MODAL DAMPING ESTIMATION FOR FLOATING FLOORS USING COMPOSITE MATERIAL ISOLATION

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Due to loading restrictions, it is often necessary to use multiple isolation materials in a single floating floor system. In addition to the transmissibility of the system, some cases require knowledge about the peak amplitude at the system’s resonance frequency. Methods for determining this peak amplitude for single isolation materials with individual loss factor have already been well documented, though this paper focuses on determining the peak for cases of multiple isolation materials with differing material loss factors. In particular, the modal loss factor of floating floor systems with various composite isolation layer combinations has been evaluated using the Modal Strain Energy (MSE) method and compared with experimental results. Both resonance frequency peak amplitude and modal loss factor predictions using the MSE method show fair agreement with measured values.

Keywords: modal strain energy method, modal loss factor

1. Introduction

Floating floor systems are widely used to achieve high sound reduction and vibration isolation. Generally, the system is comprised of an isolated slab and an isolation layer on the floor slab. The first step in designing such an isolation system is to determine the resonance frequency $f_r$ [Hz] of the system as calculated by Eq. (1).

$$f_r = \frac{1}{2\pi} \sqrt{\frac{k}{m}},$$  \hspace{1cm} (1)

where $k$ is the stiffness (spring constant) of the isolation layer, and $m$ is the mass of the isolated slab.

Fiberglass sheets are a common isolation material used in floating floor systems though this material does not have a very high load capacity. In order to support loads exceeding the load capacity of fiberglass sheets alone, the isolation layer is sometimes comprised of multiple materials, such as fiberglass sheets and isolation rubber blocks. The combination of different isolation materials leads
to the parallel arrangement of springs, i.e., the system spring constant can be obtained by summing the spring constants of each material.

Isolation performance of floating floor systems is described by the vibration transmissibility. Above $\sqrt{2} \cdot f_r$ [Hz], the transmissibility is $\leq 1$. In the architectural acoustics, the resonance frequency is usually set below 20 Hz to obtain sound reduction performance in the 63 Hz band. Although setting the resonance frequency alone is usually sufficient to design floating floor systems in practice, the estimation of the peak amplitude of the accelerance on the floating slab at the system’s resonance frequency is sometimes required. The peak amplitude is determined by the loss factor of the isolation layer. Since composite layers, such as fiberglass sheets and rubber blocks, have different material loss factors, the loss factor at the resonance frequency (i.e. “modal loss factor”) cannot be obtained via the simple calculation used with spring constants arranged in parallel. The paper focuses on the prediction of the modal loss factor using such a composite isolation.

Oberst et al. have proposed an approximate equation for the modal loss factor of laminated visco-elastic and elastic materials [1]. Johnson et al. proposed the Modal Strain Energy (MSE) method and implemented it using the finite element method to predict modal loss factors of damped complex structures [2, 3]. In this paper, a practical method of calculating modal loss factors using the MSE method for floating floor systems is discussed and compared to an example floating floor system.

2. Modal Strain Energy (MSE) method

In this section, the relationship between the modal loss factor and strain energy using the MSE method is briefly described. For the purposes of analyzing the tested example floor systems within this paper, the MSE method is applied to a two-degrees-of-freedom (2-DOF) model as shown in Fig. 1 below. The system is comprised of a base slab supported by four coil springs, and a floating slab supported by two kinds of isolation materials (fiberglass sheets and rubber blocks). Although, modelling an isolation layer and a floating slab is enough to describe the floating floor system, a base slab and coil springs are also modelled due to taking into account the stiffness of the floor slab supporting the isolation layer. For increased accuracy, future research could account for the relationship between the floor slab stiffness and the floating floor system response.

![Two-degree-of-freedom model](image)

Fig. 1  A study subject

The equation of motion of this 2-DOF model is as follows:

$$
\begin{bmatrix}
[K^*] - \omega^2 [M]
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= 
\begin{bmatrix}
k_a & -k_b \\
-k_a & k_i^* + k_2^*
\end{bmatrix}
\begin{bmatrix}
\omega^2 & 0 \\
0 & \omega^2
\end{bmatrix}
\begin{bmatrix}
m_1 \\
m_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= 
\begin{bmatrix}
f
0
\end{bmatrix},
$$

(2)
where, $\omega$ [rad/s] is angular velocity, $f$ [N] is the external force, $m_1$ [kg] is the mass of the floating slab, $m_2$ [kg] is the mass of the base slab, $x_1$ [m] is the displacement of the floating slab, $x_2$ [m] is the displacement of the base slab, $k_1^*$ [N/m] is the complex stiffness of the composite material comprised of fiberglass and rubber blocks, and $k_2^*$ is the complex stiffness of the coil springs. $k_1^*$ can be found via summing the spring constants of the individual materials: fiberglass $k_a^*$ and the rubber blocks $k_b^*$.

$$k_1^* = k_a^* + k_b^*$$ (3)

The stiffness matrix in Eq. (2) can be expanded using Eq. (3) where

$$\begin{bmatrix} k_1^* & -k_1^* \\ -k_1^* & k_1^* + k_2^* \end{bmatrix} = \begin{bmatrix} k_a^* & -k_a^* \\ -k_a^* & k_a^* \end{bmatrix} + \begin{bmatrix} k_b^* & -k_b^* \\ -k_b^* & k_b^* \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & k_2^* \end{bmatrix} = \begin{bmatrix} K_a^* \\ K_b^* + K_2^* \end{bmatrix}.$$ (4)

The complex stiffness values $k^*$ can be expressed in terms of the real spring constant $k$ and the material loss factor $\eta$ as follows:

$$k^* = k(1 + j\eta),$$ (5)

where $j$ is the complex number.

Note that the MSE method is valid when the stiffness matrix is represented as hysteresis damping, not viscous damping.

The modal loss factor at the $r$th mode $\eta^{(r)}$ is obtained via the following:

$$\eta^{(r)} = \sum \eta_i \cdot S_i^{(r)},$$ (6)

where

$$S_i^{(r)} = \frac{E_i^{(r)}}{E_{total}^{(r)}},$$ (7)

$\eta_i$ is the material loss factor, and $S_i^{(r)}$ represents the ratio of the strain energy for material $i$ to the total strain energy [2]. The strain energy per material $E_i^{(r)}$ and the total strain energy $E_{total}^{(r)}$ can be expressed as follows:

$$E_i^{(r)} = \frac{1}{2} \left( \phi^{(r)} \right)^\dagger \begin{bmatrix} K_a \\ K_b \end{bmatrix} \left\{ \phi^{(r)} \right\}$$ (8)

$$E_{total}^{(r)} = \sum E_i^{(r)},$$ (9)

Where the $r$th real eigenvector $\phi^{(r)}$ is obtained by solving the generalized eigenvalue problem of Eq. (2) neglecting the imaginary part.

By using the MSE method, the calculation can be simplified by neglecting the imaginary part of the complex stiffness when the material loss factor is small (i.e. $\eta_i < 1$). Since isolation materials used for floating floors usually have material loss factors less than 0.5, this approximation is typically valid.

Substituting the real part of Eq. (4) into Eq. (9), the total strain energy $E_{total}^{(r)}$ is obtained via:

$$E_{total}^{(r)} = \frac{1}{2} \left( \phi^{(r)} \right)^\dagger \begin{bmatrix} k_a & -k_a \\ -k_a & k_a \end{bmatrix} + \begin{bmatrix} k_b & -k_b \\ -k_b & k_b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & k_2 \end{bmatrix} \left\{ \phi^{(r)} \right\} = E_a^{(r)} + E_b^{(r)} + E_2^{(r)}$$ (10)

Thus, the $r$th modal loss factor is obtained as follows:

$$\eta^{(r)} = \eta_a \left( \frac{E_a^{(r)}}{E_{total}^{(r)}} \right) + \eta_b \left( \frac{E_b^{(r)}}{E_{total}^{(r)}} \right) + \eta_2 \left( \frac{E_2^{(r)}}{E_{total}^{(r)}} \right)$$ (11)
Finally, the accelerance of this 2-DOF model is calculated by Eq. (12).

$$A(\omega) = \sum_{m=1}^{2} \frac{-\omega^2 \cdot \phi_q^{(m)} \cdot \phi_p^{(m)}}{\lambda^{(m)} - \omega^2 + j\eta^{(m)} \lambda^{(m)}} \cdot f,$$

where $\phi_q^{(m)}$ is the $m^{th}$ mode eigenvector at the receiver point $q$, $\phi_p^{(m)}$ is the $m^{th}$ mode eigenvector at the excitation point $p$, $\lambda^{(m)}$ is the $m^{th}$ mode eigenvalue.

### 3. Experimental Validation

#### 3.1 Measurement setup

The test sample consisted of a 200 mm thick Reinforced Concrete (RC) slab supported by four coil springs with spring constants of 8.96E+04 N/m each, an isolation layer and a floating RC slab was tested at two different thicknesses of 100 mm and 150 mm. The sample size was 1200 mm x 800 mm as shown in Fig. 2. In this paper, the isolation layer was comprised of a 50 mm thick fiberglass sheet with density of 96 kg/m$^3$ and four 50 mm thick isolation rubber blocks sized 60 mm x 60 mm each. The accelerance was then measured after exciting the top slab with an impact hammer.

![Fig. 2 Floating floor test setup](image)

#### 3.2 Estimation of spring constant and material loss factor for single isolation material

First of all, the spring constants and the material loss factors for the fiberglass and rubber blocks were estimated using the test setup mentioned above. The top floating slabs were supported by the individual isolation materials as illustrated in Fig. 3. Then, the floating floor system was modelled as a 2-DOF system as illustrated in the figure.

![Fig. 3 Test setup for single isolation material and 2-DOF model](image)
By finding the resonance frequency from the experimental results, the eigenvalue $\lambda$ can be easily obtained, whereby the spring constant $k_i$ of the isolation material is obtained as follows:

$$k_i = \frac{\lambda m_i (k_2 - \lambda m_2)}{k_2 - \lambda (m_1 + m_2)}. \quad (13)$$

The material loss factor is estimated by fitting the frequency response curve calculated by Eq. (12) to the measured data by parameterizing $\eta$ in Eq. (12) until a sufficient fit with the measured curve is found. Both the measured and calculated accelerance curves are shown in Fig. 4.

The estimated spring constants and the material loss factors are shown in Table 1.

![Fig. 4 Accelerance on floating floor](image)

The material loss factor of each material are almost the same value respectively regardless of the slab thickness.

<table>
<thead>
<tr>
<th>Thickness of floating slab</th>
<th>Isolation Material</th>
<th>Spring constant [N/m]</th>
<th>Material loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 mm</td>
<td>Isolation spring</td>
<td>$3.58 \times 10^5$</td>
<td>0.04</td>
</tr>
<tr>
<td>150 mm</td>
<td>Fiberglass</td>
<td>$3.82 \times 10^6$</td>
<td>0.08</td>
</tr>
<tr>
<td>100 mm</td>
<td>Isolation rubber blocks</td>
<td>$1.71 \times 10^6$</td>
<td>0.08</td>
</tr>
<tr>
<td>150 mm</td>
<td></td>
<td>$1.79 \times 10^6$</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Figure 5 illustrates the strain energy ratios of the 100 mm floating slab as calculated by Eqs. (7) and (8) with the estimated spring constants shown in Table 1. The strain energy at the first mode for both the fiberglass and isolation rubber block tests can be seen to be dominated by the motion of the
coil springs (SP), while the strain energy at the second mode for both tests are dominated by the motion of the fiberglass (FG) or isolation rubber blocks. Therefore, the first peak in the response curves of all cases in Fig. 4 arises from the coil springs, and second peak arises from the isolation material resonance. In other words, the first modal loss factor represents the material loss factor of coil spring, and the second modal loss factor represents the material loss factor of fiberglass or rubber blocks. The fairly good agreement between the measurement and calculation curves provides evidence that the spring constant and material loss factors have been estimated fairly accurately.

![Strain energy ratio in case of 100 mm floating slab](image)

**Fig. 5** Strain energy ratio in case of 100 mm floating slab

### 3.3 Estimation for composite material

Since the estimated material loss factor of fiberglass was found to be three times as much as rubber, the MSE method was used to estimate the modal loss factor of the composite isolation layer. The composite isolation layer was comprised of a fiberglass sheet and rubber blocks as illustrated in Fig. 6. All other experimental conditions were identical with the previous sections.

![Test setup for composite isolation layer](image)

**Fig. 6** Test setup for composite isolation layer

The modal loss factor and resonance frequency are calculated from Eq.(11) by using the material properties estimated in the previous section. Calculated and measured results are shown in Table 2 below.
Table 2  Resonance frequency and modal loss factor value

<table>
<thead>
<tr>
<th>Floating Slab Thickness</th>
<th>Result Type</th>
<th>Resonance frequency [Hz]</th>
<th>Modal loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First Mode</td>
<td>Second Mode</td>
</tr>
<tr>
<td></td>
<td>Calculation</td>
<td>3.84</td>
<td>31.0</td>
</tr>
<tr>
<td></td>
<td>Measurement</td>
<td>3.85</td>
<td>29.5</td>
</tr>
<tr>
<td>150 mm</td>
<td>Calculation</td>
<td>3.55</td>
<td>28.1</td>
</tr>
<tr>
<td></td>
<td>Measurement</td>
<td>3.62</td>
<td>27.6</td>
</tr>
</tbody>
</table>

Calculated and measured resonance frequencies matched relatively well at both the first and second mode regardless of the floating slab thickness implying validity of the 2-DOF model with regards to resonance frequency.

Figure 7 illustrates the strain energy ratios calculated by Eq. (7). Since strain energy at the first mode is dominated by the coil springs as also shown in Fig. 5, the modal loss factor can be approximated from the material loss factor of the coil springs alone. On the other hand, the strain energy at the second mode is dominated by the fiberglass sheet and rubber blocks, therefore the modal loss factor needs to be determined from their material loss factors and strain energy ratios.

Figure 8 shows the calculated and measured accelerance for each floating slab thickness. The difference between calculated and measured results at the second mode may be due to another damping mechanism such as friction at the boundary between the fiberglass and the rubber blocks, though it is difficult to explain such mechanisms within this 2-DOF model. Though there is some difference between calculated and measured values, the general characteristics of the peak amplitude of second mode can be estimated by this simple 2-DOF model using the MSE method.
4. Conclusions

To predict the acceleration response of floating floor systems in practice, simplified two-degree-of-freedom (2-DOF) model was used. The 2-DOF model was comprised of floating slab, composite isolation layer and base slab. In this paper, the fiberglass sheet and the isolation rubber blocks were used for isolation layer. The material loss factors of each isolation material were obtained through the experiments individually. The Modal Strain Energy (MSE) method was used in terms of approximation of the modal loss factor at the resonance frequency of the floating floor system with the composite isolation layer. Finally, the modal loss factors were calculated and compared with the experiment results. The calculated modal loss factor and peak amplitude at the resonance frequency showed fair agreement with the experiments. Future research will be focused on further improving the accuracy of the model.

REFERENCES