This study presents a metamaterial beam consisting of periodic holes with membrane-split-ring resonators. Finite element (FE) models are constructed and used to predict the dispersion relation of the metamaterial beam. Numerical results show that both Bragg-type and resonant-type bandgaps exist in such a structure. Bragg-type bandgaps are usually occurred at high frequencies; resonant-type bandgaps can be tuned by altering the properties of local resonator. Experimental measurements are conducted to validate the FE results. It is found that when the excitation frequency is close to the local resonance, elastic wave is attenuated and the beam remains motionless.

Keywords: metamaterials, dispersion relation, membrane-split-ring resonators

1. Introduction

The reduction of structural vibration remains a major challenge in many branches in engineering field. Most common passive solutions include mass balancing, structural modification, and constrained damping layers [1-3]. Periodic structures having material or geometrical discontinuities are also viewed as good candidates for manipulating wave propagation, but they are usually ineffective at low frequencies [4-9]. Recently, the use of locally resonant structures offers an extra possibility for suppressing low-frequency vibration [10-15]. In most cases, this class of structures consists of base structures equipped with multiple substructures that can act as local resonators. When the driving frequency is close to the resonant frequency of the resonator, the main structure should remain motionless and the resonator can completely absorb external energy caused by disturbances. By adjusting the mechanical properties of local resonators, a desired frequency band with strong vibration attenuation can be easily achieved. In addition to strong wave attenuation, unusual physical properties, such as negative mass density or negative Young’s modulus, exist in such a structure. The study on its effective properties can help understand the physics behind energy transport.

In this study, we present a new design of metamaterial beam comprised of periodic cavities filled by an elastic membrane with two split rings. Flexural wave propagation of the present metamaterial beam is examined. To gain the further insight into the bandgap mechanism, the eigenmodes at key frequencies are also investigated. The FEM predictions are compared with experimental results.

2. Model

Figure 1 presents the unit cell of the metamaterial beam consisting of periodic cavities filled with membrane-mass structures acting as local resonators. The resonator comprises of a thin elastic membrane with two split rings. The host beam is made of aluminium; the split-ring mass is made of copper. The membrane used is a polyetherimide (PEI) film. The material constants and dimensions of the host beam, membrane, and split-ring mass are displayed in Table 1-3. For the purpose of
comparison, a beam with periodic cavities is also constructed. The finite element (FE) software COMSOL Multiphysics is used to predict the flexural behaviour of those beams. To insure the accuracy, a FEM model with sufficiently fine mesh is employed. The membrane tension is set as $3 \times 10^5 \text{Pa}$. The mass magnitude of each split ring is 1.02g.

![Figure 1: A unit cell of the metamaterial beam with a membrane-split-ring resonator.](image)

<table>
<thead>
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<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$L_{ho}$ (mm)</th>
<th>$B_{ho}$ (mm)</th>
<th>$T_{ho}$ (mm)</th>
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<th>$T_{nom}$ (mm)</th>
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<th>$\rho$ (kg/m$^3$)</th>
<th>$l_r$ (mm)</th>
<th>$\theta_r$ (°)</th>
<th>$W_r$ (mm)</th>
<th>$T_r$ (mm)</th>
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<tbody>
<tr>
<td>115</td>
<td>0.33</td>
<td>8890</td>
<td>8</td>
<td>60</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

### 3. Numerical results

Figure 2 displays the flexural wave band structure of the host beam. There exists a wide bandgap from 1816Hz-2129Hz. This kind of bandgap is called the Bragg-type bandgap, which is usually occurred at high frequencies. Within the bandgap, elastic waves cannot propagate freely. The corresponding mode shapes at the initial and ending frequencies of the bandgap are displayed in Fig. 3. It can be seen that at the initial frequency of the bandgap, the host beam vibrates anti-symmetrically along the y axis while at the ending frequency, a symmetric mode is generated.
The dispersion relation of the metamaterial beam is displayed in Fig. 4. In addition to Bragg-type bandgap existing in the structure, four extra bandgaps are occurred. Those bandgaps, called resonant-type bandgaps, are attributed to the local resonance behavior of the resonant unit. At the first resonance, the membrane and split rings vibrate in unison (see Fig. 5(a)); at the second resonance, two split rings present an out-of-phase vibration, as a result, the membrane is separated along the centreline (see Fig. 5(b)). For the third mode, the membrane between two split rings has a maximum amplitude and both rings rotate outward around the y axis (see Fig. 5(c)). For the fourth mode, both rings rotate clockwise (see Fig. 5(d)). The results of Figure 4 also show that the bandwidth of the 1st bandgap is much larger than others. Table 4 summarizes the bandwidth of four bandgaps. It is obvious that the 1st bandgap has the widest bandwidth. Consider a metamaterial beam with nine unit cells under a vibration excitation with a frequency spectrum of 0-2500Hz. The vibration responses of the metamaterial beam are illustrated in Fig. 6. It is note that four vibration transmittance dips (324Hz, 448Hz, 806Hz, 1100Hz) are occurred. The dip frequency coincides with the lower edge frequency of the resonant-type bandgap.
Figure 5: The mode shapes at the initial frequency of (a) the 1st bandgap (b) the 2nd bandgap (c) the 3rd bandgap (d) the 4th bandgap.

Table 4: Bandgap width for the metamaterial beam

<table>
<thead>
<tr>
<th></th>
<th>1st bandgap</th>
<th>2nd bandgap</th>
<th>3rd bandgap</th>
<th>4th bandgap</th>
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<td>Gap width (Hz)</td>
<td>58.2</td>
<td>3.4</td>
<td>7.4</td>
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</table>

Figure 6: The vibration responses of the metamaterial beam, obtained by FE method.

4. Experiment setup and results

A prototype of the metamaterial beam consisting of nine unit cells is depicted in Fig. 7. One end of the beam is clamped and the other end is connected to the exciter. A sketch of the experimental setup is exhibited in Fig. 8. Pesudo random signals with a frequency spectrum from 0 to 2000Hz are generated by an INSTEK AFG-2105 waveform generator. The signals are amplified by a DC servo amplifier (Model AMP-750W-EP) and transmitted to vibration shaker (KD-9363) which is used to excite the metamaterial beam. The vibration responses of the metamaterial beam are measured by accelerometers. Figure 9 shows the frequency response function (FRF) of the beam with resonators. The mass magnitude is the same as the one used in FE simulations. It is noteworthy that an obvious attenuation zone form 1790Hz to 2160Hz and three vibration dips (320Hz, 410Hz, and 740Hz) are occurred. The wide attenuation zone is ascribed to the structural periodicity of the beam whereas the three dips are attributed to the local resonance of the membrane-mass resonator. When comparing with FE results, the dip frequency of 1100Hz cannot be found in the experiment. The reason is possibly that unlike the first mode having a maximum vibration amplitude, the fourth mode is more...
like a rotational mode. In other words, only at the first resonance frequency, the membrane-mass resonator can absorb most of energy caused by external excitation whereas at others, the resonator cannot have a good performance for vibration reduction. Now we further investigate the energy harvesting capability of the present metamaterial beam. A PVDF patch is attached on the cell which is closest to the actuation point. A sinusoid signal with a frequency of 320Hz is applied to the one end of the beam. The experimental measurements are shown in Fig. 10. As can be seen, the generated voltage amplitude is around 0.85 V
5. Conclusions

The flexural band structure of the metamaterial beam with membrane-split-ring structures is presented in this study. The FE model is developed to predict the dispersion relation and eigenmodes of the present metamaterial beam. Numerical and experimental results show that the proposed structure has a strong capability for eliminating unwanted vibration at certain frequencies. In addition to vibration reduction, this metamaterial beam can successfully achieve energy harvesting from bending waves.

Acknowledgements

This work was supported by the Ministry of Science and Technology of Taiwan under grant MOST 106-2221-E-006-122-MY3

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