.

The equations of motion of the Timoshenko beam for the rail are

\[
\left\{ \begin{array}{l}
\rho_1 A_1 \frac{\partial^2 z_r (x,t)}{\partial t^2} + \kappa A_1 G_r \left[ \frac{\partial \varphi_r (x,t)}{\partial x} - \frac{\partial^2 z_r (x,t)}{\partial x^2} \right] = \sum_{m=1}^{N_m} \rho_m(t) \frac{\delta (x - x_{wm})}{\partial x} - \sum_{i=1}^{N_{rp}} f_{rsi} (t) \frac{\delta (x - x_{sri})}{\partial x} \\
\rho_1 I_1 \frac{\partial^2 \varphi_r (x,t)}{\partial t^2} + \kappa A_1 G_r \left[ \varphi_r - \frac{\partial^2 z_r (x,t)}{\partial x^2} \right] - E_r I_1 \frac{\partial^2 \varphi_r (x,t)}{\partial x^2} = 0
\end{array} \right.
\]  

(2)

In the Eq. (2), \( z_r (x,t) \) and \( \varphi_r (x,t) \) are the vertical and rotational displacements of the rail at the position \( x \) at time \( t \), \( \rho_r, A_r, I_r, E_r, G_r \) and \( \kappa \) are the density, cross-sectional area, cross-sectional moment of inertia, elastic modulus, shear modulus and shear factor of the rail, \( \rho_m \) and \( x_{wm} \) are the value and position of the \( m \)th wheel/rail force, \( N_r \) is the amount of the vehicle of the train, \( f_{rsi} \) and \( x_{sri} \) are the resilience of the \( i \)th fastener and its position in the global coordinate system and \( N_{rp} \) is the amount of the fasteners corresponding to a rail.

The resilient of the \( i \)th fastener is

\[ f_{rsi} (t) = K_p (x_{sri}, t) + C_p \frac{\partial z_{ri} (x_{sri}, t)}{\partial t} \]  

(3)

where \( K_p \) and \( C_p \) are the stiffness and damping of the fastener.

In order to The partial differential Eq. (2) can be solved using numerical integration by translating it into ordinary differential equation. The vertical and rotational displacements of the rail \( z_r (x,t) \) and \( \varphi_r (x,t) \) are written in the form of product of shape functions and shape coordinates, as

\[
\begin{align*}
\dot{z}_r (x,t) &= \sum_{n=1}^{N_{mR}} Z_n (x) q_n (t) \\
\dot{\varphi}_r (x,t) &= \sum_{n=1}^{N_{mR}} \Psi_n (x) w_n (t)
\end{align*}
\]  

(4)

where \( Z_n (x) \) and \( q_n (t) \) are the vertical shape functions and vertical shape coordinates, and \( \Psi_n (x) \) and \( w_n (t) \) are the regular shape functions and regular shape coordinates. The expressions of the shape functions are

\[
\begin{align*}
Z_n (x) &= \frac{2}{\rho_1 A_1} \sin \left( \frac{n \pi x}{L_t} \right) \\
\Psi_n (x) &= \frac{2}{\rho_1 I_1 L_t} \cos \left( \frac{n \pi x}{L_t} \right)
\end{align*}
\]  

(5)

In the Eq. (5), \( L_t \) is the length of the rail. Substituting Eq. (4) into (2) and using the Ritz method, the Eq. (2) is transformed into the ordinary differential equation and written as matrix form

\[ \mathbf{M}_r \ddot{\mathbf{z}}_r (t) + \mathbf{C}_r \dot{\mathbf{z}}_r (t) + \mathbf{K}_r \mathbf{z}_r (t) = \mathbf{p}_r (t) \]  

(6)

where \( \mathbf{M}_r, \mathbf{C}_r \) and \( \mathbf{K}_r \) are generalized mass, damping and stiffness matrix of the rail, \( \mathbf{z}_r (t), \dot{\mathbf{z}}_r (t) \) and \( \ddot{\mathbf{z}}_r (t) \) are the generalized displacement, velocity and acceleration vector of the rail and \( \mathbf{p}_r (t) \) is the generalized load vector applied on the rail.

According to the nonlinear Hertz elastic contact theory, the \( m \)th vertical wheel/rail force is
\[ p_m(t) = \left[ \frac{1}{G} \delta z_m(t) \right]^{3/2} \]

where \( G \) is the wheel/rail contact constant, \( \delta z_m(t) \) is the elastic compression deformation of the \( m \)th wheel/rail contact, written in the form

\[ \delta z_m(t) = z_{wm}(t) - z_r(x_{wm}, t) - z_{0m}(t) \]

in which \( z_{wm}(t) \) is the vertical displacement of the \( m \)th wheelset, \( z_r(x_{wm}, t) \) and \( z_{0m}(t) \) are the vertical displacement of the rail and the displacement irregularity of the track at the position of the \( m \)th wheel/rail contact.

The calculation efficiency of the train-track coupling dynamic model established above is improved by using an explicit integration scheme which is constructed by Zhai [26]. The forms of the explicit integration scheme are

\[
\begin{align*}
\{z\}_{n+1} &= \{z\}_n + \{z\}_n \Delta t + \left( 0.5 + \alpha \right) \{\dot{z}\}_n \Delta t^2 - \alpha \{\dot{z}\}_{n-1} \Delta t^2 \\
\{\dot{z}\}_{n+1} &= \{\dot{z}\}_n + \left( 1 + \beta \right) \{\ddot{z}\}_n \Delta t - \beta \{\ddot{z}\}_{n-1} \Delta t
\end{align*}
\]

where \( \Delta t \) is the time step, and subscripts \( n+1, n, \text{ and } n-1 \) denote integration time \((n+1)\Delta t, n\Delta t, \text{ and } (n-1)\Delta t\), respectively. \( \alpha \) and \( \beta \) are free parameters which control the stability and numerical dissipation of the algorithm.

### 2.2 Track-garage-building coupling dynamic model

In order to reduce the total number of the finite elements and improve the calculation efficiency, the soil is modelled as equivalent elastic boundary rather than being modelled using FEM. The reaction coefficients and the damper values of the elastic boundary are decided by the dynamic parameters of the soil, as

\[
\begin{align*}
k_{sv} &= \frac{\chi}{30} E \left( \frac{A_{sv}}{30} \right)^{3/4} , \quad k_{sh} = \frac{\chi}{30} E \left( \frac{A_{sh}}{30} \right)^{3/4} \\
c_{sp} &= \rho \sqrt{\frac{\lambda + 2G}{\rho}} , \quad c_{ss} = \rho \sqrt{\frac{G}{\rho}}
\end{align*}
\]

In Eq. (10), \( k_{sv} \) and \( k_{sh} \) are the vertical and horizontal soil reaction coefficient, \( A_{sv} \) and \( A_{sh} \) are the cross-sectional areas in vertical and horizontal direction, respectively; \( E \) is the elastic modulus of the soil; \( \chi \) is usually applied as 1.0; \( c_{sp} \) and \( c_{ss} \) are the damper values corresponding to the pressure wave and shear wave per unit area, respectively, \( \lambda \) and \( G \) are the Lame’s constant of the soil, \( \rho \) is the density of the soil.

The garage and the buildings belong to plate-column structure, so they can be modelled by plate elements and beam elements. The rail is modelled by Euler beam. The maximum size of the element is less than 0.5m. A part of the developed finite element model of the track-garage-building coupling dynamics is shown as Fig. 3.

![Figure 3: finite element model of the track-garage-building coupling dynamics](image-url)
3. Calculation and discussions

3.1 Excitation of the train-track coupling dynamic system

The wheel/rail forces are decided by the Eqs. (7) and (8), so the displacement irregularity of the track must be obtained before calculating the wheel/rail forces. The displacement irregularity can be regarded as the excitation of the train-track coupling dynamic system. According to the real condition of the track, the combination of the random irregularities and the irregularities induced by the track switches is used in the calculation. For the random irregularities, The AAR 6th class spectrum (wave length 1.0m~80m) and Sato spectrum (wave length 0.1m~1.0m) are used. The irregularities induced by the track switch is decided by the geometry of the track switch and can be mathematically expressed as

\[
    z_0(x) = \begin{cases} 
    h_0 \sin \left[ \frac{2\pi(x-x_0)}{L_0} \right] & \text{for } x_0 \leq x \leq x_1 \\
    h_0 \left( x-x_1 \right) / \left( x_2-x_1 \right) & \text{for } x_1 \leq x \leq x_2 \\
    h_d \left( x-x_2 \right) / \left( x_3-x_2 \right) & \text{for } x_2 \leq x \leq x_3 \\
    0 & \text{for } x_3 \leq x
    \end{cases}
\]  

(12)

where \( x_0 \) is the origin coordinate of the track switch in the track, \( h_0 \) and \( h_d \) are relative to the geometry of the track switch, referring to [3]. The displacement irregularity of the track is shown as Fig. 4.

![Figure 4: vertical displacement irregularity of the track](image)

3.2 Field test and verification

Because the residential buildings are under planning and the field test of the train induced vibration of the garage has been implemented, the track-garage coupling dynamic model is firstly established following the process mentioned in section 2.2 to predict the vibration of the garage and verify the model by comparison with the field test. The photographs of the field test are shown as Fig. 5. The test result and prediction result of the vibration of the garage at the position on the ground floor, corresponding the centre of building B1 (shown in Fig. 1), are shown as Fig. 6. The modelling process is verified by the comparison.
3.3 Vibration prediction of building

The track-garage-building coupling dynamic model is used to predict the train induced vibration of the building on the garage. The computational process is similar with that in section 3.2. The vibration of the building B1 at the first floor is shown as in Fig. 7. It is found from Fig. 7 that the vibration of the building B1 at the first floor is concentrated in the frequency range of 20~100 Hz and the maximum value is in the frequency range of centre frequency 50Hz.
4. Conclusions

The proposed prediction model for metro train induced vibration of topside building on metro garage is verified. The model is comprised of two subsystems of the train-track coupled dynamic subsystem and the track-garage-building coupling dynamic subsystem. The excitation of random irregularities and the track switch are considered.

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