STUDYING THE METHOD OF PATCH NEAR-FIELD ACOUSTIC HOLOGRAPHY BASED ON MODIFIED HELMHOLTZ EQUATION LEAST SQUARE METHOD

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In order to reconstruct the local sound field of a complex sound source accurately, a path near-field acoustic holography (PNAH) technique is proposed to optimize the HELS method. The actual sound field is regarded as the superposition of the relevant sound fields in each part of the sound source, measuring sound pressure with a small holographic aperture is used to solve the linear combination coefficients of orthogonal basis functions, so as to achieve the near-field extrapolation of holographic sound pressure. Extrapolated data is used to perform sound field reconstruction. It is equivalent to increase the size of the holographic surface by calculation to improve the sound field reconstruction accuracy. The simulation results show that this technique not only inherits the advantages of the conventional HELS method, but also can reconstruct the complex sound source or multi-source coherent sound field with high reconstruction accuracy. For the sound source type which closes to the actual project, it is easy to use.

Keywords: modified helss, date extrapolation, patch nah, acoustic field reconstruction, near-field acoustic holography

1. Introduction

In the 1980s, the near-field acoustic holography (NAH) technology was proposed by Maynard et al [1-2], which brought a breakthrough to solve the problem of acoustic radiation. The traditional fast Fourier transform (FFT)-based NAH has the advantages of mature theory, fast calculation and easy implementation. However, it requires the size of holographic aperture to be larger than the sound source size. Due to the limitation of field conditions, the requirements for large-scale sound sources cannot be satisfied. For this reason, the path near-field acoustic holography (PNAH) [3-5] technology was emerged, which not only breaks the limitation of the traditional NAH on the holographic aperture, but also has a high calculation accuracy under the small holographic aperture. It greatly facilitates the application of NAH technology.

The Helmholtz equation least-squares method (HELS) [6-9] develops the sound field approximately as a linear combination of orthogonal basis functions. The location of measurement points and reconstruction points are no restricted. The number of required measurement points is small, the computational efficiency is high, and it is easy to implement. Local reconstruction of the sound field is also allowed. However, for the reconstruction of irregularly shaped sound sources, the accuracy of the HELS method is very low. In practice, many sound sources have complex shapes and often contain multiple components, or the sound field is composed of multiple coherent sound sources. The shape of the overall sound source is irregular, and it is difficult to obtain an ideal reconstruction result using the HELS method. For the multi-source coherent sound field combined with the sound field separation technique is developed by the wave superposition method and the statistically optimal NAH feature [10], the sound field separation of coherent sound sources can be achieved under the
single holographic plane measurement conditions. However, local reconstruction of the sound field still needs further study.

Against the disadvantages of the HELS method, a modified HELS method is proposed and extrapolates data based on it to propose a new PNAH technology. Modified HELS method regards complex sound source as coherent sound field superposition which are produced by each component. The sound fields are generated by the individual parts which are represented by a linear combination of a set of independent and orthogonal spherical waves respectively. The radiation sound field can be calculated by obtaining the linear combination coefficient from the measured sound pressure. Using the modified HELS method to extrapolate the date, the extrapolation error is reduced by an iterative algorithm, and the measurement surface is equivalently enlarged. For sound field reconstruction, using the extrapolated to achieve local reconstruction of complex sound sources or multi-source coherent sound fields, which can improve reconstruction accuracy. The sound field reconstruction is a typical inverse problem. Therefore, when calculating the combination coefficients of orthogonal spherical waves, the Tikhonov regularization method [11] is used to solve ill-posed problems. The regularization parameters are determined by the generalized cross-validation GCV method [12]. The double pulse ball simulation analysis has verified the correctness, effectiveness and feasibility of the method.

2. Principle of sound field reconstruction base on HELS

An arbitrary radiator that vibrates is considered which at an angular frequency \( \omega \) in a homogeneous, compressible, inviscid fluid with a sound pressure that satisfies the simplified wave equation, the Helmholtz equation. The basic idea of the HELS method is to solve the Helmholtz equation with boundary conditions. The complex sound pressure \( p(x) \) at any field point \( x \) is expressed as a set of orthogonal basis functions.

\[
p(x) = \rho c \sum_{j=1}^{i} \zeta_j \Psi_j(x)
\]

Where \( \rho \) is the density of the medium; \( c \) is the speed of sound; \( \psi \) is the basis function; \( \zeta_j \) is the corresponding coefficient. The coordinate system is different, the basis function is also different and the spherical function is a spherical wave function under the spherical coordinate system. Since the spherical wave function can be directly obtained in the library of software such as MATLAB, a spherical coordinate system is usually used, and \( \Psi \) can be expressed as

\[
\Psi_j(x) = \Psi_{nj}(r, \theta, \varphi) = h^{(1)}_n(kr)p_{nj}(\cos \theta)e^{i\ell \varphi}
\]

Where \( h^{(1)}_n(kr) \) is the first-order ball Hankel function of order \( n \); \( p_{nj}(\cos \theta) \) is the companion Legendre function; \( k = \omega / c \) is the wave number; the relationship between variables \( j, n, l \) is \( j = n^2 + n + 1 + 1, l \) changes from \(-n\) to \( n \).

Assuming that the number of measurement points is \( M (M >> J) \). The measured sound pressure vector can be expressed as a matrix form according to the HELS method:

\[
p_n = \rho c \Psi_H \zeta
\]

In the formula:

\[
\Psi_H = \begin{pmatrix}
\Psi_1(x_1) & \cdots & \Psi_j(x_1) \\
\vdots & \ddots & \vdots \\
\Psi_1(x_A) & \cdots & \Psi_j(x_A)
\end{pmatrix}
\]

Using the least square method to eliminate the first-order error, the vector of the basis function coefficients is:

\[
\zeta = (\rho c)^{-1} (\Psi_H^* \Psi_H)^{-1} \Psi_H p_H
\]

Where the superscript ‘*’ represents the matrix conjugate transpose.
For a set of measured data is given, there is always an optimal basis function expansion term $J$, which is usually determined by iterative methods [6]. The coefficient vector $C$ is calculated by using the M1 date which choosing from M measurement date, the rest date is used for optimization, $J$ is incremented by $1\sim M_1$, and the sound pressure values of all the measuring points are reconstructed each time. The number of expansion items is $J_{\text{op}}$ when the L2 norm error of the point sound pressure is minimized. Once the number of expansion items is determined, $C$ can be calculated according to equation (5), substituting equation (1), which is, the sound pressure on the surface of the sound source and any point outside can be got.

3. Principle of PANH based on modified HELS method

3.1 Modified HELS method

When using the spherical wave function as a basis function, the traditional HELS method is more suitable for spherical or nearly spherical sound sources whose aspect ratio is approximately 1:1:1. For other shapes of sound sources, the sound source is usually included in the virtual sphere as a spherical sound source. On the one hand, the measurement needs to be performed outside the smallest spherical surface which contains the sound source, however, it will lead the evanescent waves component to be loss. On the other hand, the spherical wave function cannot describe the sound field accurately when the sound source shape is highly irregular, which results in a lower accuracy of the sound field reconstruction. In engineering practice, the shape of the sound source is relatively complex, which often contains multiple components, or there are multiple sound sources. At this time, the shape of the overall sound source is irregular, especially when the components or the sound source sound field are coherent, so that it is difficult to get ideal reconstruction results by using the traditional HELS method. For this paper proposes a modified HELS method. Assuming that the complex sound source contains $K$ components, if each part is a coherent sound source and the actual sound field can be regarded as the superposition of the sound fields of each part. According to the superposition of the coherent sound field, formula (1) can be rewritten as

$$p(x) = \rho c \sum_{k=1}^{K} \sum_{j=1}^{J} C_j^{(k)} \Psi_j^{(k)}(x)$$  \hspace{1cm} (6)

Where $\Psi_j^{(k)}$, $C_j^{(k)}$ are the kth component basis functions and expansion term coefficients, respectively. It can be seen that the main difference between the modified HELS method and the traditional HELS method is that each part of the sound source is represented by a set of independent spherical wave function combinations. The origin of the corresponding spherical coordinate system is located within the sound source of the component part. Compared with the traditional HELS method, the modified HELS method is more suitable for complex sound source or multi-source coherent sound field reconstruction. And due to the reasonable assumption of the sound field, the measurement does not need to be performed outside the smallest sphere surrounding the sound source, and the near-field measurement can be used to get evanescent wave for higher reconstruction accuracy. However, due to the sound source is divided into the various components, it is necessary to understand the internal structure of the sound source. In practice, the principle of dividing the sound sources in each section should accord to the overall structure and shape characteristics of the sound source, then selecting several major components that contribute greatly to the sound field and are easily separated in shape as part of the sound source. In addition, an appropriate basis function should be selected based on the shape of each part of the sound source.

Equation (6) is expressed in a matrix:

$$p_n = \rho c \Psi_A C_A$$  \hspace{1cm} (7)
Where $\Psi_A = [\psi_{H}^{(1)}, \psi_{H}^{(2)}, \ldots, \psi_{H}^{(K)}]_{M \times K}$ is the basis function matrix, $C_A = [(C^{(1)})^T, (C^{(2)})^T, \ldots, (C^{(K)})^T]_{K \times 1}$ is the coefficient vector. Calculating the coefficient vector is actually an inverse solution process. In order to reduce the ill-posedness of the problem, the Tikhonov regularization method is combined with a singular value decomposition (SVD) to solve the problem is a good choice.

$$C_A = (\rho c)^{-1}[\Psi_A]^{-1}P_H \quad (8)$$

Where $[\Psi_A]^{-1} = \text{VAF}_A^*_\Sigma_A^{-1}[U_A]^*$. $U_A, \Sigma_A, \text{V}_A$ is the matrix obtained by performing SVD on $\Psi_A$; $\text{F}_A^*$ is a Tikhonov regularization matrix, and regularization parameters are determined by the GCV method. Similarly, the iterative method is used to determine the optimal expansion of the basis function to calculate $C_A$. Then it can reconstruct sound pressure on the surface of the sound source and any point outside according equation (6).

### 3.2 Data extrapolation

In order to improve the accuracy of sound field reconstruction, a PNAH based on modified HELS method is proposed. Its core is data extrapolation. The relationship between sound source, reconstruction surface, holographic surface and extended surface is shown in Figure 1. The purpose of data extrapolation is to obtain the sound pressure $P_{H_2}(x_{h_2})$ at $N_2$ points on the extended surface $H_2$ from the sound pressure $P_{H_1}(x_{h_1})$ at $N_1$ measuring points on the holographic plane $H_1$. The sound field can be reconstructed according to the extrapolated sound pressure, so that the size of holographic aperture can be equivalently expanded by numerical calculation, the acoustic field data input by the algorithm can be increased. The effect of finite aperture on the reconstruction result can be reduced, and the reconstruction accuracy and efficiency can be improved.

![Fig. 1 Layout in the data extrapolation method](image)

If the sound pressure $P_{H_1}$ is measured on the holographic plane $H_1$, the coefficient vector can be obtained from equation (8) by using the modified HESL method:

$$\{C_A\}^{(1)} = (\rho c)^{-1}[\Psi_A]^{-1}P_{H_1} \quad (9)$$

Calculate the sound pressure on the extended surface $H_2$:

$$\{P_{H_2}\}^{(1)} = \rho c \Psi_B(C_A)^{(1)} = \Psi_B[\Psi_A]^{-1}P_{H_1} \quad (10)$$

Where $\Psi_B = [\psi_B^{(1)}(x_{h_2}), \psi_B^{(2)}(x_{h_2}), \ldots, \psi_B^{(K)}(x_{h_2})]_{N_2 \times K}$. Since the extended plane $H_2$ contains the measuring surface $H_1$, the part located in $H_1$ in $\{P_{H_2}\}^{(1)}$ is replaced with the measured sound pressure $P_{H_1}$. Then using $\{P_{H_2}\}^{(1)}$ as the input of equation (8) to obtain the coefficient vector $\{C_A\}^{(2)}$, and then calculating $\{P_{H_2}\}^{(2)}$. The overlapped part of $H_1$ with the measured sound pressure is replaced and enter the next iteration. The iterative process can be expressed as:

$$\{P_{H_2}\}^{(1)} = \Psi_B(C_A)^{(1)} = \text{U}_B \text{F}_A^{\alpha} \text{U}_B^* \{P_{H_2}\}^{(1-1)} \quad (11)$$

$$\{P_{H_2}\}^{(i)} = \{P_{H_1}, \{P_{H_2-H_1}\}^{(i)} \}^{(i)} \quad (12)$$

$$x_{h_2} \in H_1 \quad x_{h_2} \in H_1 \quad x_{h_2} \in H_1 \cap x_{h_2} \in H_1$$
Where, i is the number of iterations; $U_B$ is the matrix which is obtained after SVD for $\Psi_B$; $F_B^{(i)}$ is the regularization matrix in the ith iteration. When the extrapolation sound pressure from two iterations does not change much, the iteration stops. The termination condition is

$$\|P_{H_2}^{(i)} - P_{H_2}^{(i-1)}\| / \|P_{H_2}^{(i-1)}\| < \varepsilon_{sp}$$

(13)

Where, threshold of $\varepsilon_{sp}$ is $10^{-5}$. The iterative termination condition and threshold selection are found in the literature [13].

In fact, after a large number of simulations and experimental results have verified that the extrapolation results tend to be stable after a certain number of iterations, it is possible to perform tentative iterations before applying iterative termination conditions. Jia has proved this conclusion is correct and reasonable [14]. The derivation of the iterative convergence theory proves that further research is needed.

Using the extrapolated data as an input to the modified HELS method for holographic reconstruction, reconstruction of surface acoustic pressure:

$$P_R = \Psi_R [\Psi_B]^{-1} P_{H_2}$$

(14)

Where $\Psi_B$ is the basis function matrix of all reconstruction points on the reconstruction surface R.

4. Numerical simulation

To verify the effectiveness of the PNAH based on the modified HELS method, a multi-source simulation analysis was performed. Two pulsating spheres is studied as the object, each with a radius of 0.1 m, were centered at (0.3,0,0) m, (0,0,0) m, and the surface normal velocity was $2.5 \times 10^{-3}$ m/s, and the coherent sound field frequency was 1000Hz. The holographic plane $H_1$ has a location of $z_h=0.15$m, whose size is 0.3m×0.3m, containing 6×6 measuring points, and an extended surface $H_2$ containing 10×10 measuring points. The reconstruction surface R locates at $z_R = 0.11$m, the size is 1.4m×1.4m, and all grid spacings are 0.06m. A random noise with a signal-to-noise ratio of 30 dB is added to the measurement data $P_{H_1}$. Defining the root mean square error of sound field reconstruction is

$$\text{err} = \frac{\|P_R^{\text{theory}} - P_R\|^2}{\|P_R^{\text{theory}}\|^2} \times 100\%$$

(15)

Where $P_R^{\text{theory}}$ is the sound pressure of the reconstruction surface; $P_R$ is the reconstruction sound pressure.

The distribution of reconstruction surface acoustic pressure is shown in Figure 2. The sound field is reconstructed using the traditional HELS method. The distribution of the resulting sound pressure is shown in Figure 3. Comparing with the theoretical sound pressure distribution in Fig. 2, the sound field reconstructed by the traditional HELS method is completely distorted and the sound source location cannot be accurately identified. The average relative error of sound field reconstruction is 132.5%, the root mean square error is 95.87%.

Fig. 2 Contour map of Theoretical acoustic pressures

Fig. 3 Reconstructed acoustic pressures with the original HELS method
The modified HELS method is used to reconstruct the sound field. Taking the two ball centers of as the origin of the spherical wave basis function, the two sets of independent spherical wave functions are combined to represent the respective sound fields of the two ball sources. The sound pressure is measured as the sum of the radiated sound pressures of the two ball sources. The sound pressure of the three measuring points along the two axis directions in the sound pressure at the measuring point is used to calculate the number of optimal extension $J=18$, and then acquiring the corresponding expansion coefficient to perform sound field reconstruction. The reconstruction result is shown in Figure 4. As can be seen from Figure 4, the reconstruction results are greatly improved, the sound pressure of reconstruction is close to the theoretical value, and the relative error of sound field reconstruction is about 23.44%.

In order to improve the reconstruction accuracy furtherly, the PANH technique based on the modified HELS method is adopted, that is, the acoustic pressure of 36 measurement points is used to extrapolate the data to obtain 100 additional sound pressures of the hologram outside, and then the sound field is reconstructed by extrapolating the sound pressure. The result of the extrapolation is shown in Fig. 5 by using 20 iterations. The theoretical and reconstruction results of each point are shown in Figure 6. It can be seen that the reconstruction accuracy has been greatly improved and the overall average error is only 11.15%.

The above simulation results show that the PNAH based on the modified HELS method is not only suitable for reconstruction of complex sound sources or multi-source coherent sound fields, but also can improve reconstruction accuracy and stability.

5. Conclusions

(1) In order to solve the problem of low reconstruction accuracy of complex sound source or multi-source coherent sound field by the traditional HELS method, the modified HELS method was proposed in this paper. This new method has more reasonable sound field assumptions and can rebuild complex sound sources or multi-source coherent sound fields successfully.
(2) Using the results of the modified HELS method to extrapolate the date and the sound field was reconstructed using the extrapolated results. It is equivalent to increase the size of the hologram by calculation in order to improve the reconstruction accuracy of the sound field. The simulation analysis shows that this technique not only inherits the advantages of the traditional HELS method, but also can achieve the local reconstruction of complex sound source or multi-source coherent sound field with a high reconstruction accuracy. It is also easy to apply to the sound source type which closes to the actual engineering.

REFERENCES