INVESTIGATION ON THE ACOUSTIC BEHAVIOUR OF A LOCALLY RESONANT METAMATERIAL CURVED PANEL

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In order to seek for new ways to improve the sound transmission loss at ring frequency range for curved aeronautical structures, a semi-infinite curved panel, representing aircraft fuselage, is studied in this research. First, a minor modification on Koval’s classical theory for cylindrical shells is made to incorporate effective mass theory for acoustic metamaterials. Then, based on the finite element simulations, the possibility of improving the acoustic behaviour of such panels by using locally resonant acoustic metamaterial treatment is investigated. Both the host curved panel, and the host panel periodically mounted with local resonators, denoted as metamaterial curved panel, are compared from the transmission loss point of view. The observations show that the antiresonance dip appears first rather than the resonance peak in the sound transmission loss for the tested metamaterial curved panel, which may be explained by the effective impedance of the panel. The influences of the panel curvature on the dynamic behaviour of the local resonators are further discussed in comparison to the same resonant system acting on the flat panel. Both the finite element simulations and the theoretical estimations show the same trend, thus confirms the proposed explanation.

Keywords: locally resonant acoustic metamaterial, curved panel, sound transmission loss

1. Introduction

In the design of modern aerospace and aeronautical structures, the noise insulation properties often demand extra considerations due to the high excitation level and the increasing environmental requirement from the passengers. Among these issues, the airborne sound insulation is particularly challenging and of main interest of this study.

In this contribution, a curved panel is tested, with an infinite extension along the radial direction, and a simply-supported boundary condition with baffled wall on the other, in order to represent the sound transmission through an aircraft sidewall. In the context of airborne sound transmission through an aircraft fuselage, a classical mathematical model has been developed by Koval, depicting the structure as an infinite cylindrical shell under oblique incident plane waves excitation [1, 2]. Although there is a geometrical difference between the curved panel and the cylindrical shell, the same physical insights may be obtained.

Due to the panel curvature, the acoustic behaviour of the curved panel performs differently below the critical frequency compared to a flat panel [1, 3]. In particular, a severe deterioration appears in the sound transmission loss at the ring frequency, denoted as ring frequency effect, while far below the ring frequency, curved panels generally exhibit a better sound transmission loss property compared to flat panels. The specific deterioration associated with the ring frequency effect is due to a minimun
in the structural impedance, while the overall better performance is due to the increased stiffness and thus the increased structural impedance [4].

The problematic ring frequency effect of a curved panel is hard to overcome by conventional methods such as added mass or added damping, since it is a frequency purely dependent on the longitudinal wave speed and the curvature of the structure. Therefore, the concept of acoustic metamaterial is introduced in order to seek for new ways to overcome it.

Acoustic metamaterials have been intensively studied for decades due to their exotic acoustic behaviours [5–12]. For locally resonant acoustic metamaterial, the nontrivial phenomena can be achieved in a macroscopic scale on targeted frequencies, which are induced by the local resonance within each periodicity [13]. This demands that the dimension of the periodicity shall be constrained to have a much smaller size than the considered wavelength. Locally resonant acoustic metamaterials provide a new way to change the apparent dynamic properties of the panel within a designated frequency range and therefore improve the sound transmission loss behaviour. For example, it was shown to be effective in order to overcome the coincidence effect of a flat panel [10–12]. The combination of the specifically designed periodically distributed locally resonant mass spring system and the dynamic behaviour of the host panel may in this case lead to an excellent sound transmission loss property. In this paper, we explore the possibility that such an approach may be effective to address the ring frequency effect.

A minor modification of Koval’s theory is first made in order to use the effective mass theory for acoustic metamaterials. The effective structural impedance of the metamaterial cylindrical shell is thus obtained for sound transmission loss estimation. The transmission loss estimation of a curved panel, assumed to be simply supported in one dimension and infinitely extended in another, calculated with finite element simulations, although the process requires substantial computational resources. The mass-spring system is included in the finite element model in order to trigger local resonances. The system is periodically distributed on the concave surface of the curved panel, denoted as metamaterial curved panel in the following. Based on the finite element results, the ring frequency effect of the curved panel is evaluated. The dynamic behavior of the resonator influenced by the curvature is studied in comparison with the same system acting on a flat panel. The sound transmission loss of the metamaterial curved panel is compared with the theoretical estimations based on the effective impedance approach for the cylindrical shell.

2. Effective impedance approach

The infinitely extended cylindrical shell is excited by a plane wave with an incident angle $\theta$. Koval’s theory [11] for the calculation of the transmission coefficient may be interpreted as a summation of the transmission coefficient for each circumferential mode, as

$$\tau = \sum N \tau_N,$$

where

$$\tau_N = \frac{\varepsilon_N}{k_N R} \left| 1 + \frac{\cos \theta}{2 \rho_0 c_0 \alpha_N} \right|^{-2}.$$

In Eq. (2),

$$Z_N = Z_N^{sh1} + Z_N^{sh2} + Z_N^c$$

represents the impedance of the shell of the N$^{\text{th}}$ circumferential mode. Relevant variables in Eq. (2) are defined in Appendix.

In the low frequency range, the effective mass theory may be adopted for the locally resonant acoustic metamaterial [6, 7], as
where $f_{\text{res}}$ is the resonance frequency of the locally resonant system, $\delta$ is the mass ratio of the resonator to the host plate. The resulting effective impedance may therefore be expressed as

$$Z_N^{\text{eff}} = Z_N^{\text{sh1}} \left( 1 + \frac{\delta}{1 - f^2/f_{\text{res}}^2} \right) + Z_N^{\text{sh2}} + Z_N^{c}.$$  

A simplified model that may qualitatively describe the impedance of an unbounded slightly curved panel is given in [2], by

$$Z = j \omega m \left( 1 - \frac{f^2}{f_{\text{cr}}^2} \sin^4 \theta - \frac{f_{\text{ring}}^2}{f^2} \right),$$

where, for a curved panel with curvature $R$, Young’s modulus $E$, Possion’s ratio $\nu$, thickness $t$, and surface density $m$, the critical frequency and the ring frequency are $f_{\text{cr}} = c_0 \sqrt{m/D}/2\pi$ and $f_{\text{ring}} = \sqrt{Et/m(1-\nu^2)}/2\pi R$, respectively. As can be seen from Eq. (5), in comparison to a flat panel impedance [14], the influence from the ring frequency is included by the added term $-(f_{\text{ring}}/f)^2$.

Subsequently, replacing $m$ in Eq. (5) with $m_{\text{eff}}$, we have

$$Z_{\text{eff}} = j \omega m \left( 1 - \frac{f^2}{f_{\text{cr}}^2} \sin^4 \theta - \frac{f_{\text{ring}}^2}{f^2} \right) + \frac{\delta}{1 - f^2/f_{\text{res}}^2}.$$  

The sound transmission loss may therefore be calculated as

$$STL = 10 \log \left| \frac{1}{\tau} \right| = 10 \log \left| 1 + \frac{\cos \theta}{2\rho_0 c_0} Z_{\text{eff}}^{-2} \right|.$$  

3. Finite element model

Fig. 1 represents the global geometrical configuration of the curved panel. The geometric center-line of the curved panel is aligned with the $x$ axis. The structure is assumed to be infinitely extended along the $x$ axis, while baffled in the circumferential direction. The panel is assumed to be thin, isotropic and made of a homogeneous material. It is excited by a time harmonic plane wave with an elevation angle $\theta$ and azimuth angle $\varphi$. In our study, for the sake of simplification, the oblique incident angle is arbitrarily set to be $\theta = \pi/3$, $\varphi = 0$.

The semi-infinite condition is modelled with a Floquet periodic boundary condition. The mass-spring resonators are periodically mounted on the concave side of the panel. The periodicity is set to be much smaller than the considered wavelength. The coupled acoustic-structure problem is here solved using the commercial software COMSOL®.

The sound transmission of the panel may be evaluated as

$$STL = 10 \log \left| \frac{\hat{W}_{\text{inc}}}{\hat{W}_{\text{trans}}} \right|,$$  

where $\hat{W}_{\text{inc}}$ and $\hat{W}_{\text{trans}}$ are the incident and the transmitted acoustic power.

4. Results and discussion

In this section, detailed discussions concerning the acoustic behaviour of the curved panel, including the ring frequency and the curvature effect, are presented and compared with the corresponding theoretical cylindrical shell results. The local resonance as well as the induced metamaterial effects are presented in connection with the sound transmission loss. The curvature influence is evaluated in comparison with the effect of the same resonant system acting on a flat panel.
4.1 Ring frequency and curvature

The acoustic behaviour of the curved panel is critical around the ring frequency. The ring frequency effect is caused by the interaction of the bending forces and the membrane forces [4]. This interaction shifts the eigenmodes of the panel to higher frequencies. The radiation efficiency of each mode is also increased. Mathematically, the ring frequency caused by the curvature leads to a minimum for the impedance of the panel (see Eq. 5) and thus causes a critical sound transmission loss, which may be seen in Fig. 2. Fig. 2 also shows that the theoretical estimations, in general, underestimate the transmission loss at low frequencies. This is in part due to the boundary condition, which further increases the stiffness in the low frequency range. This may also explain the slight shift of the ring frequency towards lower frequencies for the curved panel.

The ring frequency effect is associated with the structural breathing mode [15]. In contrast to a cylindrical shell, the breathing mode is truncated due to the finite size of the structure in the circumferential direction. Fig. 3 illustrates the breathing mode for the finite sized panels.

As shown in Fig. 4, as the panel curvature increases, the ring frequency increases, and the acoustic behaviour in the low frequencies is further improved. The acoustic behaviour is dominated by the
stiffness below the ring frequency, and by then the mass is becoming dominant after exceeding the ring frequency. It is noteworthy that from about twice the ring frequency, the panel may behave as a flat panel.

Figure 4: Effect of curvature.

4.2 Effect of local resonance

In order to evaluate the influence of the curvature on the dynamic behaviour of the local resonators, the same resonant system is mounted on both the curved panel and the flat panel. As can be seen in Fig. 5 as the mass ratio $\delta$ increases, the working frequency range is, as expected, wider. However, for the resonator acting on the curved panel, instead of observing the resonant peak shown on the flat panel, the antiresonance appears first. The explanation may be as follows. In contrast to a resonator located in the mass-controlled region for a flat panel, below the coincidence frequency, the curved panel is in the stiffness-controlled region below the ring frequency. As the effective mass approaches the resonance frequency, the effective stiffness of the metamaterial curved panel decreases. This consequently leads to the antiresonance dip appearing first as shown in Fig. 5.

The same explanation can be seen from Eq. (5): below the ring frequency, the impedance is dominated by the ring frequency which is related to the stiffness. As the frequency reaches the
resonance frequency, the effective impedance is decreasing to eventually reach a singularity to $-\infty$. Fig. 6 shows that, the finite element simulation matches the trend of the theoretical estimation, thus confirming the proposed explanation.

The resonance frequency of the local mass-spring resonator is then tuned to the ring frequency. Transmission loss curves in Fig. 7 display a sharp improvement in the vicinity of the resonance frequency, however, at the cost of decreased performance otherwise.

5. Explanation and conclusion

The local resonator design was shown to be able to overcome the coincidence effect of flat panels [10,12]. Excellent sound transmission loss can be achieved without any antiresonance dip then, which is realized by tuning the resonance frequency to the coincidence frequency range. For a flat panel, in the coincidence region, the transmission loss behaviour is transferred from the mass-controlled region to the stiffness-controlled region. The transition of the controlled region, together
with the dynamic behaviour of the resonator, leads to a non zero effective impedance and thus to a much improved performance.

For a curved panel, at the ring frequency, however, the transition occurs by a transfer from the stiffness-controlled region to the mass-controlled region. In this case, the traditional locally resonant metamaterial treatment is therefore not suitable to overcome the ring frequency effect, besides the sharp narrow improvement of the performance at the resonance frequency. The opposite transition, i.e. from stiffness-controlled to mass-controlled, therefore requires a nontrivial local resonator, whose effective spring constant shall be negative.

6. Appendix

Variables for Eq. (2) are as follows:

$$
\varepsilon_N = \begin{cases} 
1 & N = 0 \\
2 & N \geq 1 
\end{cases},
$$

$$
Z_{sh1}^N = j\omega m \left( 1 - A_N \frac{f_{ring}}{f^2} \right),
$$

$$
Z_{sh2}^N = j\rho_0 c_0 \frac{H_{N}^{(1)} (k_u R)}{H_{N}^{(1)} (k_u R)},
$$

$$
Z_N^N = \frac{2}{\pi k_u R} \frac{1}{J_N^2 (k_u R) + N_N^2 (k_u R)} - \frac{j}{J_N^2 (k_u R) + N_N^2 (k_u R)} \left( \frac{J_N (k_u R) J_N (k_u R)}{J_N^2 (k_u R) + N_N^2 (k_u R)} + \frac{(k_z R)^4}{((k_z R)^2 + N^2)^2} \right),
$$

$$
A_N = K \left[ \frac{((k_z R)^2 + N^2 - 1)^2}{2 (1 - \nu) \left( (k_z R)^6 - (k_z R)^2 N^4 + (k_z R)^2 N^2 \right)} \right] + \frac{(k_z R)^4}{((k_z R)^2 + N^2)^2},
$$

$$
K = \frac{t^2}{12 (1 - \nu^2) R^2},
$$

with \( k_u = k \sin \theta, k_z = k \cos \theta; J_N, N_N \) are Bessel functions of the first and second kinds of order \( N \); \( H_N \) is Hankel functions of order \( N \).
REFERENCES


