VIBRATION CHARACTERISTIC OF CIRCULAT PLATE CONSIDERATION OF COMPLEX PRE-STRESS DISTRIBUTION

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The influence of the complex pre-stress on circular thin plate is investigated, with which to solve the non-uniform stress distribution problem. The differential motion equations of circular thin plate is derived. The analytical method of the circular thin plate with complex pre-stress distribution was proposed. In the end, the effectiveness of the proposed models was confirmed through numerical calculations and verification. The influence of welding residual stress distribution type on the natural frequency and mode of circular thin plate structure are discussed.

1. Introduction

The circular thin plate structure is widely used in marine, aerospace, and automotive engineering. A large amount of research effort has been devoted to study the vibration problem of circular plate with theoretical analysis, numerical calculation, and experimental investigation (Lee et al, 2008; Wu et al, 2001; Wang, 2014). The energy method based on Hamiltonian dual equations or Rayleigh-Ritz method becomes a hot issue in dynamic response analysis of the circular plate structure (Tajeddini et al, 2009; Yalcin et al, 2009). The various form stress usually exist in structure before it undertaking the work loading, it named as pre-stress or initial stress. A large amount of research effort has been devoted to study the influence of pre-stress on structural strength and fatigue (Gannon et al, 2012). The existing of pre-stress or initial stress gives a significant influence on the natural frequencies, mode shapes and dynamic response (Chen et al, 2006; Ashwear et al, 2014; Chen et al, 2016; Liu et al, 2016). Thus, it is necessary to analyze the vibration characteristic of the circular plate with non-uniform pre-stress distributions.

2. Differential equations of pre-stressed circular plate

2.1 Force analysis of element body

An element body with the size of \( dr \) and \( rd\theta \) is selected, as shown in Fig.1. The forces and moments which acting on the element body consists of two parts when the circular plate structure is vibrating: the force and moment caused by the vibration displacement; the coupling force between the vibration displacement and the pre-stress. In Fig.1, \( Q_r, Q_\theta, M_r, M_\theta \) and \( M_r\theta \) are the shear force and bending moment in the element body, respectively, the shear force and bending moment can be written as:

\[
\begin{align*}
Q_r &= -D \frac{\partial^2 w}{\partial r^2} \nabla^2 w, Q_\theta = -\frac{D}{r} \frac{\partial w}{\partial \theta} \nabla^2 w \\
M_r &= -D \left[ \frac{\partial^2 w}{\partial r \partial \theta} + \mu \left( \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \right] \\
M_\theta &= -D \left[ \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right] + \mu \frac{\partial^2 w}{\partial r^2} \\
M_r\theta &= M_\theta = -D(1-\mu) \left( \frac{1}{r^2} \frac{\partial w}{\partial \theta \partial r} - \frac{1}{r} \frac{\partial^2 w}{\partial \theta^2} \right)
\end{align*}
\] (1)
where, \( D = \frac{Eh^3}{12(1-\mu^2)} \) is the bending strength of the circular plate structure, \( \mu \) is the Poisson ratio, \( E \) is the Young modulus, and \( h \) is plate thickness.

### 2.2 Coupling force analysis

It is assumed that the complex pre-stress remains constant during structural vibration. In the cross section of element body, the neutral plane is defining as unit length (shown in Fig.1).

**Fig.1 Section force and moment caused by vibration**

In the course of structural vibrating, the distance in \( z \) direction between curve OA and curve OC to the neutral plane are \( l_{OA}^c \) and \( l_{OC}^c \), respectively. Then the distance can be written as \( l_{OA}^c = 1 + \varepsilon_r \) and \( l_{OC}^c = 1 + \varepsilon_\theta \).

Hence, the section area along the curves OA and OC in unit length can be written as:

\[
\begin{align*}
S_r &= \int_{\frac{1}{2}}^{\frac{b}{2}} l_{OA}^c \, dz = h \\
S_\theta &= \int_{\frac{1}{2}}^{\frac{b}{2}} l_{OC}^c \, dz = h 
\end{align*}
\]  

(2)

According the Eq. (2), the section area along the curve OA and OC remain constant in unit length during the structural vibrating based on the principle of equal volume. Since there is assuming that the pre-stress force remains constant during the structural vibrating, then the section tensile forces \( N_{0,r} \) and \( N_{0,\theta} \) for unit length in the direction \( r \) and directions \( \theta \) remains constant, too.

If the circular plates structure are in static equilibrium, the section tensile forces are parallel to the \( r \) axis and the \( \theta \) axis, respectively, and there satisfied with \( N_{0,r} = \sigma_{0,r} h \) and \( N_{0,\theta} = \sigma_{0,\theta} h \). In addition, without any force component in the other coordinate directions.

Since there exists displacement function \( w(r, \theta, t) \) in element body, then the section tensile forces \( N_{0,r} \) no longer parallel to \( \theta \) axis, the angle between the section tensile forces \( N_{0,r} \) and \( \theta \) axis is \( \frac{\partial w}{\partial r} \); similarly, section tensile forces \( N_{0,\theta} \) no longer parallel to \( r \) axis, the angle between the section tensile forces \( N_{0,\theta} \) and \( r \) axis is \( \frac{\partial w}{\partial \theta} \), as shown in Fig. 2.

Since there exists angle \( \frac{\partial w}{\partial r} \) and \( \frac{\partial w}{\partial \theta} \), the section tensile forces \( N_{0,r} \) has a component \( \Delta N_{0,r,z} \) in the \( z \) direction, and the section force \( N_{0,\theta} \) has a component \( \Delta N_{0,\theta,z} \) in the \( z \) direction, and those force component can be written as \( \Delta N_{0,r,z} = \sigma_{0,r} h \frac{\partial w}{\partial r} \) and \( \Delta N_{0,\theta,z} = \sigma_{0,\theta} h \frac{\partial w}{\partial \theta} \), the force \( \Delta N_{0,r,z} \) and \( \Delta N_{0,\theta,z} \) are related to the pre-stress and the vibration displacement, and is defined as the coupling force between the pre-stress and the vibration displacement, as shown in Fig.3.

The pre-stress is always perpendicular to the cross section of the element body, there not exists coupling force caused by the pre-stress force and the vibration displacement in \( r \) direction and \( \theta \) direction; in addition, there not exists any kinds of coupling moments and torques moments in any direction by the pre-stress vector. In the course of the circular thin plate vibrating, the coupling force caused by the pre-stress and the vibration displacement in the \( z \) direction have impact on the force balance equation of the element body.

### 2.3 Vibration equation of circular plate with complex pre-stress distributions
According the derivation above, it is clear that there exist coupling forces in the circular thin plate during structural vibrating which caused by coupling pre-stress and vibration displacement. Thus, the vibration equation should be modified and the coupling forces need to consider in the equilibrium equations.

Since without coupling forces are generated in $r$ direction and $\theta$ direction, thus the force equilibrium equations in the $r$ direction and $\theta$ direction are still satisfied, automatically. The force equilibrium equation in the $z$ direction and the moment equilibrium equations need to be established.

In the element body of circular plate, there exist two shear force $Q_r$ and $Q_\theta$ which caused by structural vibration in the $z$ direction (as shown in Fig.2), and two coupling force $N_{r,\theta}$, caused by the pre-stress and the vibration displacement (as shown in Fig.4). Thus, in the $z$ direction, the force equilibrium equation can be written as:

$$\frac{\partial Q_r}{\partial r} + \frac{\partial Q_\theta}{\partial \theta} + \frac{\partial \Delta N_{r,\theta}}{\partial r} + \frac{\partial \Delta N_{r,\theta}}{\partial \theta} = \rho h \frac{\partial^2 w}{\partial z^2}$$  \hspace{1cm} (3)

where, $\rho$ is the density of plate material, and $h$ is the thickness of plate structure.

Since there not exist any coupling moments or torques moments be considered in the element body, and the body force in the $r$ direction and $\theta$ direction were omitted too, the moment equilibrium equations in $r$ direction and $\theta$ direction, respectively, can be written as:

$$\frac{\partial M_r}{\partial r} + \frac{\partial M_\theta}{\partial \theta} + Q_r = 0$$ \hspace{1cm} (4)

Simultaneous Eqs. (3) and (4), and yields:

$$\frac{\partial^2 M_r}{\partial r^2} + 2 \frac{\partial^2 M_\theta}{\partial r \partial \theta} + \frac{\partial^2 M_\theta}{\partial \theta^2} - \left[ \frac{\partial \Delta N_{r,\theta}}{\partial r} + \frac{\partial \Delta N_{r,\theta}}{\partial \theta} \right] = -\rho h \frac{\partial^2 w}{\partial z^2}$$ \hspace{1cm} (5)

Substituting Eq. (4) into Eq. (5), the vibration differential equation of circular thin plate with complex pre-stress distribution can be written as partial differential equation:

$$\nabla^2 \nabla^2 w - \frac{h}{D} \frac{\partial}{\partial r} \left[ \sigma_{r,r} \frac{\partial w}{\partial r} + \frac{\partial}{\partial \theta} \left( \sigma_{r,\theta} \frac{\partial w}{\partial \theta} \right) \right] + \frac{\rho h}{D} \frac{\partial^2 w}{\partial z^2} = 0$$ \hspace{1cm} (6)

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$, $\sigma_{r,r}$ and $\sigma_{r,\theta}$ are the complex pre-stress in the $r$ direction and $\theta$ direction; $\rho$ is the density of plate material, and $h$ is the thickness of plate structure. Eq. (6) can be expressed as:

$$L(w) - C(w, \sigma_{r,r}, \sigma_{r,\theta}) = -\frac{\rho h}{D} \frac{\partial^2 w}{\partial z^2}$$ \hspace{1cm} (7)

where

$$L(w) = \frac{\partial^4}{\partial r^4} + \frac{2}{r} \frac{\partial^4}{\partial r^2 \partial \theta^2} - \frac{1}{r^2} \left( \frac{\partial^4}{\partial r^2 \partial \theta^2} - 2 \frac{\partial^4}{\partial r^2 \partial \theta^2} \right) + \frac{1}{r^4} \left( \frac{\partial^4}{\partial \theta^4} - 2 \frac{\partial^4}{\partial r \partial ^2 \theta^2} \right) + \frac{1}{r^4} \left( 4 \frac{\partial^4}{\partial \theta^4} + \frac{\partial^4}{\partial \theta^4} \right)$$

and

$$C(w, \sigma_{r,r}, \sigma_{r,\theta}) = \frac{h}{D} \frac{\partial}{\partial r} \left[ \sigma_{r,r} \frac{\partial w}{\partial r} + \frac{\partial}{\partial \theta} \left( \sigma_{r,\theta} \frac{\partial w}{\partial \theta} \right) \right]$$

are coupling factor between complex pre-stress and vibration displacement. Because the pre-stress values ($\sigma_{r,r}, \sigma_{r,\theta}$) vary with change in location, the partial derivatives cannot be ignored either.

Compared with the vibration differential equation which without consider of the complex pre-stress distribution, a coupling item $C(w, \sigma_{r,r}, \sigma_{r,\theta})$ is addition. On the other hand, compared with the circular thin plate vibration differential equation with uniform pre-stress distribution, the factor of pre-stress amplitude varies with change in location was taken into account. Thus, a new approach was required to gain the analytic solution of Eq. (7). The analytical solution can be applied to the circular plate structure with arbitrary distributed stress; it has a wider range of applications than the previous analytical methods.

### 3. Solution the motion equation

#### 3.1 Boundary condition

The physical boundary conditions of the circular thin plate include free boundary, simply supported boundary and the fixed boundary, and so on. The simply supported boundary condition is discussed in present work, the boundary condition can be written as: $w|_{r=R} = 0$ and $M_r|_{r=R} = 0$. 
3.2 General solution of vibration equation

For the isotropic materials plates’ structure, in order to simplify calculation, some hypotheses are defined: in the \(\theta\) direction, the pre-stress \(\sigma_{r,\theta}\) is a constant along the circumferential direction; in the \(r\) direction, the pre-stress \(\sigma_{r,r}\) is a function of design variable \(r\).

According to the hypotheses, the Modal decomposition method was applied, and the mode of the plate may be expressed as a sum of eigen-modes or eigen-functions at any point in time. The solution of Eq. (7) can be obtained with a form of power series expansion by using Galerkin procedure. In the polar coordinates system, the solution of the circular thin plate structure can be written as trigonometric function series:

\[
w = \sum_{q=0}^{n} W_q \cos\left(\frac{2\eta - 1}{2}\alpha\right)e^{i\omega t}, \quad \alpha = \pi/R
\]  

where \(\omega\) is the angular frequency; the \(R\) is the radius of the plate, \(W_q\) is the shape function, and \(R\) is the radius of the circular thin plate.

Substituting Eq. (8) into Eq. (7), the free vibration equation of circular thin plate with complex pre-stress distribution was derived:

\[
\sum_{q=0}^{n} \left[\left(\frac{2\eta - 1}{2}\alpha\right)^4 + \frac{1}{r^2} \left(\frac{2\eta - 1}{2}\alpha\right)^2 \right] \cos\left(\frac{2\eta - 1}{2}\alpha\right) + \frac{2}{r} \left(\frac{2\eta - 1}{2}\alpha\right)^3 - \frac{1}{r^3} \left(\frac{2\eta - 1}{2}\alpha\right) \sin\left(\frac{2\eta - 1}{2}\alpha\right) \right] W_q
\]

\[- C(w, \sigma_{r,r}, \sigma_{r,\theta}) = \omega^2 \rho h \sum_{q=0}^{n} W_q \cos\left(\frac{2\eta - 1}{2}\alpha\right)
\]

where, the mathematical expressions form of complex pre-stress \(\sigma_{r,r}\) and \(\sigma_{r,\theta}\) directly affect the type of analytical solution. Multiplying both sides of Eq.(9) by \(\cos(n\alpha r)\) and using the trigonometric function’s orthogonality, integral the function from \(r=0\) to \(r=R\) for the circular thin plate structure, then the differential equations can be obtained as:

\[
k = \int_{0}^{R} \left[\left(\frac{2\eta - 1}{2}\alpha\right)^4 + \left(\frac{2\eta - 1}{2}\alpha\right)^2 \right] \cos\left(\frac{2\eta - 1}{2}\alpha\right) W_n - \frac{2}{R} \int_{0}^{R} C(w, \sigma_{r,r}, \sigma_{r,\theta}) \cos(n\alpha r)dr = \omega^2 \rho h W_n
\]  

Express the \(C\) integration term in Eq. (10) as \(K = \int_{0}^{R} C(w, \sigma_{r,r}, \sigma_{r,\theta}) \cos(n\alpha r)dr\), which represents the effects of complex pre-stress. Because of the appearance of \(K\), the structural modes couple together, and a single structural mode can no longer be computed so that the entire coupling equation has to be solved in order to obtain the coupling modes. Thus, the choose of the expressions form for complex pre-stress is of essential, it required not only represent arbitrary pre-stress distribution, but also helps achieve structural mode decoupling. In present research, the trigonometric series was chosen, which can satisfy the above requirements. And the corresponding analytical solution of the circular thin plate can be obtained by solving \(K\).

3.3 Solution of characteristic equation

According to the different distribution forms of complex pre-stress, the Eq. (10) was solved, and the free vibration characteristic equation is gained, respectively.

3.3.1 without pre-stress distribution

If the circular thin plate without complex pre-stress distribution \((\sigma_{r,r} = \sigma_{r,\theta} = 0)\), then the integration term \(K=0\), and the free vibration characteristic equation of the circular thin plate structure can be written as:

\[
k = \omega^2 \rho h
\]  

3.3.2 Uniform pre-stress distribution

If the complex pre-stress \(\sigma_{r,r}\) and \(\sigma_{r,\theta}\) are constant, then Eq. (10) are translated into a uniform pre-stress distribution problem. Defining \(\sigma_{r,r} = \lambda\) and \(\sigma_{r,\theta} = \tau\), respectively, the integration term \(K\) can be obtained as:

\[
k = -\frac{h\lambda}{D} \left(\frac{2\eta - 1}{2}\alpha\right)^2 \sum_{q=0}^{n} \int_{0}^{R} W_q \cos\left(\frac{2\eta - 1}{2}\alpha\right) \cos(n\alpha r)dr
\]

According the Eq. (11), the pre-stress in circumferential direction does not affect the characteristic equation. And the integration term \(K\) can be written as \(K = -\frac{h\lambda}{D} \left(\frac{2\eta - 1}{2}\alpha\right)^2 W_n\). By substituting \(K\) into Eq.
the free vibration characteristic equation of the circular plate with uniform pre-stress distribution can be obtained \[
(\frac{2n-1}{2} \alpha)^2 + (\frac{2n-1}{2} \alpha)^2 W_n + \frac{2 h \lambda}{R D} (\frac{2n-1}{2} \alpha)^2 W_n = \omega^2 \rho h D W_n.
\]

### 3.3.3 Complex Pre-stress

If the welding residual stress value of \(\sigma_{0,r}\) and \(\sigma_{0,\theta}\) varies in one direction, all of them are the function of design variable \(r\), based on the general principle, the amplitude can be expanded into trigonometric function series, and the one-dimensional structural pre-stress can be expressed as:

\[
\sigma_{r,r} = \sigma_{r,r} \cos(\frac{2g-1}{2} \alpha r) \quad g = 1, 2, \ldots
\]

\[
\sigma_{r,\theta} = \sigma_{r,\theta} \cos(\frac{2j-1}{2} \alpha \theta) \quad j = 1, 2, \ldots
\]

where \(\sigma_{r,r}\) and \(\sigma_{r,\theta}\) are the amplitude of the complex pre-stress in the \(r\) direction and \(\theta\) direction components, respectively, and \(g\) and \(j\) are positive integers. Substituting Eq. (12) into integration term \(K\), and yields:

\[
K = -\frac{h}{4D} \sum_{g=0}^{\infty} W_a [\{(g + \eta - 1) \cos[(g + \eta - 1) \alpha r] \cos(\eta \alpha r) - (g - \eta) \cos[(g - \eta) \alpha r] \cos(n \alpha r)\}] dr
\]

Integral expressions of coupling terms of complex pre-stress and vibration displacement can be obtained by substituting Eq. (12) into Eq. (13):

\[
K = \left[-\frac{Rh}{4D} \sum_{n=\frac{1}{2}}^{\infty} (g + n - 1), n > g \right] - \left[\frac{h}{4D} \sum_{n=\frac{1}{2}}^{\infty} (g - n), n < g \right]
\]

Eqs. (14) means the coupling occurred among the specified modes only. Each mode is coupled with only a few specific modes, rather than with all the other modes. Therefore, for the final vibration equation, we just need to compute the terms which correspond to the specific coupling modes. What is more, after simplification the calculations do not involve integral operations any more. So the decoupling of partial modes can reduce the computation cost dramatically without accuracy loss. Then \(N\) equations can be constructed and expressed into a matrix form:

\[
(\Lambda + \Psi_g) X = 0
\]

where \(X = \{X_1, \ldots, X_{n-1}, X_n\}\); \(\Lambda\) is a sparse diagonal matrix that represents the non-stress part, and \(\Psi_g\) is a sparse non-diagonal matrix. If there exists complex pre-stress in the circular plates with, \(\Psi \neq 0\), \(\Lambda + \Psi\) is no longer a spares diagonal matrix.

### 3.4 Modal solution

Based on several typical distributions of complex pre-stresses, the free vibration characteristic equation of a simply supported circular plate structure is established. Thought the characteristic equation with complex pre-stress is more complicated than that without pre-stress and uniform pre-stress, but it is still a linear equation set, so that the determinant of the characteristic equation coefficient is zero:

\[
|\Lambda + \Psi_g| = 0
\]

The natural frequencies and modes of circular plate with complex pre-stress can be gained by Eq. (16).

### 4. Numerical verification

#### 4.1 Model description

The boundary condition of circular thin plate structure is simply supported boundary. The parameters of circular plate structure are as follows: radius of circular plate \(R=300\) mm; thickness \(h=6\) mm. The material of the circular plate structure is steel with mechanical performance as follows: density, \(\rho=7800\) kg/m\(^3\); modulus of elasticity, \(E=2.1 \times 10^{11}\) N/m\(^2\); Poisson ratio, \(\mu=0.3\).

There is a circumferential weld which locating at the \(r=200\) mm. The width of the welding stress zone is \(l=40\) mm. There exists welding residual stress near the seam welding.
5.2 welding residual stress distribution model

The parameter of welding residual stress was obtained by simulation analysis means FEM code Marc. There are three kinds of welding residual stresses for to compared. The maximum value of radial residual stress \( \sigma_{0,r} \) is 150 MPa; and the maximum value of circumferential welding residual stress \( \sigma_{0,\theta} \) is 250 MPa.

In the present study, to simplify the analysis, neglecting the variation of the welding residual stress in the plate thickness direction, and assuming that the radial welding residual stress \( \sigma_{r,0} \) remains constant along the circumferential direction. The trigonometric function was used to fit the circumferential welding residual stress and radial welding residual stress, as is shown in Fig.6. Fig. 6 shows the radial welding residual stress, the positive value is tensile stress, and the negative value is compressive stress.

After fitting the welding residual stress, the Matlab R2013 is used to analyze the free vibration of the circular thin plate structure. The influence of welding residual stress on natural frequency were compared, the first ten natural frequencies of the structure are shown in Tab.1.

<table>
<thead>
<tr>
<th>Orders</th>
<th>Natural frequency (Hz)</th>
<th>Without welding residual stress</th>
<th>With welding residual stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Case I</td>
<td>Case II</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>68.3</td>
<td>60.3</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>192.3</td>
<td>188.7</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>192.3</td>
<td>188.7</td>
</tr>
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<td>4</td>
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<td>354.2</td>
<td>347.3</td>
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<tr>
<td>10</td>
<td></td>
<td>672.4</td>
<td>667.7</td>
</tr>
</tbody>
</table>
As shown in Tab.1, due the existing of welding residual stress, the natural frequency values of the circular thin plate is changed, and detail as: in general speaking, the natural frequency values of plate is reduced since the welding residual stress; of the change trend, first order effect is most obvious; in the other order, with the increase of frequency, the relative influence by welding residual stress is deduced; in the same order, with the increase of welding residual stress value, the influence on the natural frequency is increase. the main reason is that the overall structural strength of the circular plate be deduced since the existing pre-stress, especially near the seam welding.

5.3 modes calculation
The influence of welding residual stress distribution on the modal is analyzed, and the typical order is compared. As shown in Fig.6, the structural modal shapes of the 1-6 order are compared, and the left one shows the residual stress without welding and the others are welding residual stresses.
As shown in Fig.7, due to the existing of welding residual stress, the mode shapes of the circular thin plate is changed, and detail as: because of the existence of the welding residual stress, the modes shape are no longer rules, the symmetry characteristic is lost; in the first order or sixth order modes shape, some modes of mutation are appeared in the welding residual stress distribution area; in the second to fifth order modes shape, some modal mutations are appeared in the center of the circular plate, despite the area is far from the seam welding; in the fourth order and fifth order modes shape, modal mutations are appeared with periodically in the seam welding; with the welding residual stress amplitude increases, the influence on the mode is increase. the main reason is that the overall structural strength of the circular plate be deduced since the existing pre-stress, especially near the seam welding.

5. Conclusion

The vibration equation of a circular thin plate structure with complex pre-stress (welding residual stress) distribution is derived. By defining the mode shape function, the analytical solution of the differential equations is obtained, and the influence of welding residual stress on the plate structure was compared. The results of this article has provide a novel approach to analyze the effects of welding residual stress on structural vibration problem, and it has expand the research domain of the complex pre-stress.

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