THEORETICAL PREDICTIONS AND EXPERIMENTAL INVESTIGATIONS OF SOUND TRANSMISSION THROUGH FLAT PLATES AND CYLINDRICAL SHELLS.

Taha Fatima E.
Auburn University, Department of mechanical engineering, Auburn, Alabama, USA
e-mail: fet0003@auburn.edu

Crocker Malcolm J.
Auburn University, Department of mechanical engineering, Auburn, Alabama, USA
e-mail: crockmj@auburn.edu

Flat plates and cylindrical shells are commonly used engineering structures. Flat plate structures are used as walls and windows in buildings; whereas cylindrical shells are found in many practical devices such as air-conditioning ducts and aircraft cabin walls. The structural response and sound transmission properties of these structures are important. The sound transmission through flat plates and cylindrical shells was studied both theoretically and experimentally. Statistical energy analysis (SEA) was used to predict the transmission loss of each type of structure. Experimental studies were made using microphones, accelerometers and sound intensity probes to calculate the sound transmission loss and structural response. The theoretical and experimental results were then compared to draw conclusions concerning the best ways to measure and predict the sound transmission through these structures.

1. Theoretical part

1.1 SEA overview

One of the theories that explains sound transmission through a complex structure is referred to as Statistical Energy Analysis (SEA). It has been studied by several scientists and constitutes a very important part of acoustics [1]. The analytical model study was conducted using this method which aims to predict the vibrations in a system at high frequency where it is difficult to apply the finite elements method. SEA consists of subdividing the system under study into coupled subsystems and analysing the energy flow in these subsystems under the assumption that the energy flow between coupled systems is proportional to the modal energy difference between the systems. An assumption is made that the coupling is weak and linear, as is the case for the subsystems in this study. Since the calculations used in SEA are in frequency bands (one-octave or one-third octave), the results are averages in frequency bands.

1.2 Application of SEA to the study of the plate

The critical frequency of a plate occurs when the plate bending wavelength equals the trace wavelength of grazing sound waves. The vibration of plates is higher at the critical frequency; therefore, the transmission of sound is also higher. To study the vibration of the plate below and above the critical frequency, it is necessary to study the resonant modes which can be classified into two cate-
The modal behavior of a plate is different in each of these categories. Acoustically fast modes are modes that have structural bending wave speeds greater than the speed of sound in air and are above the critical frequency. These modes have high radiation efficiencies. Acoustically slow modes have resonance frequencies below the critical frequency and bending wave speeds less than the speed of sound. Acoustically slow modes have low radiation efficiencies and can be subdivided into two classes: edge modes and corner modes. With edge modes, the bending phase speeds in one direction are greater than the speed of sound whereas in the other direction they are smaller. With corner modes, both bending phase speeds in both directions are less than the speed of sound.

The radiation resistance of a structure is a measure of the coupling of the structure with the acoustic field, or a measure of the sound power radiated by the structure for a vibration level. Determining the radiation resistance of the plate is important in further determinations of the transmission loss. The response of the plate to acoustic excitation depends on the radiation resistance and the radiation loss factor of the plate.

To determine the transmission loss, the system is subdivided into three coupled subsystems and energy flow through these systems is studied in frequency bandwidths of 1 rad/sec as described by Crocker and Price:

$$TL = N.R. + 10 \log_{10} \left( \frac{A_{p}C_{0}T_{R}}{24V_{3} ln(2\pi)} \right).$$  \hspace{1cm} (1)

Where N.R. is the noise reduction obtained from the SEA. [3]

Using the equations in the theory, a program was written to compute the noise reduction and transmission loss at each frequency for the plate as well as the response of the plate relative to mass law. The thin flat plate used in the program has the same values as the plate used for the experimental part for its different parameters. The plate used for the program had the same properties as the plate used for the experiments.

The program was run for frequencies ranging from 500 Hz to 16,000 Hz where SEA is the most accurate. At the critical frequency, a significant dip can be seen. The plate becomes transparent to sound. The transmission loss is lower than it is at all the other frequencies and does not follow “mass law” theory which does not consider the stiffness and the damping of the plate.

#### 1.3 Application of SEA to the study of the cylinder

As Lyon explains, SEA requires one to determine the parameters that will later be used to calculate the system response. Among these parameters, the most important are the natural frequencies of the cylinder that can be found theoretically, the modal density, the mode classification and loss factors which can be obtained using a powerful approach in SEA known as the wavenumber diagram. The natural frequencies of a cylindrical shell are obtained using the equation of motion derived from Love’s equation [4] and simplified by only considering the transverse vibration which gives us the Donnell-Mushtari-Vlasov equation. The wave-number diagram, also called the k-space diagram is a
A very powerful approach used in the SEA in order to simplify then plot the natural frequencies. It uses wavenumber functions, longitudinal (axial) $k_a$ and circumferential $k_c$.

![Wavenumber diagram](image)

Figure 2: Wavenumber diagram [5]

Specific modes are represented by points in the wavenumber diagram distributed on a regular 2D lattice. Each mode occupies an area of the diagram equal to the product of the distance between mode points in each coordinate direction. The modal density of a cylindrical shell is obtained from the number of modes and the bandwidth (units of mode/Hz). It has a peak value at the ring frequency. It can be defined as:

$$n(\omega) = \frac{dN}{d\omega}$$

where $N$ is the number of modes obtained by dividing the area occupied by the strip by the area of a unit mode.

In the case of a finite cylindrical shell, the transmission loss can be divided into two parts: resonant transmission and non-resonant transmission. Non-resonant transmission can be much more important at frequencies at which the resonant radiation is weak (therefore for acoustically slow modes). Resonant transmission is more important at high frequencies.

The resonant transmission can be derived from the power balance equations [ ] and is given by the formula:

$$TL_{res} = -10\log_{10}\left[ \frac{8\pi^2 c_t^2 n(\omega) R_{rad}^2}{\omega^2 m_s^2 S(2R_{rad} + R_{mech})} \right]$$

where $n(\omega)$ is the modal density in modes/Hz, $R_{rad}$ the radiation resistance of all the modes resonant in the band, $m_s$ the mass of the cylinder per unit area, $S$ the radiating surface area and $R_{mech}$ the mechanical resistance.

The non-resonant transmission loss formula depends on the frequency range being considered. Above the ring frequency, only mass-controlled modes are considered while below the ring frequency, both mass and stiffness-controlled modes are taken into account [24, 28].

The non-resonant transmission loss is given by two formulae depending on the frequency under consideration:

$$TL_{nr} = 8.33\log_{10}\left[ \frac{[\nu t h / R^2 E \rho_s / 4 \rho_s^2 c_t^2][1 - (\nu t / f)]^2 + 2.3}{\pi / 2 \sin^{-1}\left[ \nu t f / f_j \right]} \right]$$

for $\nu t < 1$, below the ring frequency, and

$$TL_{nr} = 8.33\log_{10}\left[ \frac{[\nu t h / R^2 E \rho_s / 4 \rho_s^2 c_t^2][1 - (\nu t / f)]^2 + 2.3}{\pi / 2 \sin^{-1}\left[ \nu t f / f_j \right]} \right]$$

for $\nu t > 1$ above the ring frequency.

The total transmission loss (TL) of the cylinder is calculated from the energy sum of the resonant transmission loss and the non-resonant transmission loss.

A Matlab program was used to calculate transmission loss theoretically. The implementation is different from the one used for the plate since it uses the wavenumber diagram. One first enters the specific parameters of the cylinder that are being studied, and then one starts with the bandwidth analysis which sets the frequency band considered.

For each given frequency, 300 values for $k_a$ and for $k_c$ are taken. The acoustically fast, acoustically slow, and acoustically fast and slow modes were also isolated to be able to use the correct formulae for transmission loss previously given and finally to obtain a theoretical value to compare with the experiments. The thin cylindrical shell used for the program has an outside diameter of 0.6096 m with a wall thickness of 1.27 mm. The cylindrical shell used for the program had the same...
properties as the one used for the experiments. The program was run for frequencies ranging from 500 Hz to 16,000Hz where SEA is more reliable.

2. Experiments on the flat plate

The thin flat plate under study was 1.04 m long, 0.609 m wide and 0.00127 m thick. The plate was made of galvanized steel metal with an estimated density of 7,850 kg/m³, a Young’s modulus of 200 GPa, and a Poisson’s ratio of 0.28.

The same experimental setup was used for both sets of experiments. The flat plate was clamped between two reverberation rooms of the same dimensions. Large loudspeakers and a pressure source were placed in one of the rooms, which was the source room for the noise. It was otherwise empty. The calculated critical frequency was 10,118 Hz, both these noise sources were needed to provide enough sound power so that the sound pressure level produced would be sufficiently above the background noise. The second room, or receiving room was also empty.

2.1 Transmission suite experiments on the plate

The two-room method is widely used to evaluate the transmission loss (TL) of a structure and consists of having the structure placed between a source room and a receiving room as described above. A microphone was placed in the source room to measure its sound pressure level, called the incident sound pressure level. Another microphone was placed in the receiving room to measure the received sound pressure level. The measurements were taken in different locations in both rooms to obtain a space-average of the sound pressure level in each room. Sound fields in the two room were reverberant, allowing the calculation of the noise reduction of the plate [6].

First however, the background sound pressure is measured to confirm that the sound pressure levels in each room were sufficiently above the background noise so that the results could be considered. Then, using both the loudspeakers and the turbulent flow pressure tubes as noise sources, and placing the microphone at five different locations for each room, the sound pressure level in both rooms was measured then the space-average of each of the measurements was calculated.

The noise reduction (NR) was calculated by subtracting the obtained sound pressure level from one another, using the formula: \( NR = 10 \log_{10} \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right) \) Where \( P_{\text{in}} \) is the sound pressure in the source room and \( P_{\text{out}} \) is the sound pressure in the receiving room. The noise reduction was then plotted for the desired frequency range.

2.2 Sound intensity experiments on the plate

Using the same experimental setup, a second method was applied to calculate the transmission loss of the plate. This method uses a sound intensity probe to measure the transmitted intensity in the receiving room when the plate is subjected to a noise source in the source room [7]. The sound intensity in the source room is calculated using the sound pressure level in the source room.

For the experiment, the sound pressure level was measured in the source room at five different locations of the room, space-averaged, then converted into incident intensity on the plate using the formula: \( I_{\text{in}} = \frac{P_{\text{in}}^2}{4\rho_0 c_0} \). This is valid under the assumption that the sound field in the source room is reverberant and diffuse.

3. Experiments on the cylinder

The thin cylindrical shell under study has an outside diameter of 0.6096 m with a wall thickness of 1.27 mm. The cylindrical shell is made of galvanized steel metal with an estimated 7,850 kg/m³ density, 200 GPa Young’s modulus, and 0.28 Poisson’s ratio. The overall length of the cylindrical shell is 2.0574 m with a 19.05 mm wide plywood disk placed inside both ends. The interior distance between the two plywood disks is 2.0066 m.
3.1 Transmission suite experiments on the cylinder

The first method used, also referred to as the two-room method, which consists of using two reverberation spaces separated by the structure studied (in our case the cylindrical shell) was previously described. In this method, the source room and the receiving room are also called the transmission room and the reception room. When using loudspeakers inside the cylindrical shell, the reception room is the room in which the cylinder is located, and the transmission room is the inside of the cylinder. First, however, the background sound pressure level was measured inside and outside the cylinder. To measure the background sound pressure level, the loudspeakers inside the cylinder were turned off and the microphones were placed at five different locations in the room and the cylinder. The reverberation time, previously measured is also needed to calculate the transmission loss and is then given by the formula:

\[ TL = NR + 10 \log_{10} \left( \frac{ScT_p}{24V \ln(10)} \right); \]  

3.2 Sound intensity experiments on the cylinder

The second method used to calculate the transmission loss is the intensity method. It consists of measuring the sound pressure level inside the cylinder using the microphone placed inside and the intensity measured outside the cylinder using a sound intensity probe \[8,9\]. Then the sound pressure level results calculated were used to estimate the intensity inside the cylinder. Using the previously given formula the transmission loss TL was calculated:

\[ TL = 10 \log_{10} \left( \frac{I_{in}}{I_{out}} \right); \]  

The advantage of this second method is that only one sound field is needed, while the two-room method requires two sound fields. This method is also considered to be more accurate since it does not need corrections for the surface area of the structure and the absorption of the receiving room. Because the sound field in the cylinder is considered to be a reverberant field, the incident sound intensity inside the cylinder can be calculated with the interior space-average sound pressure level results and the formula \[ I_{in} = \frac{p_{in}^2}{4\rho_0c_0} \] can be used.

4. Results and comparisons

4.1 Comparison of the results for the plate

![Graph](image)

Figure 3: Theoretical plate transmission loss (line), plate transmission loss using sound pressure (dashes), plate transmission loss using sound intensity (dots) and 6 dB slope.

The theoretical SEA results and the results obtained using both experimental methods provided convincing results in the frequency range under consideration. The dip can be seen at the critical frequency and coincides for all methods used (approximately 10,118Hz). The dip is more pronounced
for the sound intensity method. Using the sound intensity method however, seems to provide results that are in closer agreement with the theoretical SEA predictions. This was to be expected and can be explained by the fact that the intensity method is much easier to implement than the transmission suite method. Only one reverberation room is needed to use the sound intensity method and there is no need to measure the reverberation time of the reception room. Despite the difficulty of the implementation of the intensity method, the transmission loss obtained was in close agreement with the sound intensity method counterpart. It is also interesting to compare these results to the predictions of “mass law” theory which neglects the plate damping and stiffness and assumes the structure to be an infinite membrane and therefore doesn’t predict the dip at the coincidence frequency. It can be noticed that below and above the critical frequency, the plate’s response, whether it is calculated or measured agrees with “mass law” theory as it follows a slope of 6 dB per doubling of frequency. At the critical frequency however, the so-called “mass law” theory fails. This is shown in figure 3.

4.2 Comparison of the results for the cylinder

![Figure 4: Theoretical cylinder transmission loss vs transmission loss using sound pressure vs transmission loss using sound intensity.](image)

At low frequencies, the agreement in magnitude between the SEA theory and measurements is very poor []. However, it is safe to trust the two room method rather than the computational method or the sound intensity method because the assumptions made for both the SEA and the sound intensity calculation are not applicable at low frequencies. The ring frequency is apparent in all three results (approximately 2,625 Hz). The agreement is good in the region between the ring frequency and the critical frequency, where there is only a 3 dB difference between the three results. Finally, at high frequencies, the theory and the two room method seem to give the best results. The sound pressure level measured in the reception was high enough above the background sound pressure level to produce correct measurements. However it was not the case for the sound intensity measurements since the sound intensity probe fails at high frequency.
4.3 Comparison between the plate behaviour and the cylinder behaviour

4.3.1 Theoretical results

Figure 5: Theoretical cylinder transmission loss (dotted line) vs theoretical plate transmission loss (solid line).

As Figure 5 shows, the theoretical sound transmission loss obtained for both the cylinder and the plate have the similar amplitudes and follow a 6 dB per doubling of frequency slope except in the critical frequency region for the plate and the ring frequency region and the critical frequency region for the cylinder. Since the two structures studied in this project are made with the same metal material and have the same thickness, it is expected that their behavior will be similar. The ring frequency being a geometrical property of the cylinder, it is also seen that it does not appear for the plate. At low frequency, the theoretical assumptions made for the cylinder are not the same as for the plate, which explains the difference between the sound transmission loss obtained for the cylinder and for the plate at these frequencies.

4.3.2 Experimental results

Figure 6: Measurements of the cylinder transmission loss using sound pressure vs plate measured transmission loss using sound pressure.
Figure 7: Measurements of the cylinder transmission loss using sound intensity vs measured plate transmission loss using sound intensity.

The results show that the critical frequency coincided for the two structures (10,118 Hz) for both experimental methods. The best agreement is obtained at frequencies higher than the cylinder’s ring frequency. It is interesting to notice that the transmission suite method gives better results for the plate than it does for the cylinder, as also shown on Figure 7. This can be explained by the fact that for the experiments on the plate, two reverberant rooms were used for the investigation, which made the sound pressure level calculations more accurate. For the cylinder, however, the source room was the interior of the cylinder where the assumptions of a free field do not hold at low frequencies. The sound intensity method gives better results at frequencies higher than the ring frequency and lower than 12,000 Hz for the cylinder but gives good results for the whole frequency range for the plate, as shown in Figure 55. Overall, the sound intensity method seems to be the best suited for this study.

REFERENCES

3 Taha, F. E., *Theoretical predictions and experimental measurements of sound transmission through a flat plate and a cylindrical shell*, Master of Science Thesis, Graduate Program in Mechanical engineering, Auburn University, (2017).