Anthropogenic underwater noise generated by pile driving has been an issue of serious concern due to the rapid developments in offshore wind farms. The underwater noise pollution poses a threat to marine mammals. To reduce the noise, many offshore companies have developed various mitigation measures and alternatives to impact piling. One of them is the use of underwater acoustic resonators around the foundation pile. In this paper, a three-dimensional vibroacoustic model is developed in order to find the optimal configuration of the underwater acoustic resonator system and to improve the existing noise reduction potential. The model requires the proper description of the noise source, the resonator and the acoustic domain surrounding the pile. To describe the acoustic performance of the resonators for a more generic use, the frequency response function of an open-ended resonator is analytically derived based on the assumption that the resonator behaves as a linear Single-Degree-of-Freedom (SDoF) system. The derivation of the parameters of the equivalent SDoF system representing each individual resonator is based on appropriate fitting of numerical results obtained in COMSOL for a wide class of parameters. The Boundary Element Method (BEM) is then employed for determining the total pressure field in the acoustic domain in the process of pile driving accounting for the presence of multiple resonators. In this work, noise sources are represented by a distributed array of phased point sources which reproduce adequately the source of the noise field. In addition, a parametric study is presented in order to define the principal factors yielding effective noise mitigation and to obtain the optimal configuration on the predicted sound levels at the low-frequency range.

Keywords: underwater acoustics, pile driving, boundary element method, Helmholtz resonator

1. Introduction

Underwater noise generated by offshore pile driving has become a serious issue for the marine environment. With the growing demand for renewable energy, a large number of offshore wind farms are planned to be constructed in the coming future. When driving these large foundation piles into the soil, a great deal of noise is radiated from the pile-water and pile-soil interface at low acoustic frequencies, i.e. usually below 400 Hz. The underwater noise pollution generated by this percussive piling threatens marine mammals, especially the low-frequency noise, which could severely interfere with their foraging and migrating behaviour, and damage their hearing [1].

In the light of the significant research on noise prediction, the mitigation of the underwater noise can be achieved in the following two ways; by either controlling the noise at its source or by blocking effectively the noise transmission path. Based on the primary noise path in the water region,
several noise mitigation techniques have been developed, such as air-bubble curtain [2], noise mitigation screen and hydro-sound dampers. The open-ended resonator is another promising technique to mitigate the underwater noise. The open-ended resonators have the same primary mechanism as Helmholtz resonators that has been widely used in airborne noise reduction [3]. The acoustic behaviour of both open-ended resonators and air-filled balloons was investigated firstly by Lee, Wochner and et al. [4, 5]. Based on their study, the systems can both achieve a certain level of noise reduction on the target frequencies. The open-ended resonators also show a better acoustic performance than encapsulated air bubbles. The resonator considered in this paper consists of a rigid cylinder tube with an open end as shown in Fig. 1(b). When the open-ended resonator is deployed into the water with the open end facing the seabed, the air will be encapsulated and compressed in the container. This creates an analogous SDoF system. By using the specific shape and volume of the resonator, one can tune the resonator to work optimally at a target frequency. To find the optimal properties of an underwater resonator, to be compatible with the different resonator-based systems and to optimize the design of the existing techniques, a 3-D acoustically coupled prediction model is developed for the noise reduction by the application of a resonator-based noise mitigation system.

2. Model Description and Governing equations

The problem under consideration is depicted in Fig. 1(a). The sound generated from pile driving is represented by an array of phased point sources similarly to [6]. The ocean environment is modelled as an inviscid and compressible fluid, bounded by pressure release boundary at the sea surface and by the rigid boundary at sea bottom. This representation is not the most accurate one as it neglects noise transmission along the seabed-water interface [7, 8] but is expected to provide a reasonable representation of the acoustic domain away from the seabed-water interface. The Green’s function is built-up with the use of Normal Mode method for the bounded waveguide, which satisfies the boundary conditions along the vertical coordinate as well as the radiation condition at infinite distance from the source. The BEM is then used to represent the pressure field. The acoustic behaviour of the resonator is described by frequency response function, which is found by extracting the coefficient from the analogous SDoF system. The geometric dimensions of the resonator are very small compared to the targeted sound wavelength, thus, the representation by an equivalent SDoF system is a reasonable one. Thus, the present model is based on the acoustically coupled interaction between the underwater Helmholtz resonators and the noise source. For resonators of a different size, the parameters can be accordingly tuned to optimize the acoustic performance for a given pile installation scenario.

2.1 Governing equations

The inhomogeneous wave equations describing the motion of the fluid generated by a simple point source at $\vec{r}_0 = (r_0, \phi_0)$ can be expressed in the form of pressure as,

$$\nabla^2 p(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 p(\vec{r}, t)}{\partial t^2} = f(\vec{r}_0, t)$$

in which $p(\vec{r}, t)$ is the pressure, $f(\vec{r}_0, t)$ is the source term, $c$ is the sound speed at the fluid domain, $\vec{r} = (r, z, \phi)$, and $\nabla^2$ is the Laplacian operator defined in the cylindrical coordinate system. We assume the sound speed and density of the water constant over the water depth. By introducing frequency-time Fourier transform pair,

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega)e^{i\omega t} d\omega, \quad \tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$$

one obtains the well-known Helmholtz equation, in which $f(t)$ is indicated as an examined quantity, i.e. the pressure of the fluid. Therefore, the pressure equation in frequency domain reads:

$$\nabla^2 \tilde{p}(\vec{r}, \omega) + k^2 \tilde{p}(\vec{r}, \omega) = S_\omega \delta(\vec{r} - \vec{r}_0)$$

in which $\omega$ is the angular frequency and $k$ is the wave number.
where $S_\omega \delta(\vec{r} - \vec{r}_0)$ is the source term, the source strength $S_\omega$ is defined as volume-injection amplitude. The fluid motion must satisfy the homogeneous Helmholtz equation, the radiation condition at infinity and the boundary conditions at the sea surface and the sea bottom:

- Pressure release boundary for the sea surface at $z = z_1$: $	ilde{p}(r, z_1, \phi, \omega) = 0$;
- Perfectly rigid boundary for the sea bottom at $z = z_2$: $\vec{v}_z(r, z_2, \phi, \omega) = 0$;
- Radiation condition at infinity: $\lim_{r \to \infty} r \left( \frac{\partial \tilde{\phi}}{\partial r} + ik \tilde{\phi} \right) = 0$.

### 2.2 Solution to Governing equations

By using the BEM, the pressure field can be expressed by the superposition of noise source and resonators through integrals [9]. The Green’s function should inherently satisfy the sea surface and seabed boundary conditions, and the radiation condition. By doing this, the Green’s function could provide the contribution of the pressure at field point $\vec{r}$ from a simple point source at $\vec{r}_0$. Therefore, only the pile surface needs to be discretized since the boundary integral formulation inherently satisfies the wave equation throughout the volume as well as boundary conditions and radiation condition at infinity. For the sake of simplicity, we assume that the pressure on the surface of the open end of each resonator is equally distributed (the validity of this assumption needs always to be checked), thus we could take the first integral as the surface area multiplied by the pressure contribution at the center point:

$$
\tilde{p}(\vec{r}, \omega) = \sum_{n=1}^{M} \left\{ \int_{S_R} \left[ G_\omega(\vec{r}, \vec{r}_n) \frac{\partial \tilde{p}(\vec{r}_0)}{\partial \vec{n}_0} - \tilde{p}(\vec{r}_0) \frac{\partial G_\omega(\vec{r}, \vec{r}_n)}{\partial \vec{n}_0} \right] dS_0 \right\} - \int_{V} \tilde{f}(\vec{r}_S) G_\omega(\vec{r}, \vec{r}_S) dV_S
$$

where $\vec{n}_0$ is the normal vector to the open end, $V_S$ is denoted as the volume of the source, $S_0$ indicates as the open surface area of the resonator. It is worth to mention that the scattering effect is not included in this model. Therefore, the other surface of the resonator is not considered in the integral. However, this can be included in the formulation above with minimum computational effort. Green’s function is the solution to the inhomogeneous Helmholtz equation, expressed as:

$$
G_\omega(\vec{r}, \vec{r}_0) = \sum_{n=1}^{\infty} \left[ \frac{i}{2D} \sin(k_{zm_1} z_0) \sin(k_{zm_1} z) H_0^{(2)}(k_{rm_1} r') \right]
$$

Figure 1: Geometry of the model: (a) 3-D description of the domain; (b) configuration of the individual resonator.
where $k_{zm} = \frac{2m+1}{2D_0} \pi$, $k_{zm_1} = \sqrt{k_z^2 - k_{zm_1}^2}$, $r'$ is the range of a point in the field with respect to the source, $r' = \sqrt{r_0^2 + r^2 - 2r_0r \cos(\phi_0 - \phi)}$, $H_0^{(2)}$ is the Hankel function of the second kind. By

By substituting the location of the field point as $\vec{r} = \vec{r}_0 = \vec{r}_m = (r_{m1}, \tau_m, \phi_m)$, the boundary integral equation for a number of resonators is formulated with each resonator denoted by the index "$m$":

$$
\tilde{p}(\vec{r}_m, \omega) = \sum_{n=1}^{M} \left\{ \int_{S_n} \left[ G_\omega(\vec{r}_m, \vec{r}_n) \frac{\partial \tilde{p}(\vec{r}_n)}{\partial \vec{n}_0} - \tilde{p}(\vec{r}_n) \frac{\partial G_\omega(\vec{r}_m, \vec{r}_n)}{\partial \vec{n}_0} \right] dS_0 \right\} - \int_V \tilde{f}(\vec{r}_S) G_\omega(\vec{r}_m, \vec{r}_S) dV_S
$$

(6)

where $\vec{r}_m$ is the location of the resonator number $m$, $S_n$ represents the surface area of the resonator with index "$n$". Substituting the spatial derivative of the pressure by using frequency response function, and moving the terms related to number $m$ resonator to the left-hand side of the equation,

$$
\left( 1 - \alpha S_n \varrho^2 H(\omega) - \frac{\partial G_\omega(\vec{r}_m, \vec{r}_m)}{\partial \vec{n}_0} \right) \tilde{p}(\vec{r}_m, \omega) = \sum_{n=1}^{M} \alpha S_n \left( \frac{\partial G_\omega(\vec{r}_m, \vec{r}_m)}{\partial \vec{n}_0} \right) \tilde{p}(\vec{r}_n, \omega) + \tilde{p}(\vec{r}_n) \frac{\partial G_\omega(\vec{r}_m, \vec{r}_n)}{\partial \vec{n}_0} + S_n G_\omega(\vec{r}_m, \vec{r}_S)
$$

(7)

where $\alpha$ indicates the end correction factor for the open-ended resonator, $H(\omega)$ is the frequency response function derived from the results in COMSOL and $\varrho$ is the density of the water. The above equation is valid for all the resonators in the array, the linear algebraic equations for $M$ resonators are formulated as:

$$
\mathbf{L}^R \mathbf{\tilde{p}} = \mathbf{q}^R
$$

(8)

$$
\begin{bmatrix}
\mathcal{L}^R_{1,1} & \mathcal{L}^R_{1,2} & \cdots & \mathcal{L}^R_{1,M} \\
\mathcal{L}^R_{2,1} & \mathcal{L}^R_{2,2} & \cdots & \mathcal{L}^R_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
\mathcal{L}^R_{M,1} & \mathcal{L}^R_{M,2} & \cdots & \mathcal{L}^R_{M,M}
\end{bmatrix}
\begin{bmatrix}
\tilde{p}(\vec{r}_{11}, \omega) \\
\tilde{p}(\vec{r}_{12}, \omega) \\
\vdots \\
\tilde{p}(\vec{r}_{M,\omega})
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{q}(\vec{r}_{1,1}, \omega) \\
\mathbf{q}(\vec{r}_{1,2}, \omega) \\
\vdots \\
\mathbf{q}(\vec{r}_{M,\omega})
\end{bmatrix}
$$

(9)

where $\mathbf{q}_m = S_n G_\omega(\vec{r}_{m1}, \vec{r}_S)$, the operator $\mathbf{L}^R$ can be formulated as $\mathbf{L}^R = \mathbf{A}^R + \mathbf{B}^R$.

$$
\mathbf{A}^R =
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix},
\mathbf{B}^R =
\begin{bmatrix}
\mathcal{B}^R_{1,1} & \mathcal{B}^R_{1,2} & \cdots & \mathcal{B}^R_{1,M} \\
\mathcal{B}^R_{2,1} & \mathcal{B}^R_{2,2} & \cdots & \mathcal{B}^R_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
\mathcal{B}^R_{M,1} & \mathcal{B}^R_{M,2} & \cdots & \mathcal{B}^R_{M,M}
\end{bmatrix}
$$

(10)

where $\mathcal{B}^R_{m,n} = -\alpha S_n G_\omega(\vec{r}_m, \vec{r}_n) \varrho^2 H(\omega) + \frac{\partial G_\omega(\vec{r}_m, \vec{r}_n)}{\partial z}$. Thus, the pressure at $\vec{r}_m (m = 1...M)$ is obtained as $\hat{p} = [\mathbf{L}^R]^{-1} \cdot \mathbf{q}^R$. Upon substitution of the pressure at the location of each resonator back into the boundary integral equation Eq. 4, the pressure at any point in the field can be obtained.

### 3. Results and Discussion

#### 3.1 The representation of the sound radiation from pile driving

In order to qualitatively represent the sound radiation from pile driving, a method of using an array of phased point sources is used in present paper introduced by Reinhall and Dahal [6]. The pulse delay in the point source is in order to account for the waves travelling down along the pile at the supersonic speed, $c_{p} = 5048 m/s$.

For $i$-th point source in the array, $s_{i}(\vec{r}, f)$ can be expressed as $G(\vec{r}, \vec{r}_i, f)A(f)e^{i2\pi f \tau_i}[6]$. $\tau_i$ represents the time delay for each point source $i$, which is equal to source depth $z_i$ divided by the supersonic
speed $c_p$. $A(f)$ is the empirical amplitude weighing spectrum, which is used to estimate the sound radiation from pile driving. The superposition of the array of point sources can be used to represent the sound source $S_1(\vec{r}, f)$ as $\sum_{i=1}^{N} S_i(\vec{r}, f)$ [6]. It gives the complex amplitude spectrum of the first arrival of the sound generated from the phased point sources, which represents the form of the pressure cone. For the sake of simplicity, the other subsequent arrivals are not taken into account in this study. In this case, we choose 39 phased point sources along the pile axis with the spacing 1 meter, the water depth for the field is 40 meter. Before the inverse Fourier transform, the pressure field in the frequency domain is zero-padded in order to achieve a fine time-step interpolation in time domain.

As shown in Fig. 2, the pressure field is obtained in time series. The results show that no signal can be emitted before $t = \tau_i$ for each point source $i = 1...N$. The signal in this case is determined as a single wave with a frequency of 300Hz. As shown in Fig. 2, the longitudinal impulse waves travel down the pile with a supersonic speed $c_p$, which forming the Mach cone with an inclined angle around $17^\circ$ in the water column. It can be seen that the inclination of the wavefront has been qualitatively reproduced, compared with the FE-model [6], WI-model [10] and vibro-acoustic model [11]. Thus, based on the comparison of the results, one can conclude that this method is able to represent the main characteristics of the sound radiation from pile driving. This method can also allow us to predict the sound pressure level at large distance from the pile axis.

3.2 Parametric Study

By choosing the proper parameters for the resonator curtain, one can optimise the design of a resonator-based noise mitigation system. Therefore, to investigate the principal factors that influence the performance of the resonator-system, we apply the present model into four different scenarios in the waveguide with 10-meter water depth. The influence of the internal resistance of the resonator will be investigated through the comparison of resonators with different damping coefficients. The performance of the resonators with multiple resonance frequencies is also discussed. To a large extent, the void fraction of the resonator system could also determine the intensity of the noise reduction, i.e. by changing the number of the resonators or the distance between the resonator curtain and the noise source. The resonator array will be deployed at the target locations for different cases.

3.2.1 Influence of the internal resistance and resonance frequency of the resonator

The influence of the internal resistance of the resonator will be investigated by comparing resonators of different damping. In the configuration examined hereafter, the two resonators are placed...
at \((r, z, \phi) = (1.5m, 5m, 0^\circ)\) and \((r, z, \phi) = (1.5m, 6m, 0^\circ)\), respectively. A single point source is placed in the middle of the domain. First, the frequency response function for resonators with different flow resistivities needs to be computed by the COMSOL model. By fitting the magnification factor of the open-ended resonator from COMSOL, the effective properties of the equivalent SDoF system can be determined. As shown in Fig. 3(d), in order to describe the acoustic performance of a resonator with \(R_f = 5000Pa\cdot s/m^2\), the parameters can be founded through the fitting. As can be

concluded from Fig. 3 (a) and (b), the resonator with lower damping causes clear destructive interference at the non-target frequency, while the resonator with higher damping can achieve increased noise reduction at the target frequency and mitigate the destructive interference. In reality, we know that the damping coefficient is relatively difficult to measure but still we could increase the damping by adding porous material such as foam material to achieve a better performance.

As shown in Fig. 3 (b) and (c), the combination of two resonators in the array with two different resonance frequencies can achieve a wider attenuation spectrum than two identical resonators. Also, the destructive interference can be eliminated by adding another resonator in the vicinity of the non-target frequency. Therefore, the behaviour of resonators with multiple resonance frequencies gives us insight in how to improve the performance of the system, which can be achieved by finding an optimal combination of a series of resonators with resonance frequencies covering the target frequency range.

### 3.2.2 Influence of the position of the resonator system

Based on the representation of the sound radiation from pile driving, the wave field is investigated further by implementing the resonator-system for the predictions of the noise reduction level. The domain contains an open-ended resonator-system in 10-meter water depth, with 19 point sources spacing 0.5 meters. Various positions and configurations of the resonator-system are discussed in

![Figure 3](image-url)
order to find the critical parameters that have large influence on the noise reduction. In the first scenario, the resonator-arrays are placed at $\Delta r = 1.5m$ and $\Delta r = 10m$, respectively. Each resonator-system consists of 20 arrays equally distributed around the source axis as a resonator curtain, in which 20 resonators are placed on each array. For the sake of simplicity, 10 types of resonators are used in total. However, in the practical case, the acoustic behaviour of the resonator will depend on the water depth and the configuration. In other words, the number of the types of resonators can be estimated as $N_{\text{depth}} \cdot N_r$, in which $N_r$ indicates the number of different configurations of resonators, and $N_{\text{depth}}$ represents the number of different water depths for one resonator with a certain configuration. As shown in Fig. 4(a), the black line with asterisks indicates that the level of noise reduction is higher when the resonator system is placed near the source axis ($\Delta r = 1.5m$), compared to the red line in which the system is placed at the further away ($\Delta r = 10m$). The number of the resonators for two cases is identical, when the system is placed at a longer horizontal distance from the source axis, the interval between every two arrays of resonators is increased. Therefore, the acoustic waves can pass through the resonator curtain which, in turn, reduces the effectiveness of the system altogether.

### 3.2.3 Influence of the number of resonators in the system

Following the discussion on the position of the resonator system, the different number of resonators used in the system is compared in Fig. 4(b). We can conclude that the more resonators are used in the system, the higher noise reduction level can be achieved. The blue line indicates the baseline spectrum with the moving point sources solely. The ambient noise without both point sources and resonator-system is assumed to be 90 dB in this study. The blue line with circles shows that a resonator-system containing 800 resonators can achieve a noise reduction level more than 20 dB around 140 Hz. Based on the discussion above, we can conclude that one of the primary parameters that dominating the performance of noise reduction is the void fraction of the resonator-system, in other words, the position of the system and the number of the resonators used in the system. The results from this study show that the resonator-system placed in the vicinity of the pile is preferable. The increase of the number of the resonator element can improve the acoustic performance of the resonator-system.

### 4. Conclusion

The present model aims at a first order estimation of the required amount of resonators for targeted noise reduction level by improving the configuration of the resonator-system through evaluating the
optimal properties. The result of this study indicates that choosing a resonator with proper internal resistance can improve the efficiency of the noise reduction. The destructive interference is also found in this study. It shows that although the mitigation of the sound takes place at our target frequency, the resonator could shift part of the energy to an adjusted frequency. This can be mitigated by using resonators of different properties. By doing this, a wider band of noise reduction can be achieved and the destructive interference can be mitigated. The Mach cone formed in this model shows a qualitative agreement with the results from other models in past research. Based on this description of the noise source, the noise reduction levels with the use of the complete resonator system with different positions and configurations are predicted. However, there is still future work required in order to improve the present model with respect to i.e. acoustics of the layered medium at the seabed, multiple reflection and refraction effects.

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