NONLINEAR DYNAMIC BEHAVIOR STUDY OF AN ELASTIC PLANAR CABLE NET VIBRATION ISOLATOR BASED ON SDOF MODEL

Shao Xiaolin, Zhang Kun

Reactor engineering research sub institute, Nuclear power institute of China, Chengdu, China

email: xiaolin_shao@163.com

Huang Xiuchang, Jiao Sujuan and Hua Hongxing

Institute of vibration, shock and noise, Shanghai Jiaotong University, Shanghai, China

For the purpose of controlling the vibration from momentum wheels of a satellite, a new type of vibration isolator made up of elastic planar cable net was proposed. Experiments were conducted to verify the isolator performance and study its parameter influence. Such a structure has a nonlinear dynamic property, which means there exist multiple resonant frequencies. So in order to avoid its negative influence on the original function of momentum wheels, the response of the system is analyzed with a SDOF model method. Then the parameter influence on the system performance is studied by varying density of the net grid and pretension force of the cable. Such an analysis is meant to build a design guidance of cable net isolator and make its further research more efficient.

Keywords: cable net isolator; nonlinear response

1. Elastic planar cable net vibration isolation system

1.1 Introduction of elastic planar cable net vibration isolation structure

In order to fulfill the requirement of suppressing momentum wheels interference on a certain type of satellite, a new type of vibration isolator, using elastic planar cable net, was designed to set momentum wheel units and satellite bearing cylinder apart, so as to isolate the vibration from momentum wheel units. Thus the working environment of the sensitive payloads mounted on the other side of cylinder is ensured. The structure of the system is as shown in Fig. 1.

Figure 1: Elastic planar cable net vibration isolation system

1—elastic planar cable net, 2—momentum wheel, 3—intermediate support, 4—bearing cylinder
Elastic planar cable net consists of a number of crossed cables which are prestressed. Cables go through the intermediate support, and are fixed at their two ends on the wall of bearing cylinder. The tensioned cable net has three translational and three rotational DOF, thus it can provide six DOF elastic supports, and isolate vibration in all six DOF.

1.2 Experimental verification of elastic planar cable net vibration isolation performance

Experiments were conducted to verify the actual vibration isolation function of the cable net. The experiment system is as shown in Fig. 2.

![Vibration isolation test system of Elastic planar cable net](image)

Figure 2: Vibration isolation test system of Elastic planar cable net

The tennis racket is fixed on the vibroplatform and strings are tensioned on the racket with 58 pounds pretension force. A 20kg mass is mounted on the racket net by 4 supports with the strings embedded in the intersecting parallels grooves on the bottom surface of them, which is as shown in Fig. 3 (a). In order to analyze the influence of intermediate support on vibration isolation performance, the 4 supports are mounted in two different ways, which are marked by points with different colours in fig. 3 (b).

![Fixed form of the supports](image)  ![The location of supports](image)

Figure 3(a): Fixed form of the supports  Figure 3(b): The location of supports

During the experiment, the vibroplatform is excited by a frequency sweep signal between 0Hz to 200Hz with small vibration amplitude. Two acceleration sensors mounted on the mass and vibroplatform are used to measure vibration transmissibility from the vibroplatform to the mass in Z direction (vertical to the cable net plane). The experiment results are as shown in Fig. 4.
Fig. 4 shows that the fundamental frequency of the system in the vertical direction with supports fixed in the outer track of the net is 13.5\,Hz, and the transmissibility decays for 19.7\,dB near 30\,Hz. On the other hand, the fundamental frequency of the system in the vertical direction with supports fixed in the inner track is about 9\,Hz, and the transmissibility decays for 22\,dB near 30\,Hz. Therefore, the elastic planar cable net has a well performance in vertical vibration isolation. Furthermore, its performance can be affected by the mounting position of the intermediate support. Since the stiffness is smaller for the inner track mounting system, a lower fundamental frequency can be reached. So it shows a better isolation behavior when the supports are fixed in the inner track compared to the outer track.

2. Elastic planar cable net nonlinear equivalent dynamic equation

After verifying the vibration isolation function of the elastic planar cable net, a further analysis is needed to study its own characteristics.

2.1 Static analysis of string

Elastic planar cable net is made up of a number of crossed strings which are pretensioned. Suppose the original length of the string is $2l_0$, with cross section area $A$ and elastic modulus $E$. The string will take a lateral displacement when a concentrated load is exerted on its midpoint, and its deformation should be symmetric, which is as shown in Fig. 5.

Suppose the midpoint displacement is $z$ under the vertical concentrated force whose amplitude is $P$. Then $P$ can be expressed as:

$$P = \frac{2EAz}{l_0} \left(1 - \frac{l_0}{\sqrt{l_0^2 + z^2}}\right) \quad (1)$$

2.2 Equivalent dynamic equation of elastic planar cable net

The elastic planar cable net vibration isolation structure is as shown in Fig. 6(a). The whole mass of the system can be equivalently considered as a concentrated mass on each node of the net. So the equivalent mass on each node, added with the bearing mass on the net, can be expressed as:
\[ M_p = \frac{2A_P \rho P L_P}{N_P + 1} + \frac{M_{up}}{(N_P + 1)} \]  \hspace{1cm} (2)

In Eq. (2) \( A_P \) is the cross section area of a single string, \( \rho P \) is the density of the string material, \( L_P \) is the outer ring diameter of the net, \( N_P \) is the number of strings in each direction, \( M_{up} \) is the whole mass of momentum wheel units carried on the net.

Planar cable net, shown in Fig. 6(a), is a complicated multi DOF system. It is difficult to study its nonlinear response with analytic methods directly. Thus an equivalent simplification of the system is necessary. According to reference [5], the cable net can be equivalent to a SDOF structure as shown in Fig. 6(b) when analyzing its vertical dynamic response. And the relations between parameters of the equivalent model and the original structure are described in reference [6].

![Figure 6: The planar cable net isolator and its equivalent model](image)

Assume the equivalent model has the following parameters: concentrated mass \( M \), damping \( c \), the vertical reaction force from the crossing strings \( P_{cable} \), and load \( P \). So the following equation can be conducted:

\[ M \dddot{z} + c \ddot{z} + P_{cable} = P \]  \hspace{1cm} (3)

\( P_{cable} \) should satisfy Eq. (3). Expanding \( P_{cable} \) in Taylor series with respect to \( z \) and neglecting the higher order terms, the new relationship between vertical load and displacement can be written as:

\[ P_{cable} = \frac{4EA}{l_0} \left( 1 - \frac{l_0}{L_n} \right) z + \frac{2EA}{L_n^3} z^3 \]  \hspace{1cm} (4)

Taking into account that the load \( P = P_0 \cos(\omega t) \), the dynamic equation of the system vibration is:

\[ \dddot{z} + \frac{c}{M} \ddot{z} + \frac{K}{M} z + \frac{2EA}{L_n^3 M} z^3 = \frac{P_0}{M} \cos(\omega t) \]  \hspace{1cm} (5)

Then assume:

\[ 2\xi_0 = 2\epsilon \mu = \frac{c}{M}, \quad \omega_0 = \frac{K}{M}, \quad \epsilon = \frac{2EA}{L_n^3 K}, \quad F = \frac{P_0}{M} \]  \hspace{1cm} (6)

Eq. (9) can be rewritten as:

\[ \dddot{z} + 2\xi_0 \dot{z} + \omega_0^2 z + \omega_0^2 \epsilon z^3 = F \cos(\omega t) \]  \hspace{1cm} (7)

Eq. (10) is a typical forced damped Duffing oscillation equation, where \( \omega_0 \) is the natural frequency of the derived system. When the frequency of the exciting force belongs to different ranges, different kinds of resonance, namely, fundamental, 1/3 Subharmonic and 3rd superharmonic resonance, may occur, which can make the system oscillate violently.

### 3. Numerical example

The process of conducting solutions of Duffing equation is omitted, since the pages are limited. Use 60Si2Mn, whose elastic modulus and density are \( E=206\text{GPa} \) and \( \rho=7.85\times10^3\text{kg/m}^3 \) respectively, as the string material of elastic planar cable net. The diameters of the single string and bearing cylinder are 1mm and 1100mm respectively. The total mass, including momentum wheels units and intermediate supports, is supposed to be 210kg. According to reference [7], the damping ratio of the
Cable net should be $\zeta=2\%$. The amplitude of the interference force is small when the satellite is on-orbit. So take the value of amplitude as 5N.

In order to study the influence of the density of the net grids and pretension force on the system vibration characteristics, 3 working conditions, which are listed in Table 1, are chosen to analyze the nonlinear dynamic response of the cable net.

Table 1: Working condition of elastic planar cable net vibration behavior analysis

<table>
<thead>
<tr>
<th>No.</th>
<th>Number of cables per direction ($N_p$)</th>
<th>The relationship between the original length and length after pretension of the cable (pretension force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>$(L_0)_p = (L_0)_p / 1.2$</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>$(L_0)_p = (L_0)_p / 1.2$</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>$(L_0)_p = (L_0)_p / 1.1$</td>
</tr>
</tbody>
</table>

The natural frequencies of the derived system for each condition are listed in Table 2.

Table 2: Natural frequency of derived system

<table>
<thead>
<tr>
<th>No.</th>
<th>Natural frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.5168</td>
</tr>
<tr>
<td>2</td>
<td>38.4036</td>
</tr>
<tr>
<td>3</td>
<td>24.4070</td>
</tr>
</tbody>
</table>

3.1 Fundamental resonance

The fundamental resonance curve of amplitude with respect to frequency can be drawn with the data of first order approximation solution, which is as shown in Fig. 7, as well as the numerical solution calculated by improved Euler method.

![Figure 7: Fundamental resonance response curve](image)

Some conclusions can be drawn from Fig. 7:

1. The fundamental resonance will occur near the natural frequency of the derived system, where the amplitude of amplitude will increase rapidly.
2. Increasing the number of the cables will lead an increase in fundamental resonance frequency, as well as the amplitude of it.
3. A smaller pretension force will lead a decrease in fundamental resonance frequency, but an increase in amplitude.
4. The numerical solution accord with the curve well, which means the first order approximation solution can describe the response of the system well.
3.2 1/3 subharmonic resonance

First examine the condition which must be satisfied when the 1/3 subharmonic resonance is to occur. The interval condition which the amplitude of exciting force must meet for 1/3 subharmonic resonance occurring is as shown in Fig. 8.

![Figure 8: Excitation condition for 1/3 subharmonic resonance](image)

From Fig. 8, it is obvious that the interval conditions cannot be satisfied for the whole three working conditions. Therefore the 1/3 subharmonic resonance does not exist for the system. The amplitude-frequency curve of the first approximation is as shown in Fig. 9.

![Figure 9: Response curve near triple natural frequency](image)

The system has a small response amplitude near the 3 times natural frequency. And increasing the number of cables will lead an increase of the response amplitude. On the other hand, decreasing the pretension force will lead a small decrease of the response amplitude.

3.3 3rd superharmonic resonance

The amplitude-frequency curve of 3rd superharmonic resonance is as shown in Fig. 10.
Some conclusions can be drawn from Fig. 10:
(1) The sub-resonance will occur near the 1/3 times the natural frequency of the derived system, where the amplitude will increase rapidly.
(2) Increasing the number of the cables will lead an increase in sub-resonance frequency, as well as the amplitude.
(3) A smaller pretension force will lead a decrease in fundamental resonance frequency, but an increase in amplitude.
(4) The amplitude of the 3rd superharmonic resonance is small and has little influence on the system.

The amplitude-frequency curve of the second term in the first order approximation solution near 1/3 times the natural frequency is as shown in Fig. 11.

From Fig. 11, it is obvious that the amplitude of response is small near 1/3 times the natural frequency. Increasing the number of cables and decreasing the pretension force will both lead an increase in response amplitude. Because the amplitude of 3rd superharmonic resonance is quite small, the response is dominant by the second term of the first order approximation solution near 1/3 times the natural frequency.
4. Conclusion

For the requirement of controlling the disturbance from the momentum wheels, a new type vibration isolator, using elastic planar cable net, is proposed. The isolator sets momentum wheel components and satellite bearing cylinder apart, as to isolate the vibration from all six DOFs. Experiments were conducted to verify the vibration isolation function of the cable net and test the influence of substructure parameters on its isolation performance. While analyzing the cable net, the nonlinear property is found in such a structure. For a nonlinear system, there exist phenomena such as jumping and sub-resonance when the excitation meets certain requirements. Such a phenomenon may cause a large displacement vibration of the momentum wheel units which are mounted on the cable net. So an analysis about the nonlinear dynamic behavior of the system itself is needed to ensure the attitude controlling function of the momentum wheels. Then an equivalent nonlinear SDOF model is used to investigate the nonlinear dynamic response of the cable net, which is subject to harmonic excitation. On the other hand, the influence of cable net parameters on dynamic response of the structure is analyzed, through varying density of the net grid and pretension forces. Such an analysis will make the design of cable net isolator and parameter optimization more convenient.

REFERENCES