ON ENERGY HARVESTING VIA BI-STABLE PIEZOELECTRIC VIBRATION STRUCTURE

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Bi-stable oscillator is one of the nonlinear vibration structures. The mechanical energy from the environment vibrations or vibrating machines can be transformed to electric energy through the mechanism of piezoelectric energy harvesting. To improve the performance of the energy harvesting, the nonlinear bi-stable oscillation system was investigated by using the Duffing equation. The nonlinear bi-stable oscillation system has one unstable and two stable equilibrium points. When the bi-stable system is excited with certain amplitude and frequencies of inputs, the system presents a bi-stable behavior, called “Snap-through” phenomenon. The oscillator with “Snap-through” phenomenon can produce a larger vibration amplitude and thus induce the piezoelectric materials to generate more electric energy than the pure linear mono-stable system. The purpose of this research is to experimentally validate the conditions of Snap-through phenomenon related to the input amplitude, input frequencies and magnetic force. The performance of energy harvesting was also evaluated in case of nonlinear bi-stable system with compared to the pure linear oscillation system.

Keywords: Bi-stable, Non-linear vibration, Piezoelectric structure, Energy harvesting, Duffing equation

1. Introduction

Piezoelectric vibration energy harvester is to use the electromechanical coupling characteristics of piezoelectric materials to transform the environmental vibration energy into the electric energy. The traditional vibration energy harvesting systems are usually limited by the operating frequency and thus cannot be widely utilized. In recent years, therefore, many studies of energy harvesting have been conducted to expand the effective bandwidth and increase the efficiency of piezoelectric energy harvester.

In the study of Moon and Holmes [1], the magnet-cantilever beam system presented a chaotic vibration behavior under a periotic excitation due to the nonlinear magnetic force. In 2001, Glynne-Jones et al. [2] used the piezoelectric film to design the vibrating element, and then the external vibrating energy can be collected through the piezoelectric film. The design of a double-layered
piezoelectric beam was proposed to achieve the optimal energy harvesting by adding the tip mass of the cantilever beam [3]. Erturk et al. [4] presented a piezoelectric vibration energy harvester using a magnetic cantilever beam. Their study demonstrated that the nonlinear vibrating structure due to the magnetic forces has wider effective frequency band than the linear piezoelectric beam. In 2010, a stochastic resonance phenomenon was studied on broadband piezoelectric vibration system [5]. An elastic support external magnet was utilized to achieve the bi-stable vibration for piezoelectric energy harvesting [6].

In this research, a sandwich cantilever beam with attached piezoelectric patches was utilized to form the vibrating structure. A magnet was installed on the tip of the cantilever beam. The other external magnet provided the repulsive force to form a nonlinear stiffness. The vibration and electric voltage responses in time and frequency domains were investigated by varying the parameters of distance between magnets, excitation amplitude and excitation frequencies. The energy harvesting performance was evaluated in cases of mono-stable, chaos and snap-through conditions through the experiment. The energy harvesting of linear vibration structure was also compared.

2. Theoretical Model

2.1 Duffing equation

The theory of Duffing equation that was proposed by Georg Duffing [7] depicts the dynamic behavior of nonlinear stiffness due to the nonlinear force, th is,

\[ F_s = k_1y \pm k_3y^3 \]  

(1)

where \( y \) is the spring deformation, \( k_1 \) and \( k_3 \) are the linear and nonlinear spring constants. In such a case of nonlinear stiffness, the equation of dynamical motion is expressed as

\[ \ddot{x}(t) + c\dot{x}(t) + \alpha x(t) + \beta x^3(t) = Y \omega^2 \cos(\omega t) \]  

(2)

where \( c, \alpha, \beta, Y, \omega \) and \( x \) are the damping constant, linear stiffness, nonlinear stiffness, excitation amplitude, excitation frequency and displacement response. Once a dynamic system can be formulated as the Duffing Equation, the system has two stable equilibrium points and one unstable equilibrium point. As shown in Fig. 1 [8], the potential energy of the nonlinear bi-stable system may have different trajectories (a, b and c) depending on the different excitation amplitude. The corresponding time responses and phase plots are shown in Fig. 2. For a certain excitation frequency, a small excitation amplitude causes small periodic response, called mono-stable condition, as shown in Fig. 2(a) and (d). With increasing the excitation amplitude, the vibration response becomes chaotic behavior, as shown in Fig. 2(b) and (e). Once the excitation amplitude is large enough, the vibration response can cross over the two stable equilibrium points periodically, and presents the snap-through condition, as shown in Fig. (c) and (f).

![Fig. 1: Potential energy with respect to displacement that Duffing equation depicts [8]](image-url)
2.2 System mathematical model

As shown in Fig. 3, the energy harvester consists of a piezoelectric cantilever beam with magnet tip. The other magnet attached on the host frame provides a repulsive magnetic force to the beam tip. The host frame is excited by an external vibrating displacement. The mathematical model can be expressed as

\[ m_{eq} \ddot{x}_{oc} = -k_{eq} (x_{oc} - x_{host}) - c(\dot{x}_{oc} - \dot{x}_{host}) + F_{mag} - 2F_{piezo} \]

\[ 2Ae_{33}^{s} \dot{V} = 2 \frac{Ae_{33}^{i}}{L_{p}} \dot{x} - \frac{1}{R_{i}} V \]

where \( x_{oc} \), \( m_{eq} \), \( c \) and \( k_{eq} \) represent the displacement, equivalent mass, damping constant, equivalent stiffness of the oscillator, respectively. \( x_{host} \) represents the displacement of host frame and \( F_{mag} \) is the magnetic force. \( F_{piezo}, A, e_{33}^{s}, d_{p}, L_{p}, e_{33}^{i} \) represent the force exerted by piezoelectric patches, cross-section area of the piezoelectric patch, dielectric constant of the piezoelectric material, thickness of piezoelectric patch, length of the piezoelectric patch and piezoelectric constant of the patch, respectively. \( R_{i}, V, x \) represent the load resistance, voltage output of the piezoelectric patch and the displacement of the piezoelectric patch tip, respectively.

3. Experimental Case Study and Result Analysis

This experiment is to investigate the differences in the electric energy harvesting performance of the nonlinear bi-stable system compared with the linear vibration system. The experiment consists of three parts. The experiment (I) is to explore the nonlinear vibration responses and the corresponding energy harvesting performance with respect to the influence of excitation frequencies. In the experiment (II), the nonlinear vibration behavior and the corresponding energy harvesting performance are evaluated by varying the excitation amplitude. The influence of the nonlinear force/system stiffness is investigated to assess the vibration response and the corresponding energy harvesting by changing the distance between the magnets in the experiment (III).
3.1 Case 1: Frequency response to bi-stable piezoelectric vibration system

For such a nonlinear vibration system, the response frequencies are normally different from the excitation frequencies and the response magnitude is not proportional to the excitation amplitude. In this experimental case, the excitation displacement amplitude was set at 10 mm and the excitation frequency is varying from 1 Hz to 15 Hz with an increment of 0.5 Hz. The distance between the magnets \(d\) was set as 8 and 12 mm. The acceleration of oscillator and the host frame and the corresponding voltage of piezoelectric patch were recorded by the signal acquisition device. The frequency response of the electric energy harvesting \(A\) is defined as the root-mean-square value of the piezoelectric voltage response to the root-mean-square of the velocity response of host frame,

\[
A = \frac{\text{RMS}[V(t)]}{\text{RMS}[v_{host}(t)]}
\]  

Fig. 4 shows the frequency response results. It is observed that the magnitude of the bi-stable system in the case of \(d=8\) mm at 5.5 Hz increases rapidly compared with the linear system. The associated phase plot is shown in Fig. 5. It can be noted that the nonlinear bi-stable system presents snap-through phenomenon since the displacement responses oscillate across the two stable equilibrium points periodically. At the frequency of 5.5 Hz, the frequency response magnitudes are 22.32 (Vs/m) and 22.29 (Vs/m) for \(d=8\) and 12 mm, while the linear system has frequency response of 12.72 (Vs/m). Therefore, the piezoelectric energy harvester has 75% higher voltage output in the nonlinear bi-stable system than in the linear system.

![Fig. 4: Frequency response of linear and nonlinear bi-stable systems](image)

Fig. 4: Frequency response of linear and nonlinear bi-stable systems (excitation amplitude= 20 mm)

At the excitation frequency of 7 Hz, it is observed that the bi-stable system has obvious frequency response decreasing in the case of 12 mm magnets gap. The possible reason is that the bi-stable system in this case \((d=12\) mm\) enters the unstable region at 7 Hz. The frequency response of the bi-stable system in the case of \(d=8\) mm, however, is close to that of linear system. Fig. 6 shows the phase plots of the bi-stable system \((d=8\) and 12 mm\) and linear system. It can be found that the vibration displacement in the case of \(d=12\) mm periodically oscillates between the two equilibrium points and thus it is classified into (excitation-induced stability) EIS phenomenon.

![Fig. 5: Phase plot of oscillator responses at excitation frequency of 5.5 Hz; Equilibrium points -- *: magnets gap of 12 mm; *: magnets gap of 8 mm](image)
While the excitation frequency is 14 Hz, the frequency responses of the linear system, bi-stable system of cases d=8 and 12 mm are 6.68 (V/s/m), 16.0 (V/s/m) and 4.27 (V/s/m), respectively. By observing the phase plots of the cases as shown in Fig. 7, it is noted that the bi-stable system in the case of d=12 mm presents mono-stable phenomenon and the case of d=8 mm presents a chaos condition at 14 Hz. Therefore, the electric energy harvesting performance in the case of d=8 mm is twice more than that of the linear system, while the case of d=12 mm is 37% lower than that of the linear system.

3.2 Case 2: Excitation magnitude effect on bi-stable piezoelectric vibration system

It is obvious that the factors for presenting the snap-through phenomenon consist of the excitation frequency as well as the excitation amplitude. In this experiment case, the gap between the magnets is set as d=8 mm, and the excitation frequency is set from 2 to 15 Hz with increment of 0.5 Hz. The vibration response and the electric voltage are measured in cases of the excitation amplitude of 5, 7.5 and 10 mm, respectively. Fig. 8 shows the frequency responses of the electric voltage with respect to the oscillator velocity.

Fig. 6: Phase plot of oscillator responses at excitation frequency of 7 Hz; Equilibrium points -- *: magnets gap of 12 mm; *: magnets gap of 8 mm

Fig. 7: Phase plot of oscillator responses at excitation frequency of 14 Hz; Equilibrium points -- *: magnets gap of 12 mm; *: magnets gap of 8 mm

Fig. 8: Frequency response of bi-stable system with different excitation amplitude (magnets gap d=8 mm)
In general, a larger excitation amplitude results in a higher frequency response value. At the frequency of 5.5 Hz, the frequency response increases drastically in the case of excitation amplitude of 10 mm. By observing the phase plot as shown in Fig. 9, it is apparent that the oscillator presents the snap-through phenomenon in the case of excitation amplitude of 10 mm, while the other two cases (7.5 and 5.0 mm) present mono-stable condition. When the excitation frequency is 9.5 Hz, the frequency responses of the three cases are close. The phase plots (Fig. 10) shows that the case of 10 mm presents chaos phenomenon, and the other two cases are under mono-stable conditions.

3.3 Case 3: Magnet gap effect on bi-stable piezoelectric vibration system

As investigated in the previous cases, the nonlinear bi-stable oscillator can present the snap-through phenomenon under a certain range of excitation amplitude as well as excitation frequencies. The vibration behaviour is mainly characterized by the nonlinear magnetic force. In this experimental case, the oscillator and output voltage responses are investigated by changing the gap distance between the magnets. The excitation frequency is set to 12 Hz and the magnet gap distance is set \( d = 13 \) and 8 mm. The responses are measured by varying the excitation amplitude from 5 to 10 mm with the increment of 0.5 mm.

Fig. 11: Frequency response of output voltage under excitation frequency of 12 Hz

Fig. 11 shows the frequency responses of the output voltage under the excitation frequency of 12 Hz and varying excitation amplitude (x-axis) in case of magnet gap \( d = 13 \) and 8 mm. It is that the output voltage response increases apparently from 7.5 mm to 8.0 mm in the case of \( d = 8 \) mm. Through analyzing the phase plots, the oscillator response presents snap-through phenomenon in
the case of $d=8$ mm while the excitation amplitude is larger than 8 mm. However, the response in the case of $d=13$ mm still presents the mono-stable condition within the same excitation amplitude.

4. Conclusion

In this research, a nonlinear bi-stable piezoelectric system was performed to evaluate the effectiveness of electric energy harvesting. Three experimental cases were designed to investigate the relationship among the excitation amplitude, excitation frequency, magnetic force, oscillator response and the output voltage. According to the excitation frequency and amplitude, the bi-stable oscillator may present mono-stable, chaos, EIS or snap-through phenomena. By investigating the different conditions, the higher energy harvesting performance can be obtained once the bi-stable oscillator presents the snap-through phenomenon. Compared with the linear system, the nonlinear bi-stable system has broader bandwidth for effective energy harvesting as well.

REFERENCES