EXPERIMENTAL IDENTIFICATION OF STATIC AND DYNAMIC STIFFNESS OF POLYMERIC VIBRATION DAMPERS

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Polymeric dampers are frequently used in the automotive field in order to filter engine produced vibrations and thus protecting electronic devices. An accurate knowledge of damper stiffness is therefore important for a proper design of such mechanical filters. Since viscoelastic materials exhibit a significant stiffness–frequency dependency, dynamic stiffness is to be characterized for automotive vibration dampers. In the present paper, a method for broadband identification of the dynamic complex stiffness is described; since the proposed method is reliable at medium and high frequency only (500–1500 Hz), an additional static measurement has been performed. Therefore, the resulting method is a combination of static and dynamic measurements and it does not require specific DMA equipment: all experiments have been carried out using typical vibration testing equipment.

Keywords: Master curve, viscoelasticity, automotive damper.

1. Introduction

Vibration control is a relevant design requirement for several industrial applications, often achieved by means of passive damping technologies involving viscoelastic materials. Among them, elastomeric vibration isolators are widely adopted, consisting of a low stiffness and high damping connection between a vibrating support and the item to be fastened. Due to polymeric elements, to characterize the dynamic behavior of a damper it is of paramount importance estimating stiffness and damping changes with excitation frequency and temperature [1, 2, 3].

Most commonly, experimental results presented in the literature deal with measurements of the viscoelastic properties of prismatic rubber elements [4] or beam–like specimens [5]. Although being dealt with several papers, there is still a lack of well–designed experimental work dealing with the measurement of damping properties of polymeric vibration isolators. In this case the characterization is commonly carried out at low frequency by means of a Dynamic Mechanical Analysis (DMA), nonetheless this approach cannot be applied at higher frequency [6].

In the present study, the polymeric vibration damper is modelled as a single degree of freedom system (SDOF), using a shaker as source of vibration. The viscoelastic properties of the system are described by its dynamic stiffness, a complex function of the frequency, whose real (storage modulus) and imaginary (loss modulus) parts are experimentally estimated. The proposed method is based on a direct measurement of the energy loss in hysteretic cycles, and it is suitable for simple implementation using common instruments for vibration measurement.

2. Testing equipment

Damping measurements are carried out on the specimen of polymeric vibration damper shown in Fig. 1, using the test rig shown in Fig. 2. The test rig is composed of a large steel mass and a Bruel & Kjear 4808 electrodynamic shaker. Both the mass and the shaker are suspended with low stiffness
supports. The shaker exerts a force on an aluminium disk which is connected to the large mass by means of the test specimen (i.e. the polymeric damper shown in Fig. 1). Instrumentation consists of three accelerometers and a dynamic load cell. Measured points are:

- $a_0$: mono–axial accelerometer on the shaker;
- $a_1$: mono–axial micro–accelerometer on the right side of the damper (in Fig. 2);
- $F_1$: dynamic load cell on the right side of the damper (in Fig. 2);
- $a_2$: mono–axial accelerometer on the steel mass.

The shaker is connected to the aluminium disk by means of a stinger, in order to avoid lateral loading of the load cell and of the test specimen.

Figure 1: Polymeric vibration damper.

Figure 2: Electrodynamic shaker experimental setup.

3. Analysis of experimental data

In order to characterize the dynamic stiffness of the polymeric damper, a cyclic loading is applied by means of the shaker, and the values of forces and accelerations are measured during 50 hysteresis loops. In the following, characterization in terms of viscous damping and of complex modulus will be shown.
As excitation signal, a stepped sine is adopted with varying frequency from 150 to 1050 Hz (logarithmically spaced). For each frequency, a sufficient amount of periods are neglected in order to avoid transient results, and 50 periods are stored. The acceleration signal is integrated numerically in order to obtain a velocity; the relative velocity $v$ is measured as the difference between mass velocity $v_2$ and disk velocity $v_1$. Thanks to measured force $F_1$, the work lost in a cycle can be estimated as:

$$W_d = \int_0^{2\pi} F(t)v(t)\,dt$$

and the value of the equivalent viscous damping is computed according to:

$$c_{eq} = \frac{\int_0^{2\pi} F(t)v(t)\,dt}{\int_0^{2\pi} v^2(t)\,dt}$$

Notice that eq. (2) involves measured signals only; integrals are performed by Simpson’s rule.

Figure 3: Hysteresis loops, left 320.4Hz, right 583.0 Hz.

Figure 4: Relative displacement amplitude (left) and Equivalent viscous damping vs. frequency (right).
Figures 3 and 4 show the frequency range in which the proposed method is reliable. At low frequency there is interaction between the stinger and the system, so that the response is nonlinear. Conversely, at frequencies over 500 Hz, the response is linear, and a clean hysteresis curve can be observed in Figure 3(right).

In Fig. 4, results of three different runs are shown: the three runs differ for the controlled variable \( (x_0 \text{ or } x_1) \) and for the length of the stinger. Despite being characterized by three very different levels of excitation, see Fig. 4(left), the measured value of the viscous damping is the same for the three measurements above 500 Hz, as in Fig. 4(right). Obviously the measured \( c_{eq} \) for run 3 is more noisy, due to the worse signal to noise ratio.

Thanks to performed measurements, it is possible to compute the frequency dependence of the real stiffness \( k \) and of the loss factor \( \eta \), which are needed to characterize the polymeric damper. Figure 5(left) shows that real stiffness is increased from 500 Hz to 1000 Hz by 40 %. The average error between run 1 and run 2 is 2% (note that the stinger length has been changed). The value of the loss factor \( \eta \) is shown in Fig. 5(right). Above 500 Hz, the measured values are in agreement with the expected behaviour of a polymer where real stiffness is increasing. At 1000 Hz the loss factor is 0.26, which is equivalent to a dimensionless damping of 0.13.

![Figure 5: Real (left) and Imaginary (right) parts of dynamic stiffness vs frequency.](image1)

![Figure 6: Normalized work per cycle (W/\(x^2\)) vs frequency.](image2)
An additional investigation has clarified the work/frequency dependency. Figure 6 shows that lost work per cycle normalized over the square oscillation amplitude is increasing less than linearly with frequency. Obviously, values below 500 Hz are not reliable and were disregarded.

Figure 7 shows a hysteresis loop on the force/displacement diagram. This plot is obtained by numerically integrating the measured accelerations twice. The shown measurements are clean, and the 50 cycles are almost overlapping, therefore the whole procedure of measurement and data processing is robust and repeatable.

Figure 7: Work lost per cycle at 1050 Hz (left) and related time history (right).

4. Identification technique

A proper choice of the material constitutive model plays a fundamental role in the identification of a mechanical structural system exhibiting internal dissipation. With reference to Fig. 5, such model should be able to fit the frequency dependent experimental data of both the storage modulus $k$ and the loss factor $\eta$. Since the basic Kelvin–Voigt and Zener constitutive models generally are not sufficient to this purpose, the Fractional Zener constitutive model may be considered [3]. This model, as shown in Fig. 8, generalizes the standard Zener model with substitution of a Newton element (damping coefficient $c$) with a Scott–Blair element (fractional damping coefficient $c_f$).

The time–domain constitutive equation of the Fractional Zener model reads:

$$f(t) = \frac{k_2}{k_1 + k_2} \left[ k_1 + c_f \frac{d^\alpha}{dt^\alpha} \right] x(t)$$

(4)
yielding the following expression of the (complex) dynamic stiffness:

\[ k(w) = k_0 \frac{1 + (iw \tau_s)^{a}}{1 + (iw \tau_s)^{b}} \]  

(5)

where \( a \in [0, 1] \) is the non–integer (fractional) derivative order, \( k_0 \) is the static stiffness, \( \tau_s \) and \( \tau_r \) are the creep retardation time and the relaxation time respectively [3], defined according to:

\[ k_0 = \frac{k_i k_z}{k_i + k_z}, \quad \tau_s = \frac{c_f}{k_i}, \quad \tau_r = \frac{c_f}{k_i + k_z} \]  

(6)

Consequently, the storage modulus \((k)\) and the loss modulus take the form:

\[
\begin{align*}
\text{Re}[k(w)] &= k_0 \frac{1 + c_w w^2 (t_s^* + t_r^*) + w^2 t_s^* t_r^*}{1 + 2 c_w w^2 t_s^* + w^2 t_r^*} \\
\text{Im}[k(w)] &= k_0 s_w w^2 (t_s^* - t_r^*) \frac{1 + 2 c_w w^2 t_s^* + w^2 t_r^*}{1 + 2 c_w w^2 t_s^* + w^2 t_r^*}
\end{align*}
\]  

(7)

with:

\[ \lim_{w \to 0} \text{Re}[k(w)] = k_0, \quad \lim_{w \to 0} \text{Im}[k(w)] = c_w, \quad \lim_{w \to \infty} \text{Re}[k(w)] = \lim_{w \to \infty} \text{Im}[k(w)] = 0 \]  

(8)

The energy loss per cycle of oscillation (with maximum strain amplitude \( x_0 \) due to a steady–state harmonic loading) is proportional to the loss modulus:

\[ W(w) = p x_0^2 \text{Im}[k(w)] \]  

(9)

while the maximum strain potential energy is proportional to the storage modulus:

\[ V(w) = \frac{1}{2} x_0^2 \text{Re}[k(w)] \]  

(10)

hence the loss factor \((\eta)\), defined as:

\[ h(w) = \frac{\text{Im}[k(w)]}{\text{Re}[k(w)]} = \frac{W(w)}{2pV(w)} \]  

(11)

gives a relative measure of the energy loss per cycle of oscillation.

To get a consistent model identification for the polymeric damper, however, reliable experimental data in the low frequency range are needed, in addition to those obtained at higher frequencies with the proposed method. Therefore in the next future dynamically measured properties will be mixed with standard low frequency measurements in order to characterize the damper in terms of a minimum number of parameters.

5. Conclusions

In the present work a test rig for characterizing dynamic stiffness and damping of a polymeric damper is presented. While standard quasi–static procedures (DMA) provide information at low frequency, the current method is capable to provide data at frequencies in the range 500–1000 Hz, which are of high interest in the automotive field. Conversely, the method is not reliable at low frequencies, due to the occurrence of instabilities in the connecting stinger. Therefore …

A great advantage of the proposed method is that it can be implemented using standard vibration testing equipment, such as a small shaker, mono–axial accelerometers and a dynamic load cell.

Measured real stiffness and loss factor curves are in agreement with the literature, and therefore they can used to fit a rheological model of the polymeric material, which will be useful for modelling purposes. In the present work, an overview of constitutive models have been proposed; in the next future, dynamically measured properties will be mixed with standard low frequency measurements in order to characterize the damper in terms of a minimum number of parameters.
REFERENCES