THE PREDICTION AND CONTROL OF DYNAMIC INTERACTIONS BETWEEN TALL BUILDINGS AND HIGH-RISE VERTICAL TRANSPORTATION SYSTEMS SUBJECT TO SEISMIC EXCITATIONS

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Excitations due to earthquake result in large responses of tall buildings. Those in turn affect modular vertical transportation systems such as high-rise elevators. In this paper a nonlinear model of dynamic interactions between the building structure and a high-rise elevator system is developed and discussed. The natural frequencies of tall buildings become near the natural frequencies of the car/counterweight suspension-guide rail in traction-drive elevator systems. This results in resonance interactions with car/counterweight suspension and compensating ropes that sway and are getting entangled with other equipment in the well. The paper shows that the application of transverse tuned mass dampers (TMD) can reduce the dynamic responses in the system. Thus, the TMD can be deployed as a protective measure to mitigate the associated hazards.

Keywords: buildings, elevators, seismic, vibrations, control.

1. Introduction

Ground motion excitations induced by earthquakes can result in large seismic responses of high-rise buildings in the modern mega cities [1]. The fundamental natural frequencies of buildings often fall within the frequency range of long-period seismic excitations.

This in turn results in large resonant responses and severe damage to the buildings and/or to their non-structural installations such as high-rise vertical transportation systems (lifts – elevators [2-3]). In this paper an extended analytical and numerical study to predict the dynamic responses of a cable-mass–tuned mass damper (TMD) system representing a simplified high-rise elevator model [4-5] is presented. The model is then used to demonstrate the effectiveness of the TMD strategy to control the dynamic performance of the system under long period earthquake excitations.

2. Cable–mass–tuned mass damper (TMD) system

The cable–mass–TMD system illustrated in Fig. 1 is mounted within a vertical cantilever host structure subject to ground motion $s_i(t)$. The mass $M$ is suspended on the cable of time-variable length $L(t)$ and is constrained horizontally within the host structure by a spring–viscous damper element of effective coefficient of stiffness $k$ and damping coefficient $c$. The upper end of the cable is passing through $O$ and the height of the structure is $AB = Z_0$. The structure undergoes bending elastic deformations $\bar{w}(z,t)$ where $0 \leq z \leq Z_0$, with the displacements at the top end defined as $\bar{w}(Z_0,t) = \bar{w}_0(t)$. The cable moves vertically within the host structure at transport speed $V$ and...
acceleration $a$. The mean quasi-static tension, mass per unit length, modulus of elasticity and cross-sectional area of the cable are denoted as $T' = [M + m(L - x)](g - a)$, $m$, $E$, and $A$, respectively. The Eulerian spatial coordinate $x$ is measured from the upper end of the cable downwards as shown. The lateral dynamic displacements of the cable are denoted as $v(x,t)$. They are coupled with the longitudinal displacements denoted as $u(x,t)$. The lateral and longitudinal motions of mass $M$ are denoted as $v_M(t)$ and $u_M(t)$, respectively.

![Figure 1: Cable – mass – TMD model.](image)

3. Mathematical model

In the analysis to follow the cable - mass - TMD system is treated as a nonstructural modular component of the building and the assumption is made that its dynamic response does not affect the bending deformations of the structure. Then, the bending deformations of the structure are introduced as a base excitation.

3.1 Equations of motion of the structure

The equation of motion of the structure is formulated as

$$m_i(z)w_i + C w_i + Lw = 0.$$  \hspace{1cm} (1)
where \( w(z,t) \) are overall displacements of the structure, \( m_s(z) \) represents the linear mass density of the structure, \( L = \frac{\partial^2}{\partial z^2} \left[ EI(z) \frac{\partial^2}{\partial z^2} \right] \) is the spatial operator related to the elastic potential energy in bending, \( C = \beta \frac{\partial^2}{\partial z^2} \left[ I(z) \frac{\partial^3}{\partial z^2 \partial t} \right] \) is the damping operator with \( \beta \) denoting a constant damping parameter, to account for the strain rate (internal) damping of the structure, \( EI(z) \) denotes the bending stiffness and \( I(z) \) is the cross-sectional area moment of inertia. The overall displacements of the structure can be expressed as

\[
w(z,t) = \bar{w}(z,t) + s(t). \quad (2)
\]

so that equation of motion (1) is re-written as

\[
m_s(z) \bar{w}_n + C \bar{w}_n + L \bar{w} = -m_s(z) \ddot{s}(t). \quad (3)
\]

For complex structures a closed-form solution of Eq. 1 is not admitted. An approximate solution can be sought instead in the following form

\[
\bar{w}(z,t) = \sum_{n=1}^{N} \Psi_n(z) \varphi_n(t). \quad (4)
\]

where \( \Psi_n(z) \) is a set of comparison functions and \( \varphi_n(t) \) are generalised coordinates. Using Eq. 4 in Eq. 3 leads to the following

\[
M \ddot{\varphi} + C \dot{\varphi} + K \varphi = F(t). \quad (5)
\]

where \( M = [m_m] \), \( m_m = \int_0^{Z_0} m_v \Psi_r \Psi_n dz \), \( C = [c_m] \), \( c_m = \int_0^{Z_0} \Psi_r C \Psi_n dz \), \( K = [k_m] \), \( k_m = \int_0^{Z_0} \Psi_r L \Psi_n dz \), \( r,n = 1,\ldots,N \), and \( F = \left[ F_r \right] \), \( F_r = -\ddot{s}_0(t) \int_0^{Z_0} m_v \Psi_r dz \) is a vector of generalized excitation forces.

### 3.2 Equations of motion of the cable - mass system

The equations governing the undamped dynamic response of the cable - mass system are developed by the application of the Hamilton’s principle which requires that

\[
\int_{t_1}^{t_2} \left( \delta E - \delta \Pi - \delta \Pi_{nc} \right) dt = 0, \quad \delta w(x,t) = 0 \text{ at } t = t_1, t_2. \quad (6)
\]

where \( E \), \( \Pi \) and \( \Pi_{nc} \) denote the kinetic energy and the potential energy of the system, and the work due to non-conservative forces acting upon the system, respectively. The mathematical model of the system is then formulated as

\[
\begin{align*}
\frac{D^2 u}{Dt^2} - E A v_x &= 0, \quad m \frac{D^2 v}{Dt^2} - T_M v_x + m(g-a)(x v_{xx} + v_t) - E A (v_{xx}) &= 0, \\
M \ddot{v}_{xx} + T(L) v_x |_{x=L} + kA + cA - k_d (z_d - v_M) - c_d (\ddot{z}_d - \ddot{v}_M) + E A \varepsilon |_{x=L} v_x |_{x=L} &= 0, \\
m_a \ddot{z}_d + k_d (z_d - v_M) + c_d (\ddot{z}_d - \ddot{v}_M) &= 0, \quad (M + m_d) \ddot{u}_M + E A \varepsilon |_{x=L} = 0.
\end{align*}
\]
where $\varepsilon = u_x + \nu^2/2$ represents the axial strain, $D(\cdot)/Dt = (\cdot)_t + V(\cdot)_x$ and $(\cdot)_t$ and $(\cdot)_x$ represent partial derivatives with respect to time $t$ and $x$, respectively, $T_{ML} = (M + m_d + mL)(g-a)$, $T' = [M + m(L-x)](g-a)$ and $\Delta$ is the deformation of the spring $k$ – damper $c$ system. For tensioned members such as steel wire ropes (SWR) the lateral frequencies are much lower than the longitudinal frequencies. Thus, considering that the excitations frequencies are much lower than the fundamental longitudinal frequencies the longitudinal inertia of the ropes can be neglected in the first equation in (7). This equation can be integrated to give $u_x = \varepsilon(t) - \nu^2/2$ where $\varepsilon(t)$ represents the quasi-static axial strain in the rope [5].

### 3.3 Base excitation and resonance response due to ground motion

In the proposed approach the bending deformations of the host structure are treated as a base excitation for the cable – mass – spring system. Thus, it is assumed that the influence of the cable system dynamics on the structural response can be neglected. In the scenario when the structure is subject to fundamental resonance an approximate solution of Eq. 3 can be sought by using the polynomial shape function $\Psi'(\eta) = 3\eta^2 - 2\eta^3$ in Eq. 4, where $\eta = z/Z_0$. In order to accommodate the base excitation in the equations of motion (7) the overall lateral displacements of the cable – mass - spring system are expressed as

$$v(x,t) = \overline{v}(x,t) + V_0(x,t), \quad 0 \leq x \leq L(t).$$

where the first term in Eq. 8 is expressed as

$$\overline{v}(x,t) = \sum_{n=1}^{N} \Phi_n \left[ x; L(t) \right] q_n(t).$$

where $\Phi_n$ are orthogonal trial functions and $q_n(t)$ represent time-dependent generalised coordinates. It can be assumed that the variation of length $L$ with time is small. Thus, $L$ is considered a slowly varying function in time meaning that the change of $L(t)$ over a period corresponding to the fundamental frequency of the system is small compared to $L$ [6]. In order to represent this fact a slow time scale defined as $\tau = ct$, where $c \ll l$ is a small parameter, is introduced. Considering that $L = L(\tau)$ the trial functions satisfy the homogenous boundary conditions and are defined as $\Phi_n(x; L(\tau)) = \sin \left[ \sigma_n(L(\tau))x \right]$, $n = 1, 2, \ldots, N$, with $N$ denoting the number of terms/modes taken.

The slowly varying eigenvalues $\sigma_n$ are defined by the frequency equation given as

$$\left( k - \frac{M}{m} T_{Md} \sigma_n^2 \right) \sin(\sigma_nL) + T_{Md} \sigma_n \cos(\sigma_nL) = 0, \quad T_{Md} = T'(L)$$

The second term in Eq. 8 is given as

$$V_0(x,t) = s_i(t) + \left( 1 + \frac{\Psi_L - 1}{L(\tau)} \right) \overline{\omega}_0(t), \quad \Psi_L = \frac{Z_0 - L(\tau)}{Z_0}; \quad \overline{\omega}_0(t) = \overline{\omega}(Z_0,t)$$

By using Eq. 8 in Eqs 7 the dynamic response of the system is described by the following set of nonlinear ordinary differential equations
4. Numerical simulation and results

A numerical simulation case study has been conducted to demonstrate the dynamic behaviour and the TMD action. The model of the mass–cable system comprising the mass \( M = 2800 \text{ kg} \) suspended on 6 steel wire ropes has been considered. The ropes have mass per unit length \( m_r = 0.872 \text{ kg/m} \) and longitudinal stiffness \( E A = 22.889 \text{ MN/m}^2 \), each.

Fig. 2 shows the variation of the first four lateral natural frequencies of the system (red solid lines) when the length of the ropes changes from about 20 m to \( L_{\text{max}} = 208.66 \text{ m} \), and the spring constant is \( k = 32.723 \text{ kN/m} \). In the scenario considered in the simulation it is assumed that the ground motion is harmonic and is exciting the fundamental mode of the building structure of the same frequency. The frequency of the excitation is assumed as \( \omega_0 = 4.71 \text{ rad/s} \) \((0.75 \text{ Hz}) \), with a peak acceleration value taken as 0.1 m/s\(^2\) and the response of the structure results in the amplitude of 0.06 m at the top level (at \( z = Z_0 \), corresponding to the maximum value of \( \Phi_0(t) \) determined from Eq. 5).

Curve veering phenomena can be observed in the frequency diagram when two eigenvalues approach each other closely and suddenly veer away [7]. For example, the veering regions of the 1st and the 2nd, the 2nd the 3rd and the 3rd and 4th natural frequency loci correspond to the lengths \( L \) of about 40 - 60 m, 90 - 110 m and 140 – 160 m, respectively. It is also interesting to observe that the flat sections of the 1st, 2nd and 3rd natural frequency curves correspond to the modes of the mass \( M \) motion.

The frequency value can be estimated as \( \omega_M = 4.83 \text{ rad/s} \) \((0.77 \text{ Hz}) \), and is near the excitation frequency \( \omega_0 \) (represented by the black dashed horizontal line in Fig. 2).

Considering that the frequency of base excitation becomes tuned to the fundamental mode of the system when the length of the suspension ropes \( L \) is approximately 45 m the parameters of TMD are determined as \( m_d = 325.4 \text{ kg} \), \( k_d = 6022 \text{ N/m} \) and \( c_d = 516.9 \text{ Ns/m} \). The equations of motion are then solved by using a single mode approximation in the expansion (9) resulting in \( \Phi(x,t) = \Phi_0 \left[ x; L(\tau) \right] q_0(t) \), where \( r = 1 \) is used. The 4th-5th order Runge-Kutta algorithm numerical is used to solve the equations of motion numerically assuming the damping ratio \( \zeta_1 = 0.25 \), when the mass is ascending slowly from the lower level upwards at speed of 1.25 m/s and the length of the ropes changes from about 65 m to 8.6 m, respectively.
Fig. 3, Fig. 4 and Fig. 5 show the lateral response $r_M$ of the primary mass vs. time, and the corresponding forces acting upon the host structure vs. time, and the longitudinal displacements $u_M$ of the mass, with TMD action (red solid lines) and without TMD action (dashed blue lines).

The plots demonstrate the resonance behaviour of the system suffering from large dynamic responses. It is evident that the resonance oscillations and forces are becoming attenuated by the TMD action by about 33%. In addition, the application of TMD at the primary mass results in the attenuation of longitudinal vibrations of the system that are nonlinearly coupled with lateral response.

![Figure 2: The natural frequencies of the cable – mass system.](image)

5. **Concluding remarks**

Seismic-induced ground motions have adverse influence on the performance of vertical transport installations. The analysis and results presented in this paper show that under resonance conditions the application of a passive TMD system is effective in reducing the dynamic responses and dynamic forces acting upon the system components. The level of reduction predicted in the case study discussed above is about 33%. The application of active tuned mass damper strategy to enhance the control performance could be considered.
Figure 3: Lateral displacements of the primary mass.

Figure 4: Force acting upon the host structure.
Figure 5: Longitudinal displacements of the primary mass.

REFERENCES


