NUMERICAL INVESTIGATION ON ACOUSTIC IMPEDANCE VARIATION ALONG UNIFORMLY DISTRIBUTED MULTI-SLIT RESONATORS

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Acoustic impedance is the most important characteristic of acoustic liners. For an locally reacting liner, the impedance spectrum over all the panel is usually assumed to be the same. Under this assumption, the liner panel in a duct or a nacelle could be macroscopically treated as a single impedance value at each specific frequency. However, in real application, different parts of liner may encounter unequal acoustic and flow environment such as different acoustic modes and sound pressure levels, even if the liner structure is uniformly distributed. Consequently, the acoustic response could be varied at different locations. The main purpose of present paper is to investigate the impedance distribution of a uniform locally reacting liner in details and its relationship with incoming sound wave frequency and sound pressure level. A series of eight Helmholtz resonators with grazing incident waves were simulated by a two dimensional DNS solver. Two disparate methodologies are adopted to determine the acoustic impedance which are referred as to the straightforward method and the definition of impedance method. Numerical results show that representing the liner with an averaged impedance leads to considerable errors of the reconstructed sound fields when the nonlinear effect is important. Therefore, a piecewise function is derived and calibrated by the DNS results for accounting for the resistance variation over the liner length. Comparison shows the new model could achieve better results.

Keywords: acoustic impedance, high intensity, multi-slit resonators

1. Introduction

Acoustic liners are recognized as one of the most effective passive control strategy in commercial aviation industry. Commonly used liners are locally reacting which consist of honeycomb cells covered with perforated facing sheet and rigid backing sheet. The acoustic performance of liners are determined by the their impedance. It is defined as the ratio of sound pressure and particle velocity. The intrinsic parameter is strongly associated with the liner geometric parameters. Besides the liner structure, the flow conditions and the incident sound power can also significantly affect the acoustic impedance. In laboratory, grazing flow impedance tube (GFIT) is a typical facility for investigation of liners in a grazing flow environment with grazing incident sound waves as well. By measurement
of acoustic pressure data at different locations in the duct, the impedance can be derived by diverse impedance eduction methods\textsuperscript{[1] [2] [3] [4] [5]}. Over the past forty years, great efforts have been made to develop impedance eduction methodologies in GFIT. Such as \textit{in situ} method proposed by Dean \textsuperscript{[1]}, single-mode method (SMM) proposed by Armstrong et al.\textsuperscript{[2]}, straight forward method (SFM) proposed by Jing et al. \textsuperscript{[3]} and iterate inverse method developed by NASA\textsuperscript{[4]} and mode matching method (MMM) provided by Elnady and Bodén \textsuperscript{[5]}.

Nearly all the impedance eduction methods macroscopically treat the locally reacting liner panel in a duct or a nacelle as a single impedance value at each specific frequency. It is an averaged value for engineering application. The \textit{in situ} method is an exception. With acoustic pressure data on the surfaces of the facing sheet and the backing sheet, the local acoustic impedance of a single cell can be calculated immediately. However, more information requires more microphones mounted into the tested liner which is inconvenient for arrangement. Meanwhile, the intrusive measurement would affect the local acoustic field and reduce the measurement accuracy. In practice, only a limited number of microphones are used to measure the impedance. Therefore, it is still an approximation to represent the liner panel with an averaged impedance value.

In real applications, different parts of liner may undergo unequal acoustic and flow environment due to reflection and absorption, even if the liner structure is uniformly distributed (Fig. 1). Previous investigations confirmed that the impedance, especially resistance, is strongly associated with grazing flow velocity and incident sound intensity as well. Consequently, the acoustic impedance value could vary gradually along the liner surface. However, this phenomenon is neglected on most occasions. It is still an open question whether the averaged impedance assumption is accurate enough to represent the acoustic performance of a liner under high intensity grazing incident waves.

![Figure 1: Impedance variation along the tube due to different acoustic environment.](image)

This paper is aimed to explore the acoustic impedance variation over the length of the liner surface by numerical simulation. A multi-slit-resonator liner model is investigated. Acoustic responses to different sound pressure levels and frequencies of the incident sound waves are analyzed. Two distinct methods are adopted to extract the global and local acoustic impedance for comparison. The details of numerical model and computational algorithm are given in the second section. Numerical results are analyzed and discussed in the third section. Conclusions and overlook are given in the last section.

## 2. Numerical model and computational algorithm

### 2.1 Numerical Model

The main objective of this paper is to estimate the impedance variation along locally reacting acoustic liner surface with unified structures in the presence of grazing incident sound waves. A liner sample mounted in a grazing flow impedance tube (GFIT) is an appropriate configuration for the purpose of this investigation. A multi-slit-resonator liner is chosen as the computational model in this paper. The geometric parameters are from the previous investigation conducted by Tam et al.\textsuperscript{[6]}. The acoustic liner composes of eight identical cavities and a panel with an equal number of
evenly distributed slits as shown in Fig. 2. The cavities are $W = 1.95$ inch width and $H = 2.2$ inch depth. The slits are $D = 0.05$ inch width and the perforated facing sheet is $T = 0.062$ inch thick. The Helmholtz resonance is near 500 Hz. Four discrete frequencies of the incoming waves are considered, from 250 Hz to 1000 Hz with interval of 250 Hz. The sound pressure levels are increased from 110 dB to 140 dB with a step of 5 dB.

![Figure 2: Computational Domain.](image)

### 2.2 Governing Equations and Computational Algorithms

The two-dimensional compressible Navier-Stokes (N-S) equations are employed as governing equations, which can be represented in a conservation form as,

$$
\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = \frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y}
$$

These equations are non-dimensionalized with respect to the ambient density $\rho_\infty$, the ambient speed of sound $c_\infty$, the thickness of the perforated plate $L_\infty = T$ and the ambient viscosity $\mu_\infty$. The dynamic viscosity $\mu$ is calculated by Sutherland's law. The Reynolds number is defined as $Re = \rho_\infty c_\infty L_\infty / \mu_\infty$, which is 36720. The Prandtl number is 0.72. The spatial discretization is based on the 4th order dispersion-relation-preserving (DRP) scheme proposed by Tam & Webb. An 11-point artificial selective filter proposed by Bogey & Bailly is adopted in numerical simulation. In addition, an absorption boundary condition based on the perfectly matched layer method (PML) of Hu et al. has been implemented. An optimized multi-dimensional interpolation scheme is adopted. More details and validations of the in-house code can be found in previous paper.

### 2.3 Determination of Acoustic Impedance

The acoustic impedance of the liner is derived by two distinct strategies. Firstly, a straightforward method for wall impedance eduction in a flow duct proposed by Jing et al. is used to derive the averaged impedance of the liner. In this method, the test liner is treated as locally reacting liner with a uniform impedance $\zeta_{\text{global}}$. In this paper, 32 evenly distributed sampling points are used to provide the complex pressure field information based on the numerical simulation results. Details of the methodology can be found in the reference paper.

In the second method, the definition of impedance is adopted to calculate the local impedance of each resonator directly. The acoustic impedance is defined as the complex average sound pressure $\tilde{p}(\omega)$ divided by the volume velocity $\tilde{q}(\omega)$, it is associated with average properties on a specific surface,

$$
Z(\omega) = \frac{\tilde{p}(\omega)}{\tilde{q}(\omega)}
$$

where $\tilde{q}$ is the surface integral of the normal component of the particle velocity (direction points into the lined surface is positive).

$$
\tilde{q} = \int_S \boldsymbol{u} \cdot d\boldsymbol{S}
$$
where $S$ is the slit surface. By these formulas, the acoustic impedance of each resonator could be derived separately to find the impedance variation along the sound propagation direction.

After the acoustic impedance is determined by the two methods, the Helmholtz equation is solved by a finite element solver in COMSOL Multphysics to validate the impedance value. The pressure boundary condition at source plane is given by the DNS result.

### 3. Numerical Results and Discussion

#### 3.1 DNS Results

The sound pressure level distributions along the wall opposite to 8-cavity liner are depicted in Fig. 3. These figures give the acoustic performance of the liner under grazing incident planar waves with different sound pressure levels and frequencies. When the incoming wave is 120 dB, at 500 Hz the sound pressure level decays almost linearly within the liner section as shown in Fig. 3a. It indicates that the acoustic impedance of the liner is uniform over the liner length. Reflected waves are also found which is caused by the discontinuity between liner and hardwall. At non-resonant frequencies, the liner panel behaves like a solid wall. The insertion loss is extremely small.

Figure 3b illustrates the sound pressure level distributions of configurations with 130 dB incoming wave. If the impedance of the liner remains the same and uniform as the case of 120 dB incident wave, the SPL distribution profiles should be identical to the corresponding curves in Fig. 3a albeit shifting up 10 dB. However, the numerical results show that there are discernible differences among the SPL profiles at the same frequency which confirms that the acoustic impedance is changed due to the higher intensity sound wave. Furthermore, the linear relationship between SPL decay and distance from the liner leading edge is broken, which indicates the acoustic impedance is varied over the liner panel length. Continuing to increase the intensity of incoming wave from 130 dB to 140dB, the non-linear phenomenon is more strong as shown in Fig. 3c. It is also found the acoustic performance is becoming worse at resonance. The sound attenuation at 750 Hz increases a little bit since non-linear absorption happens as well. At 250 Hz and 1000 Hz, there is nearly no attenuation at all.

![Figure 3](image-url)

**Figure 3:** Comparison of SPL distribution along the wall opposite to the 8-cavity liner.

Figure 4 displays how a specific sound pressure level of incoming wave at resonance decays in the lined duct. Both of the SPL and relative phase distribution along the wall opposite to the 8-cavity liner are given. The SPL profiles under 125 dB are nearly the same which indicates the sound energy is dominantly attenuated by a linear procedure. At 115 dB and 120 dB, the relative phase profiles nearly overlap with each other. As the intensity of incident wave increasing, the profiles start to change. The SPL drops at a slower rate at the beginning of the liner and then at somewhere the declining rate becomes approximately the same as the profiles of the lowered incoming wave intensity.

Previous investigations have pointed out that there are two sound energy dissipation mechanisms of liner which are shear gradients of the unsteady boundary layer flows and shedding of micro-vortices.
from the mouth of the resonator. The incident sound pressure level determines which mechanism dominates. Therefore, in a grazing incident lined duct, the sound dissipation by the liner could be separated into two regions. When the sound wave propagates through liner section, the attenuation is achieved mainly by a nonlinear procedure at the beginning. Then as the acoustic energy is dissipated and reflected by the resonators, the nonlinear effect gradually fades away. In the low intensity incident region, the sound energy is absorbed primarily by viscosity and the liner behaves linearly.

The instantaneous vorticity fields shown in Fig. 5 confirm that the cells of the liner exhibit different responses as well. At 130 dB, strong vortex shedding occurs at the first two slits. No shed vortex is found in the other slits. At 140 dB, the vortex shedding can be found in the first six resonators. The strength gradually decreases and disappears toward the last two resonators.

With the data of SPL and phase distribution along the wall opposite to the liner, the global averaged impedances are extracted by the straightforward method[3]. Figure. 6 illustrates comparisons of normalized acoustic impedance of different excitations. The global resistance changes with frequency and sound pressure level of incident wave. As shown in Fig. 6a, the global resistance increases slightly as frequency increases. Meanwhile, it increases significantly as sound pressure level rises for
all the simulated frequencies. As expected, the acoustic reactance changes with frequency. However, it is not sensitive to the sound pressure level.

![Normalized resistance](image1)

![Normalized reactance](image2)

Figure 6: Global specific impedance extracted by SFM.

In this study, the derived impedances are validated by a finite element duct propagation model. With the boundary conditions of source plane (given by DNS results), impedance of lined section (given by straightforward method) and solid wall, the sound field is calculated in frequency domain. The results are used to examine how accurate the reconstructed sound fields are by representing the liner with an averaged impedance value.

Figure 7 shows the comparisons of FEM and DNS results. First, at non-resonant frequencies (250 Hz, 750 Hz, 1000 Hz), the results agree with each other very well. However, discernible difference can be observed in the profiles at resonance especially for intense incoming waves. From Fig. 6(a), the resistance becomes larger when SPL increases. For high intense incoming waves at resonance, the sound pressure level drops quickly along the liner surface which makes different sections of the liner experiencing different attenuation procedure. As a consequence, the resistance should vary over the liner length. When the sound pressure level is larger than 130 dB, the variation is relative significant which makes the sound attenuation more complex. Both linear and nonlinear effect are including in the procedure. It leads to errors when a global acoustic impedance value is used to calculate the sound field. In other words, the averaged impedance value is insufficient to represent the acoustic performance of the liner.

![SPL distribution](image3)

Figure 7: Comparison of SPL distribution along the wall opposite to the 8-cavity liner.

In order to evaluate the acoustic impedance variation along sound propagation direction at resonance, the local acoustic impedance of each slit is calculated by its definition in Eq. (2). Figure 8 gives the resistances and reactances at each upper surfaces of the eight slits. These values of acoustic impedances are much smaller than the corresponding global impedances. At 120 dB, the resistance
and reactance axial profiles generally stay constant except for the first resonator. At 130 dB, the resistance descents gradually from the first resonator to the fourth one and then keeps a stable value close to the resistance at 120 dB. The reactances of the first and the last resonator also exhibit some difference comparing with the other resonators. At 140 dB, the resistance of the first slit increases to 0.02 and then decreases one slit by one slit over all the liner length. The reactance also demonstrates some variation.

![Figure 8: Specific impedance of each slit with 500 Hz incident wave.](image)

Based on the numerical results, a formula is constructed to account for the acoustic resistance variation along the liner surface caused by sound attenuation. As given in Eq.(4),

\[ \zeta(x) = \begin{cases} 
\theta_{115dB} + \beta (x_{115dB} - x) + i\chi_{115dB}, & x_{LE} \leq x < x_{115dB} \\
\theta_{115dB} + i\chi_{115dB}, & x_{115dB} \leq x \leq x_{TE} 
\end{cases} \]

(4)

where \( \beta = 2.8767 \) is a coefficient accounting for the impedance variation due to nonlinear effects. \((\theta_{115dB}, \chi_{115dB})\) is the acoustic impedance derived from 115 dB incoming wave, \(x_{LE}\) and \(x_{TE}\) are the coordinates of liner leading edge and trailing edge respectively, \(x_{115dB}\) is the position where the sound pressure level is attenuated to 115 dB. In this case, it is assumed that the decrease of the resistance only happens when the sound pressure level is higher than 115 dB and it drops linearly with the distance to the leading edge of the liner. Figure 9 provides comparisons of FEM results with the boundary condition of constructed impedance value and global impedance value. According to the comparisons, the SPL profiles is closer to the DNS results comparing with the SPL distribution computed by global impedance \(Z_{global}\), especially for high intensity incident wave.

![Figure 9: Comparison of SPL distribution along the wall opposite to the 8-cavity liner.](image)
4. Conclusion

Numerical simulation of a uniformly distributed multi-slit liner is conducted to investigate the acoustic performance of different sections of the liner. Numerical results show that the liner operates on uneven SPL of the incident waves over the panel length. As a consequence, the impedance, especially resistance, would vary along the liner surface. If the excited sound wave is intense enough, the impedance eduction methods which model the liner as an averaged impedance value could lead to a considerable error. In order to represent the liner with varied impedance relating to the local incident sound pressure level, a piecewise function is constructed. In this study, only resistances is considered since it is found that the reactance is relatively independent with the incident sound intensity. Comparison of SPL distribution along the wall opposite to the liner is given which shows the new function can achieve better results. Moreover, the feature could be exploited for the design of a liner with two sections to achieve better performance in both nonlinear dominant region and liner dominant region.

Acknowledgments

This work is supported by grants from the National Science Foundation of China (51476005, 91752204).

REFERENCES