Propagation of acoustic pulses through a forest may be described using radiative transfer theory under a modified Born approximation, resulting in an analytical expression for the diffuse intensity as a function of time and dominant frequency. The simple case of an impulse in an infinite homogeneous forest of diffuse scatterers is first considered, and then the effects of successively including non-diffuse scatterers, ground reflections in a forest of finite height, and finally a realistic forest model are analyzed. These theoretical findings are then compared with experimental results.

Keywords: forest, pulse, diffuse, reverberation

1. Introduction

The propagation of acoustic waves through a forest can be highly complicated and difficult to predict in a deterministic manner. This difficulty is due to the fact that forests generally consist of a large number of scattering objects, each with its own characteristic size, location, orientation, and acoustic impedance. Furthermore, the distribution of these scatterers is essentially random from forest to forest, such that specific results obtained from the study of one forest do not necessarily apply to another forest, even of the same type and maturity. Thus, instead of using a deterministic (usually pressure-based) approach, the random-like nature of forests instead suggest using statistical methods. In particular, we have found that a three-dimensional multiple scattering theory [1] provides a useful description of acoustic propagation in forests.

Three-dimensional multiple scattering theory has been used to estimate the effective acoustic wavenumber in a forest, incorporating the effects of trunks, branches, and the canopy layer [2]. Radiative transfer theory, which is a consequence of the multiple scattering theory and is described below, was used to describe steady state propagation of initially planar acoustic waves through a forest modeled with four layers: ground, trunk, canopy, and open air [3]. Of particular note, Ostashev et al. showed using radiative transfer theory that acoustic propagation through a system of discrete scatterers (such as a forest) may be cast in a form that is equivalent to acoustic propagation through a turbulent atmosphere [4], which has been extensively discussed in the literature [5]. The purpose of this paper is to demonstrate how radiative transfer theory may be applied to the problem of acoustic pulse propagation in forests.

Since the submission of the abstract for this paper the authors have had an article published in the Journal of the Acoustical Society of America that discusses the contents of this talk in detail [6]. Thus, this paper presents the overall results and big-picture perspective, and those interested in the details of the derivations are directed to the above-mentioned journal article.
2. Radiative Transfer Theory

Radiative transfer theory is based on three-dimensional multiple scattering theory and is general enough that it may be used to fully (in a deterministic manner) describe the propagation of acoustic fields. One of the largest benefits to using radiative transfer theory is that it may be naturally approximated to account for statistical descriptions of the propagating domain. This is accomplished using differential scattering ($\sigma_d$) and total ($\sigma_t$) cross section distributions (hereafter just cross sections without the word “distribution” for conciseness), which describe how incident energy is scattered and absorbed by any given region of the propagating domain. In other words, the nature of the scattering bodies and their distribution throughout the domain are represented by the cross sections.

2.1 Equation for the Diffuse Intensity

The full radiative transfer equation can be difficult to solve, but several useful and often applicable assumptions and approximations make it tractable. The most important assumption is that the scattered field following any scattering event is significantly weaker than the incident field. Under this assumption we may model any given propagation path as originating at the source, propagating to a scattering location, scattering, and then propagating to the measurement location. Another important assumption is that the source may be treated as a narrowband pulse, such that the cross sections for a single frequency may be used to describe the scattering of the pulse. Then, for measurement locations far from the source a narrowband pulse may be reasonably approximated by a narrowband “impulse”, or a delta-function with a characteristic frequency. Forests are assumed to be statistically homogeneous and approximated as a uniform scattering system from the ground to the canopy. Finally, the atmosphere is assumed to be static and homogeneous and the overall sound speed through the forest is considered to be equal to the sound speed in the absence of the forest.

Using the approximations and assumptions described above, the diffuse intensity at a measurement location $\vec{R}$ due to a point narrowband impulse source at $\vec{R}_0$ may be written as

$$I_d(\tau) = \frac{I_0}{2\pi} e^{\alpha H(\tau - 1)} \int_{-1}^{1} du \int_{-\pi}^{\pi} d\phi \frac{e^{-\gamma_1 - \gamma_2}}{\tau^2 - u^2} \frac{\sigma_d(\tau, u, \phi)}{\sigma_0}, \quad (1)$$

where $I_0 = p_0^2 c_0 t_0 \sigma_0 / Z_0 L^2$ is a characteristic diffuse intensity, $p_0$ is the characteristic pressure amplitude, $c_0$ is the sound speed in air, $t_0$ is a characteristic time scale of the wave motion, $\alpha = \sigma_0 L$ is a normalized decay constant, $L$ is the distance between the source and the measurement positions, $\tau = c_0 t / L$ is a normalized time, and $\sigma_0$ and $\sigma_0$ are characteristic differential and total cross sections. The quantities $\gamma_1$ and $\gamma_2$ represent the total energy lost through the propagation to and from the scattering site, respectively, and may be written mathematically as

$$\gamma_1 = R_1 \int_{0}^{1} \sigma(v(\vec{R}_0) + s \vec{R}_1) ds \quad \text{and} \quad \gamma_2 = R_2 \int_{0}^{1} \sigma(v(\vec{R} + s \vec{R}_2)) ds, \quad (2)$$

where $\vec{R'}$ is the location of the scattering event, $\vec{R}_1 = \vec{R'} - \vec{R}_0$, and $\vec{R}_2 = \vec{R} - \vec{R'}. The quantities $u$ and $\phi$ are (along with $v$, which has already been integrated over) coordinates in a prolate spheroidal coordinate system $(x, y, z)$ defined relative to the Cartesian coordinate system with the source and receiver locations at $(x, y, z) = (\pm L/2, 0, 0)$, such that

$$x = \frac{L}{2} u e, \quad y = \frac{L}{2} \sqrt{u^2 - 1} \sqrt{1 - u^2} \cos \phi, \quad \text{and} \quad z = \frac{L}{2} \sqrt{u^2 - 1} \sqrt{1 - u^2} \sin \phi. \quad (3)$$

Prolate spheroidal coordinates were chosen because surfaces of $\vec{R'}$ that yield constant arrival times are prolate spheroids with the source and measurement locations at the foci.

While this work is primarily focused on the diffuse intensity, it should be noted that the coherent intensity may also be evaluated. Indeed, it is easier to calculate as it propagates directly from the
source to the measurement position, remains a delta function, and only depends on the total cross section. In particular,

\[ I_{\text{coh}}(\tau) = \frac{e^{-\gamma_0} p_0^2 t_0 \delta(t - L/c_0)}{Z_0} = \frac{I_0 e^{\alpha - \gamma_0}}{L \sigma_{d0}} \delta(\tau - 1), \]  \tag{4} \]

where

\[ \gamma_0 = L \int_0^1 \sigma_t \left( \vec{R}_0 + s(\vec{R} - \vec{R}_0) \right) ds. \]  \tag{5} \]

2.2 Diffuse Intensity for Forest Models

The diffuse intensity for a wide variety of forests may be modeled as special cases of Eq. (1). In this section the cases of an infinite forest of small scatterers, a diffusely scattering forest of finite height, and a simple real forest model will be sequentially considered.

2.2.1 Infinite Forest of Small Scatterers

The differential scattering cross section of any object may be approximated in the long-wavelength limit as

\[ \sigma_d = A(1 + D \cos \theta_s)^2, \]  \tag{6} \]

where \(\cos \theta_s = \vec{R}_1 \cdot \vec{R}_2/R_1 R_2\), \(A\) is the magnitude of the differential scattering cross section, and the quantity \(D\) is a dimensionless quantity representing the relative strength of dipole scattering to monopole (or diffuse) scattering [7]. If the scattering object is a rigid sphere then \(D = -3/2\), and if \(D = 0\) the scattering object is a diffuse scatterer. Then, assuming that the total cross section is isotropic and homogeneous, such that \(\sigma_t = \sigma_{t0}\), and defining \(\sigma_{d0} \equiv A(1 + D)^2\), Eq. (1) may be calculated for small scatterers as

\[ I_d(\tau) = I_0 e^{-\alpha(\tau-1)} \left[ \frac{1}{\tau} \ln \left( \frac{\tau + 1}{\tau - 1} \right) \right] \left[ 1 - \frac{2D}{1 + D} \frac{\tau^2 - 1}{\tau^2} + \frac{3D^2}{2(1 + D)^2} \frac{(\tau^2 - 1)^2}{\tau^4} \right] - \frac{4D}{\tau^2(1 + D)} + \frac{D^2}{(1 + D)^2} \frac{\tau^2 - 3}{\tau^4} H(\tau - 1). \]  \tag{7} \]

For \(\tau \gg 1\) the diffuse intensity may be approximated as

\[ I_d(\tau) \approx I_0 \frac{2e^{-\alpha(\tau-1)}}{\tau^2} \left( \frac{1 - D}{1 + D} \right)^2, \]  \tag{8} \]

such that the decay of the diffuse intensity goes as \(e^{-\alpha\tau/\tau^2}\) for \(D \neq 1\). For \(D = 1\) the diffuse intensity goes as \(e^{-\alpha\tau/\tau^6}\).

A graph of \(I_d(\tau)/I_0\) for several values of \(D\) and for \(\alpha = 0.1\) (which corresponds roughly to a 300-Hz source in a sparse pine forest with \(L = 60\) m) is shown in Fig. 1. The diffuse intensity for all values of \(D\) diverge as \(\tau \to 1\), except for \(D = -1\) which approaches a constant, and decay as \(e^{-\alpha\tau/\tau^2}\) as \(\tau \to \infty\), except for \(D = 1\) which decays as \(e^{-\alpha\tau/\tau^6}\). The main difference between the different values of \(D\) is the structure of \(I_d(\tau)\) near \(\tau = 1\) and the constant of proportionality for the decay.

2.2.2 Finite Forest of Diffuse Scatterers

Real forests are not infinite in extent and always have at least one reflecting surface, the ground. Incorporating the effects of a finite height of the forest and the ground leads to three classes of propagation paths: no reflection paths, single reflection paths, and double reflection paths. The
Figure 1: (Based on Fig. 4 in Ref. [6]) Normalized diffuse intensity as a function of the normalized time \( \tau = c_0 t/L \) for an infinite homogeneous, isotropic field of small scatterers. The quantity \( D \) denotes the relative strength of the monopolar (diffuse) and dipolar scattering modes.

single reflection paths go from the source to the ground, then to the scattering position, and then to the measurement location, or from the source to the scattering position, then to the ground, and then to the measurement location. The double reflection paths propagate to the ground both before and after the scattering event. The diffuse intensity may then be separated into four quantities:

\[
I_d(\tau) = I_{0R}(\tau) + I_{1R,b}(\tau) + I_{1R,a}(\tau) + I_{2R}(\tau),
\]

where \( I_{0R}(\tau) \) is the diffuse intensity due to paths with no reflections, \( I_{2R}(\tau) \) is the diffuse intensity due to paths with two reflections, and \( I_{1R,b}(\tau) \) and \( I_{1R,a}(\tau) \) are associated with paths with one reflection (b) before and (a) after the scattering event.

Without providing the details of the geometry for lack of space, the four diffuse intensity components in a forest of finite height may be written as

\[
I_{0R}(\tau) = \frac{I_0}{\pi} e^{-\alpha(\tau-1)} H(\tau-1) \int_{-1}^{1} du \int_{\Phi_0}^{\Phi_R} d\phi \frac{\sigma_d}{\sigma_{d0}} \frac{1}{\tau^2 - u^2},
\]

\[
I_{1R,b}(\tau) = \frac{I_0}{\pi} e^{-\alpha(\tau-1)} H(\tau-1) \int_{-1}^{1} d\bar{u} \int_{\Phi_0}^{\Phi_R} d\bar{\phi} \frac{\sigma_d}{\sigma_{d0}} \frac{|\mathbb{R}_2|^2}{\tau^2 - \beta^2 \bar{u}^2},
\]

\[
I_{1R,a}(\tau) = \frac{I_0}{\pi} e^{-\alpha(\tau-1)} H(\tau-1) \int_{-1}^{1} d\bar{u} \int_{\Phi_I}^{\Phi_R} d\bar{\phi} \frac{\sigma_d}{\sigma_{d0}} \frac{|\mathbb{R}_1|^2}{\tau^2 - \beta^2 \bar{u}^2},
\]

\[
I_{2R}(\tau) = \frac{I_0}{\pi} e^{-\alpha(\tau-1)} H(\tau-1) \int_{-1}^{1} du \int_{\Phi_I}^{\Phi_R} d\phi \frac{\sigma_d}{\sigma_{d0}} \frac{|\mathbb{R}_1|^2 |\mathbb{R}_2|^2}{\tau^2 - u^2},
\]

where \( \beta = \sqrt{1 + 4h^2/L^2} \), \( h \) is the height of the source above the ground, \( H \) is the total height of the forest, and the \( \Phi \) variables are the \( \phi \) values for the limiting planes: ground, top of forest, and bottom of an image forest. The variables \( \mathbb{R} \) denote the pressure reflection coefficient for different reflection events. In particular, given a normalized ground impedance \( Z \) and characteristic wavenumber \( k \), we
Figure 2: (Based on Fig. 6 in Ref. [6]) Normalized diffuse intensity components and sum total due to point source in a 30-m diffusely scattering forest as a function of the normalized time $\tau = c_0 t / L$.

It is important to notice that in the limit that $\tau \to \infty$ we may approximate

$$I_{0R}(\tau), I_{1R,a}(\tau), I_{1R,b}(\tau), I_{2R}(\tau) \to 2I_0 \frac{H}{L} \frac{\sigma_{\infty} e^{-\alpha(\tau-1)}}{\tau^3},$$

(11)

where $\sigma_{\infty}$ is the differential scattering cross section in the backward direction far from the source and measurement location, and so

$$I_d(\tau) \to 8I_0 \frac{H}{L} \frac{e^{-\alpha(\tau-1)}}{\tau^3},$$

(12)

which is independent of the normalized ground impedance $Z$, and decays as $e^{-\alpha \tau / \tau^3}$.

Plots of each diffuse intensity component and the total diffuse intensity as described in Eqs. (9) are shown in Fig. 2 using $h = 5$ m, $H = 30$ m, $f = 150$ Hz, $c_0 = 340$ m/s, $Z = 4.8 + 4.5$ (typical of a mixed deciduous forest). As may be seen, the total diffuse intensity decays as $I_{0R}(\tau)$ until $\tau \approx 1.01$ when the reflected paths first arrive. After the first reflected paths arrive there is a sharp rise in the
Figure 3: (Based on Fig. 10 in Ref. [6]) Normalized diffuse intensity components and sum total due to point source a 30-m forest as described in Ref. [3] as a function of the normalized time $\tau = c_0 t / L$.

total diffuse intensity due to the single-reflection paths. The double-reflection paths do not noticeably contribute to the total diffuse intensity until near $\tau = 1.34$, where $I_{2R}$ reaches a maximum. By $\tau = 10$ all of the diffuse intensity components are decaying as $e^{-\alpha \tau / \tau^3}$, as described above.

2.2.3 Simplified Real Forest Model

Ostashev, et al. describe a forest composed of trees that are modeled as finite cylinders and derived differential and total scattering cross sections [3]:

$$\sigma_d = \nu k^2 H^2 l^2 \left| \text{sinc} \left( \frac{k l}{2} (\cos \theta_{p2} - \cos \theta_{p1}) \right) \sum_{n=0}^{\infty} B_n \cos n \psi \right|^2,$$

$$\sigma_t = \frac{4 \nu H}{k} \Re \left\{ \sum_{n=0}^{\infty} \epsilon_n J_n' \left( \frac{k b \sin \theta_p}{H_n^{(1)}} \right) \right\},$$

where $\epsilon_n = 1$ for $n = 0$ and $\epsilon_n = 2$ for $n \neq 0$ is the Neumann factor, $b$ is the trunk radius, $J_n$ is the $n$th order Bessel function of the first kind, $H_n^{(1)}$ is the $n$th order Hankel function of the first kind, and $\theta_p$ is defined such that $\cos \theta_p = \hat{R} \cdot \hat{z}$, $\hat{z}$ is the vertical direction. Further,

$$B_n = \frac{\epsilon_n}{2} J_n' \left( \frac{k b \sin \theta_{p1}}{H_n^{(1)}} \right) \left[ \sin \theta_{p1} J_n \left( \frac{k b \sin \theta_{p2}}{H_n^{(1)}} \right) - \sin \theta_{p2} J_n' \left( \frac{k b \sin \theta_{p1}}{H_n^{(1)}} \right) \right],$$

$sinc(x) = \sin(x)/x$, $l$ is the finite length of the cylinders (equal to $H$ in this model), $\psi$ is the horizontal component of the scattering angle, and $\theta_{p1}$ is the propagation angle for the $\hat{R}_i$ portion of the propagation path. The reference differential scattering and total cross sections are chosen to be cross sections for $\theta_{p1} = \theta_{p2} = \pi/2$. These formulas are appropriate for planar incident waves. We will neglect any curvature of the incident waves for simplicity at the expense of accuracy near $\tau = 1$ where $R_1$, $R_2 \gg 2b$, $l$.

Figure 3 shows the normalized diffuse intensity due to a point source in a forest that is described using the model by Ostashev, et al. [3]. As with the diffuse forest model described in Sec. 2.2.2, the incorporation of a reflecting ground leads to four components of the total diffuse intensity. Each of these components along with the total diffuse intensity is shown in Fig. 3. In contrast to the diffuse intensity of the diffusely scattering forest, the diffuse intensity initially decreases rapidly before leveling off and then decaying slowly. The final decay follows the same trend as the diffusely scattering...
forest, \( e^{-\alpha \tau}/\tau^3 \), because far from the source and measurement positions the propagation paths are all approximately parallel to the ground and the scattering angle is a constant 180°. Thus, the asymptotic behavior of any homogeneous and transverse-isotropic forest of finite height and infinite extent will be the same.

3. Experimental Comparison

Measurements of gunshots in a mixed deciduous forest in Pomfret, VT taken and reported by Albert et al. in Ref. [8] provide an opportunity to verify the theoretical results presented above. The measurement consisted of a hand-fired pistol shooting 0.45 caliber blanks from the top of a 10 m cliff. The microphones relevant to the present discussion were placed at a range of 60 and 205 m from the pistol. The measurements were taken on two separate occasions, one with leaves on the trees and one with the leaves off of the trees. From the measured waveforms, \( N = 9 \) of them (five with leaves and four without leaves) were used to estimate both the coherent and diffuse intensity fields at the measurement locations. The time-dependent complex pressures \( p_i(t) \) were estimated by

\[
p(t) \approx x_i(t) + i\mathcal{H}\{x_i(t)\},
\]

where \( x_i(t) \) is the \( i \)th measured time waveform and \( \mathcal{H}\{\cdot\} \) is the Hilbert transform. The coherent and diffuse intensities are then estimated by

\[
\begin{align*}
I_{coh}(t) & \approx \frac{1}{NZ_0} \left| \sum_{i=1}^{N} p_i(t) \right|^2 \\
I_d(t) & \approx \frac{1}{NZ_0} \sum_{i=1}^{N} |p_i(t)|^2 - I_{coh}(t).
\end{align*}
\]

See Ref. [6] for additional details of the signal processing used to obtain the results presented here.

The experimentally estimated coherent and diffuse intensities for the 60-m and 205-m measurement locations are presented in Fig. 4. As expected, at both of the measurement locations the coherent intensity is greater than the diffuse intensity at the initial arrival (\( \tau = 1 \)) and then drops well below the diffuse intensity for later arrival times. The limited number of measurements used for the ensemble averages do not provide enough information around the initial arrival for a meaningful comparison of the experimental data with the models. However, the late arrival information of the experimental data may be used to fit the asymptotic behavior of the finite-forest models: \( I_d(\tau) \sim e^{-\alpha \tau}/\tau^3 \) and provide estimates of \( \alpha = \sigma_{t0}L \), and so the characteristic total cross section. The fitted values of \( \alpha \) at the 60 and 205 m measurement locations are, respectively, 1.25 and 5.99. These values correspond to \( \sigma_{t0} = 0.021 \text{ m}^{-1} \) and \( 0.029 \text{ m}^{-1} \). The fact that these estimates of the characteristic total cross section are so similar suggests that they may indeed be physically meaningful. The true value of \( \sigma_{t0} \) is probably near 0.025 m\(^{-1}\) for this portion of forest.
4. Conclusions

Radiative transfer theory has been used to provide a statistical description of both the coherent and diffuse intensity fields in a homogeneous forest of various types. A common feature of the models for forests of finite height is the asymptotic behavior of the late-arrival diffuse intensities, which depends only on the time of arrival, distance from the source, and the total cross section of the forest. Fitting the models to experimentally obtained data sets provides a way to estimate the total cross section of a forest. Additional characterizations of forests may be obtainable with improved experimental data sets.

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