The maximum speed of electrical motors for E-Mobility applications keeps increasing. The Power demand is fulfilled with small motor volumes at high top speeds of up to 30,000 rpm. To reach the required torque to accelerate the car, a transmission with high ratio is used. Low design noise for the gear stages are mandatory. Current gear design approaches for automotive transmissions are aimed at input speeds of up to 8,000 rpm. It is known that the noise behavior of a gear stage is not acceptable if the resonance speed of the mesh is reached, at which the gears are oscillating against each other and very high noise emissions are to be expected. This leads to the question if the traditional, sub-resonance speed design for gear meshes is obsolete for future E-mobility applications.

This paper discusses different gear designs for sub- and super-resonance operation in an electrical drivetrain. The gear mesh design for a number of variants is covered, a test transmission is used to measure the excitation behavior of the gears under speeds of up to 30,000 rpm.

Keyword: NVH, Gears, Transmission, design, E-Mobility
2. Gearbox noise sources

Gear noise is commonly attributed to the following influence parameters:
- Variation of mesh stiffness
- Flank geometry modifications
- Tooth deformation
- Gear alignment deviations
- Surface conditions and manufacturing deviations of the meshing flanks
- Friction influences

Variation of the gear mesh stiffness leads to excitation of the gearbox structure and along the transfer path to airborne noise. All resonances are important since these amplify even small excitations. The resonance of the gear pair itself is critical which is an oscillation of the gears against each other. High dynamic load increases in the mesh are usually the consequence of operation in the gear main resonance.

The excitation amplitude of the gear mesh force is usually dependent on the difference between mesh frequency that is related to the running speed and the gear resonance frequency that is related to the gear geometry. The aim is to ensure low noise behaviour and low dynamic overloads by designing for the given operating conditions.

3. Geometric properties of the gear mesh

For parallel axis gears, the involute flank form has many advantages and is common in application. The tooth force in the mesh is oriented tangentially to the base circle of both gears and is not varying in direction under constant operation. Mathematically, the involute provides a smooth transition between two consecutive teeth. The number of teeth in contact with the mating gear is changing over time while running through the mesh. The medium number of teeth in contact over time determines the profile contact ratio and the overlap ratio.

However, the loaded gear contact is subject to deformations of the teeth transmitting the load, so the ideal mathematical conditions do not apply. Considering the tooth stiffness and contact conditions leads to considering a changing mesh stiffness. This consideration results in the parametric excitation that is typical for gears.

Gear main resonance may be determined by simulation. To support the first design approach, a simpler model is used. This comprises of the two gear masses and the connecting mesh stiffness [11]. That provides the possibility to get an overview over many configurations. Heider [6] used that approach for a study over gear stages in regard to center distance and mesh frequency. The mesh frequency of a gear mesh is dependent on the running speed of the shaft and the number of teeth on the gear. Since the changing mesh stiffness is not sinusoidal over the path of contact, also multiples of the mesh frequency are exciting the structure, see eq. (1). Sometimes a limitation is to keep below resonance also with the higher order harmonics. Then only a max. speed of half the resonance speed or even less is allowed. A simplified approach to evaluate the resonance frequency acc. to Heider is shown in eq. (2). The results from a parameter variation are documented in Fig. 1.

Mesh frequency (harmonics for x >1)

\[ f_x = \frac{n_z}{60} \cdot x \quad \text{with} \quad x \in \{1, 2, 3, \ldots\} \]  

Resonance frequency of gear pair
\[ f_E = 190 \times \frac{1000}{a} \times (i + 1.5) \]  

Figure 1: Approximation of mesh eigenfrequencies in respect to center distance and ratio. Valid for cylindrical steel gears, mesh stiffness \( c_r = 20 \text{ N/(mm \, \mu m)} \) – see Heider [6]

Figure 1 shows that with smaller center distance and with increasing ratio the gear resonance frequency increases. The Level curve’s numbers denominate the resonance frequency \( f_E \) in Hz. The Colour indicates load: darker colour corresponds to higher transmittable torque moment. If higher input speeds are required, the design efforts should aim at moving the geometry in direction of smaller center distances. However, the figure also shows that transmittable load decreases with smaller center distance. This might pose a limitation.

To perform a rough evaluation, we focus on two main limitation of load carrying capacity for gears: pitting limit and tooth root breakage limit. Maximum contact pressure is a well established value to rate pitting load capacity [8]. Figure 2 shows a sketch of the contact pressure in transverse section, important parameters are tooth force, tooth width and radii of curvature, see eq. (3).

Figure 2: Sketch of tooth contact in transverse section
For tooth root breakage, tooth root stress is used as a limiting value in rating methods e.g. ISO 6336 [8]. Eq. (4) shows important influence parameters, tooth force, tooth width, lever of force acting on the tooth flank and the relevant tooth root chord. The resulting nominal stress is an input value for the further calculation (notch influence and further parameters).

\[
\sigma_b = \frac{M_b}{W_b} = \frac{F_n \cdot \cos \alpha \cdot h_F}{h_F b \cdot s_{Fn}}
\]  

\( \sigma_b \left[ \frac{N}{\text{mm}^2} \right] \): bending stress  
\( M_b \left[ \text{Nm} \right] \): bending moment around tooth root center  
\( W_b \left[ \text{mm}^3 \right] \): bending resistance  
\( F_n \left[ \text{N} \right] \): Tooth force  
\( \alpha \left[ \text{rad} \right] \): pressure angle  
\( h_F \left[ \text{mm} \right] \): lever  
\( b \left[ \text{mm} \right] \): tooth width  
\( s_{Fn} \left[ \text{mm} \right] \): relevant tooth root chord  

Both values, maximum contact pressure and nominal tooth root stress, only specify the load influence. The material influence including size, hardness, surface etc. is not discussed here.

To illustrate the implication made by these equations in the following a comparison between two single gear gearboxes with the same ratio shall be made that differ in center distance. All geometric properties shall be half in size in gearbox one compared to gearbox two, namely center distance, see
eq. (5), tooth width and tooth dimensions. According to Fig. 1 this would about double the resonance frequency of gearbox one in comparison to gearbox two, see eq. (6).

\[
a_1 = \frac{1}{2} a_2 \\
(5)
\]

\[
f_{c1} \approx 2 \cdot f_{c2} \\
(6)
\]

About double the input speed would seem feasible and about half the torque moment to keep the input power constant. Looking at load capacity, tooth width is reduced by half, radii of curvature are reduced by half because of the reduced base diameter of the gears and that also leads to a torque moment resulting from the tooth force that is reduced by half. That means, allowable torque moment is reduced by a factor of 8. The same exercise for tooth root stress arrives at a factor of 8 as well. Figure 4 shows that rough trend for allowable torque moment over center distance.

![Figure 4: Trend of transmittable torque moment over center distance](image)

In terms of load carrying capacity, a change in center distance means a change in transmittable torque moment by the power of 3, see eq.(7).

\[
T_1 \approx T_2 \left( \frac{a_1}{a_2} \right)^3 \\
(7)
\]

An increase in input speed will only be compensated by reduced gearbox size if large amendments to transmittable torque moment are made. As soon as this is not possible any more, operation in resonance frequency and beyond will have to be accepted.

4. Design approach for low noise gears

A standard design approach is to minimize the transmission error of the gear mesh [2], [3]. Loaded transmission error is the change of deformation of the mesh while rolling through the contact path. The amount is usually in the range of fractions of microns up to a few microns [1]. It is influenced by small quantities in tooth flank geometry. A common optimization measure is modifying the flank geometry from the involute for only a few microns [7], [5], [10]. The optimization is done by iterative approaches since a direct compensation is difficult to manufacture [9].

The transmission error is the result of a quasi-static calculation. There is no influence of inertia and no resonance behaviour included. It allows for fast and numerous iteration in optimization. The
results are valid for the boundary condition that the application and operation conditions are far away from resonance speed. The validity of that design concept for very high speed gears shall be analysed by measurement. For that purpose, gear design are used that Gwinner [4] published (see Fig. 5 and Table 1).

![Figure 5: Gear geometry concepts for a high speed concept taken from Gwinner [4]](image)

Table 1: Gear geometry data for a high speed concept taken from Gwinner [4]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Index</th>
<th>Ref.</th>
<th>Low-Loss</th>
<th>Sub-critical</th>
<th>Super-critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre distance</td>
<td>a/mm</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
</tr>
<tr>
<td>Pinion teeth</td>
<td>z₁/₁₀</td>
<td>18</td>
<td>27</td>
<td>13</td>
<td>27</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>i⁻⁻⁻⁻</td>
<td>4.94</td>
<td>4.93</td>
<td>5.08</td>
<td>3.7</td>
</tr>
<tr>
<td>Modulus</td>
<td>mₑ/mm</td>
<td>1.42</td>
<td>1.03</td>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td>Normal press. angle</td>
<td>a₁⁰⁻⁻⁻⁻⁻</td>
<td>17.5</td>
<td>22.5</td>
<td>15</td>
<td>17.5</td>
</tr>
<tr>
<td>Tooth width</td>
<td>b/₁₀⁻⁻⁻⁻</td>
<td>19</td>
<td>20.5</td>
<td>21</td>
<td>20</td>
</tr>
<tr>
<td>Helix angle</td>
<td>β⁻⁻⁻⁻⁻⁻⁻⁻</td>
<td>29</td>
<td>19</td>
<td>19</td>
<td>24</td>
</tr>
<tr>
<td>Trans. Contact ratio</td>
<td>εₑ⁻⁻⁻⁻⁻⁻</td>
<td>1.65</td>
<td>1.0</td>
<td>1.52</td>
<td>1.59</td>
</tr>
<tr>
<td>Overlap ratio</td>
<td>εᵦ⁻⁻⁻⁻⁻⁻</td>
<td>2.06</td>
<td>2.06</td>
<td>1.09</td>
<td>2.04</td>
</tr>
<tr>
<td>Tooth loss factor</td>
<td>Hᵥ⁻⁻⁻⁻⁻⁻⁻⁻</td>
<td>0.22</td>
<td>0.07</td>
<td>0.22</td>
<td>0.18</td>
</tr>
</tbody>
</table>

The designs for the high speed stage yield sufficient load carrying capacity to allow for variations: The reference design shall remain subcritical even at 30,000 rpm. The low-loss design is a special tooth form aimed at low power loss due to friction in the mesh. The subcritical design has a low number of teeth on the pinion to limit excitation frequency, the supercritical design has a high number of teeth on the pinion to run through the main resonance frequency.

All gears are optimized according to the approach using the quasi-static transmission error as indication value as described above.

As a first validation of the considerations, the reference design was subject to a new and extended measurement compared to earlier publications. The test geartrain and a picture of the test transmission are shown in Fig. 6 and Fig.7.
Figure 6: Sketch of test gear train – green stage is in focus [4]

Figure 7: view into the test transmission [12]

Figure 8 shows the measurement results in an order diagram. The pinion order and multiples are clearly visible. More orders may be identified that can be related to the second gear stage. At about 22.000 rpm the 36th input shaft order which is the second order of the high speed stage seems to meet the resonance frequency. This resonance does not meet the pinion order (18th) in the speed range up to 30.000 rpm. So the measurement proves the basic design which aimed for a subcritical layout. Furthermore, the pinion orders are not dominant compared to the further orders, so the general design approach seems basically feasible. Any remaining optimization potential may be experimentally proven with some additional test gear designs in the future.

Figure 8: Acceleration measurement of test transmission, focus is high speed stage
Summary

Basic considerations show that smaller gear center distance yield higher resonance speeds. For high speed electrical motors transmissions with smaller center distances still allow subcritical design of the gear stages. However, the transmittable torque moment changes much faster with a change in center distance than resonance speed. Reducing gearbox size to keep established design method valid will only work up to certain point. Then the designer has to cope with higher dynamic excitations because of running up to the resonance frequency or running through it in operation.

Exemplary measurements show that to some extent established design methods will be useful even when coming close to resonance speed.

REFERENCES