A STUDY ON BAND GAPs OF ACOUSTIC METAMATERIALs WITH PARALLEL OSCILLATORS

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In this study, the band gaps and transmission of a multi-resonator lattice system are investigated. Different from the traditional mass-spring-mass system, the unit cell of the lattice system consists of multiple separate masses. And each of the oscillators are not in contact with each other, all of them are connected to the basic mass by linear springs. First, the band structure of the modified acoustic metamaterial is calculated by using Bloch’s theorem under the assumption of infinite lattice. The results show that more band gaps can be obtained compared with the conventional acoustic metamaterials, and the width of the band gap is also greatly improved. It is found that the inherent frequency of the oscillators has a great influence on the width of the pass band. Besides, increasing the number of the oscillators in one unit cell with mass graded, much more band gaps can be obtained. By adjusting the mass of the oscillators and the stiffness of the springs to lead the inherent frequency of the oscillators are similar, it is easy to obtain dense band gaps in a certain area, thus greatly stopping the propagation of the wave. Finally, the transmission of the proposed mode is analysed.

Keywords: bandgap, acoustic metamaterial, spring-mass, wave propagation

1. Introduction

Wave propagation phenomenon in periodic structure has been found and investigated for several decades. Frequency ranges in which wave can propagate or not are referred to as pass bands or band gaps. Bragg Scattering mechanism strongly depends on the size of periodic constant, although the wave attenuation in the band gap is larger, the band gap frequency is relatively high. Liu et al. constructed a sonic crystal with coated lead sphere, which exhibits spectral bandgaps two orders of magnitude smaller than the relevant wavelength called locally resonant mechanism [1]. This pioneering work directly leads to the generation of acoustic metamaterials (AMs). Since then, much more attention has been paid to AMs with negative effective mass. Essentially, the external vibration is weakened by local resonance of subsystems between certain frequency bands, thus, elastic waves are forbidden to propagate forward.

For one-dimensional (1D) AMs, a great amount of theoretical and experimental research effort has been made to broaden the band gap width. Another way for the design of multi-resonator AMs is increasing the number of internal masses parallel arranged in each AMs unit cell [2,3]. In this study, oscillators are parallelly arranged in each unit cell, and the band gap and wave transmission of such lattice system will be investigated. It is found that the inherent frequency of the oscillators has a great influence on the width of pass band. The aim of this study is to obtain dense band gaps in a certain area by adjusting the mass of oscillators and the stiffness of springs.
2. Infinite lattice model with two parallel resonators

![Infinite lattice model with two parallel resonators](image)

In order to simplify the subsequent calculation, model considered in this section is an infinite lattice system with two resonators in each cell, as shown in Fig. 1. Through the analysis of this particular case, the influence of mass, stiffness as well as inherent resonant frequency on the band gap structure is studied.

For the elastic wave propagation in this infinite lattice system, governing equations of motion can be written as \[4,5\]

\[
\begin{align*}
(1) & \quad m_1 \ddot{u}_1^{(i)} + k_1 (2u_1^{(i)} - u_1^{(i-1)} - u_1^{(i+1)}) + k_2 (u_2^{(i)} - u_2^{(i-1)}) + k_3 (u_3^{(i)} - u_3^{(i-1)}) = 0 \\
(2) & \quad m_2 \ddot{u}_2^{(i)} + k_2 (u_2^{(i)} - u_1^{(i)}) = 0 \\
(3) & \quad m_3 \ddot{u}_3^{(i)} + k_3 (u_3^{(i)} - u_1^{(i)}) = 0
\end{align*}
\]

where, \(u_n^{(i)}\) represents the displacement of mass “\(n\)” in the \(i\)-th unit cell. Waveform of the displacement for stationary response is

\[
\begin{align*}
\dot{u}_n^{(i)} & = A_n e^{i(qv - \omega t)} \\
\dot{u}_n^{(i+m)} & = A_n e^{i(qv+mql - \omega t)}
\end{align*}
\]

Substituting Eq. (4) and Eq. (5) into Eqs. (1), (2) and (3) yields

\[
\begin{bmatrix}
-\omega^2 m_1 + 2k_1 (1 - \cos(ql)) + k_2 + k_3 & -k_2 & -k_1 \\
-k_2 & -\omega^2 m_2 + k_2 & 0 \\
-k_3 & 0 & -\omega^2 m_3 + k_3
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2 \\
A_3
\end{bmatrix} = 0
\]

In order to have a non-trivial solution, the determinant of coefficient matrix in Eq. (6) has to be zero. Dispersion equation can be then obtained as

\[
\begin{align*}
-m_1 m_2 m_3 \omega^6 + (2 \cos(ql) k_1 m_2 m_3 + 2 k_1 m_2 m_3 + k_2 m_2 m_3 + k_3 m_2 m_3 + k_1 m_1 m_3 + k_2 m_1 m_3 + k_3 m_1 m_3) \omega^4 \\
+ (2 \cos(ql) k_1 k_2 m_3 + 2 \cos(ql) k_1 k_3 m_2 + 2 \cos(ql) k_2 k_3 m_1 - 2k_1 k_2 m_2 - 2k_1 k_3 m_1 - 2k_2 k_3 m_3 \omega^2 - 2 \cos(ql) k_1 k_2 m_3 + 2k_1 k_2 k_3 = 0
\end{align*}
\]

In all the above equations, \(m_1\) is the mass of main oscillator, \(m_2\) and \(m_3\) are the mass of internal additional oscillators. Spring stiffness is defined as \(k_1, k_2\) and \(k_3\). These unit cells are uniformly placed with a periodic distance constant \(l\). \(A_n\) is complex wave amplitude, \(q\) is wavenumber and \(\omega\) is angular frequency.

Equation (7) is an even function of \(\omega\) of 6-th order, then three non-negative solutions can be obtained for each given wavenumber \(q\). Thus, it can be speculated that there will be three branches corresponding to three real solutions, and they will form two band gaps.

The data of each figure in Fig. 2 are collected in Table 1. In Fig. 2(a), (b) and (c), there are only one oscillator. In Fig. 2(c), there are two oscillators with the same mass and spring stiffness. Compared with Fig. 2(a), it can be seen that when the total mass of oscillator is constant, the oscillator divided into two parallel parts can broaden width of band gap. The passband width is very small, close to zero. However, the position of band gap tends to the high frequency. Compared with Fig. 2(b),
it can be seen that when an oscillator with the same mass and spring stiffness is added, the band gap width will increase greatly, and the band gap will move to lower frequency and the width of the passband is very small. In Fig. 2(e), there are two oscillators with same inherent frequency. Compared with Fig. 2(a) and Fig. 2(c), the parameters of two oscillators are same. The band gap range completely contains the sum of band gap range in two graphs. Compared with Fig. 2(d), Band gap width does not increase with the increase of mass, but band gap position is more inclined to low frequency. In Fig. 2(f), the inherent frequency of two oscillators are different, the passband width is large. Compared with Fig. 2(e), the only difference is spring stiffness $k_3$, although the band gap width becomes larger, its position tends to high frequency. Fig. 2(g) shows the passband width will increase with the increase of inherent frequency, and the width is close to zero when the inherent frequency is same. Fig. 2(h) shows the variation trend of total band gap width with difference of natural frequency.

Table 1: Simulation parameters used in numerical calculation

<table>
<thead>
<tr>
<th></th>
<th>$m_1$(kg)</th>
<th>$k_1$(n/m)</th>
<th>$m_2$(kg)</th>
<th>$k_2$(n/m)</th>
<th>$m_3$(kg)</th>
<th>$k_3$(n/m)</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>0.056</td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.028</td>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0.028</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>0.028</td>
<td>70</td>
<td>0.028</td>
<td>70</td>
<td></td>
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<tr>
<td>e</td>
<td>0.028</td>
<td>70</td>
<td>0.028</td>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>0.056</td>
<td>70</td>
<td>0.028</td>
<td>70</td>
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</tr>
</tbody>
</table>

(a) (b) (c) (d)
From the above analysis and comparison, it can be found that the addition of parallel oscillator can effectively expand the total band gap width, and generate the band gap in much lower frequency. As long as the inherent frequency of oscillator is kept the same, the pass band width between the band gaps will be very small. In addition, if the total mass of oscillator is constant, the total width of band gap will be greatly increased if the oscillator is divided into the same parallel parts.

3. Infinite lattice model with multiple parallel resonators

The model accounted for in this part is an infinite lattice model with multiple resonators, as shown in Fig. 3. Similar to the governing equation in the above section, system matrix can be expressed as
In order to have a non-trivial solution, the determinant of coefficient matrix of Eq. (8) has to be zero.

In Fig. 4, it shows the band gaps of infinite lattice model with three parallel resonators. It can be seen that when the inherent frequency of oscillators is the same, optical mode and third mode are basically overlapping and the corresponding frequency is the inherent frequency of oscillator. Compared to Fig. 2(d), an extra same oscillator is added in Fig. 3(a), the band gap range is extended, especially in high frequency, and completely contains the band gap range in Fig. 2(d). Compared to Fig. 2(e), it is found that inherent frequency and mass of oscillators are kept constant, the width and position of band gap will not change when increasing the number of oscillator. In Fig. 3(c), compared to Fig. 3(b), the oscillators have same inherent frequency and half mass and stiffness of spring. The range of band gap is obviously reduced. In Fig. 3(d), oscillators have the same inherent frequency and different mass and spring stiffness. In Fig. 3(e) and (f), there are ten oscillators in one unit cell, five obvious band gaps can be observed, which indicates the existence of overlapping modes. Compared with previous band gaps, with the increase of oscillator number, lower frequency band gap can be obtained, and the total width of band gap has great change, especially in high frequency range.
4. Finite lattice model with multiple parallel resonators

Finite lattice model with multiple parallel resonators will be studied in this section, number of unit cells in this model is $n$. As the calculation results shown in second section, for elastic wave propagation in this limited lattice model, the equations of motion for the $i$-th unit cell can be expressed as [6]

$$(-\sigma^2 m_i + k_1 + k_2 + \cdots + k_p)U_i^{(i)} - k_1 U_i^{(i-1)} - k_2 U_i^{(i+1)} - k_3 U_i^{(i)} - \cdots - k_n U_i^{(i)} = 0$$  \hspace{1cm} (9)

$$U_n^i = \frac{k_{2i+1}}{-\sigma^2 m_{2i+1} + k_{2i+1}} U_{n-1}^i$$  \hspace{1cm} (10)

with

$$\tau = 20 \log \left( \frac{U_n^i}{U_1^i} \right)$$  \hspace{1cm} (11)

and in the equations above, $\sigma$ is the forcing frequency (rad/s) and $\tau$ is the transmittance.

In order to simplify the calculation, ten unit cells in limited lattice model is considered in this paper. In Fig. 5, it shows the comparison of finite lattice model transmittance. The three graphs correspond to changing the total mass, stiffness, and natural frequency, respectively. Results are in good agreement with the previous analysis. Besides, in Fig. 5(b), it can be seen that transmittance shifts to the left as the number of vibrators increases. In Fig. 5(c), it is interesting to see that when the number of oscillator increases, band gap overlap range is more and more. The model of five parallel oscillators almost completely contains the band gap range of the model with three parallel oscillators.
Fig. 5 Comparison of limited lattice model transmittance
5. Conclusion

In this paper, design of increasing the number of oscillators parallel arranged in each unit cell is studied. Numerical results show that this design can provide a wider band gap range selection by adjusting the total mass of oscillators, spring stiffness, as well as the inherent frequency. It can shed some light for the generation of much lower and wider band gaps.

REFERENCES